# DISTRIBUTED CONCATENATED ALAMOUTI CODE DESIGNS FOR ONE-WAY RELAY NETWORKS USING UNIQUELY-FACTORABLE PSK CONSTELLATION

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**ABSTRACT**

This paper develops a novel uniquely-factorable constellation pair (UFCP) by carefully factorizing phase-shift keying (PSK) constellation. With this unique factorization, a new distributed concatenated Alamouti code is proposed for a one-way relaying network consisting of two single-antenna terminals and one relay having two antennas. This design allows the relay to transmit its own information while forwarding the source information which it has received to the destination. By making use of the Alamouti coding scheme twice and jointly processing the signals from the two antennas at the relay node, such a code design also renders the equivalent channel between source and destination be a product of the two Alamouti channels and therefore, is called distributed concatenated Alamouti space-time block code (STBC). In addition, the asymptotic symbol error probability (SEP) formula is derived with the maximum likelihood (ML) detector, showing that the optimal diversity gain function is achieved and proportional to $\ln \mathtt{SNR}/\mathtt{SNR}^2$.

## 1. INTRODUCTION

Cooperative diversity is a promising technology which can be used for enhancing coverage and improving reliability of wireless communication systems [1–11]. By sharing the use of their antennas, the in-cell mobile users create a virtual array through distributed transmission and signal processing. Since this arrangement forms a distributed MIMO system, the diversity techniques for the MIMO systems have been naturally extended to such relaying networks for the design of so-called distributed STBC [7, 9, 12–17]. The currently-available relay networks with distributed STBCs allow the relay to forward whatever it has received from the source node to the destination and do not allow it to transmit its own information. However, in a practical communication process, it is often necessary to allow relay node to send information to the terminal node, e.g., transmitting channel state information (CSI) or control sequence. Conventionally, this task can be accomplished by allocating orthogonal subchannels, e.g., time slots or frequency bands to it, which operates roughly at a packet level. However, under a certain of strictly-constrained delay systems, this could be problematic. Hence, in this paper, we consider a one-way relaying network consisting of two single-antenna terminals and one relay having two antennas. For such a system, our design objective is to allow the source and the relay to transmit information simultaneously at the symbol level by making use of the PSK constellation and Alamouti coding scheme. To this end, the main idea of this paper is to develop a novel UFCP from the $2^r$-ary PSK constellation by utilizing the concept of uniquely-factorable constellation pair (UFCP) recently proposed in [18, 19]. Here, it should be mentioned that this unique factorization is closely related to that of coprime PSK constellations, which was originally proposed in [20–22] for the design of full diversity noncoherent STBC. Our code design is also closely related to those in [23, 24]

**Notation**: Column vectors and matrices are boldface lowercase and uppercase letters, respectively; the matrix transpose, the complex conjugate, the Hermitian are denoted by $(\cdot)^T, (\cdot)^*, (\cdot)^H$, respectively; the $i$-th entry of $\mathbf{b}$ is denoted by $b_i$; the columns of an $M \times N$ matrix $\mathbf{A}$ are denoted by $\mathbf{a}_1, \mathbf{a}_2, \cdots, \mathbf{a}_N$. $\gcd(a, b)$ means the greatest common divisor of $a$ and $b$; $a|b$ means $a$ divides $b$; notation $a \nmid b$ means $a$ does not divide $b$; $a \equiv b \mod m$ means $m|(a-b)$; Notation $\mathbb{E}[\cdot]$ denotes the expected value of the expression in brackets; Notation $\mathbf{I}_N$ denotes the $N \times N$ identity matrix.

## 2. CONSTRUCTION OF UFCP WITH PSK CONSTELLATIONS

In this section, we first develop the concept of UFCP generated from PSK constellations. Then, we show that such factorization can be appropiately and uniquely obtained by factorizing $2^r$-PSK constellations and then solving specific Diophantine equations.

### 2.1. UFCP generated from PSK constellations

The uniquely-factorable distributed concatenated Alamouti code to be proposed in this paper is based on the concept of UFCP [18, 19, 25–27]. For clarity of exposition, we briefly introduce it below:

**Definition 1** *A pair of constellation $\mathcal{X}$ and $\mathcal{Y}$ is said to be a UFCP, which is denoted by $\mathcal{Y} \sim \mathcal{X}$, if there exist $x, \tilde{x} \in \mathcal{X}$ and $y, \tilde{y} \in \mathcal{Y}$ such that $xy = \tilde{x}\tilde{y}$, then, we have $x = \tilde{x}, y = \tilde{y}$.* ■

The first main result in this paper is stated as the following theorem:

**Theorem 1** *If we let two sets $\mathcal{X}$ and $\mathcal{Y}$ be*

$$\mathcal{X} = \left\{ \exp\left( \frac{j2\pi m}{2^p} \right) \right\}_{m=0}^{2^p-1}, \tag{1a}$$

$$\mathcal{Y} = \left\{ \exp\left( \frac{j2\pi n(2^p-1)}{2^r} \right) \right\}_{n=0}^{2^q-1}, \tag{1b}$$

*where $r = p + q$, then such a pair of $\mathcal{X}$ and $\mathcal{Y}$ constitutes a UFCP.* ■

*Proof*: Let $x = \exp\left(j\frac{2\pi m}{2^p}\right)$, $\tilde{x} = \exp\left(j\frac{2\pi \tilde{m}}{2^p}\right)$, $y = \exp\left(j\frac{2\pi n(2^p-1)}{2^r}\right)$, and $\tilde{y} = \exp\left(j\frac{2\pi \tilde{n}(2^p-1)}{2^r}\right)$, where $0 \le m, \tilde{m} \le 2^p - 1$, $0 \le n, \tilde{n} \le 2^q - 1$. By the property of the

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PSK constellation, $xy = \tilde{x}\tilde{y}$ is equivalent to $m2^q + n(2^p - 1) \equiv \tilde{m}2^q + \tilde{n}(2^p - 1) \mod 2^r$, or equivalently

$$(m - \tilde{m})2^q + (n - \tilde{n})(2^p - 1) \equiv 0 \mod 2^r. \quad (2)$$

Since $2^q|2^r$, we attain that $(n - \tilde{n})(2^p - 1) \equiv 0 \mod 2^q$. Notice that $(2^p - 1, 2^q) = 1$. Hence, we have $2^q|(n - \tilde{n})$. Since $0 \leq n, \tilde{n} \leq 2^q - 1$, we obtain $n = \tilde{n}$ and as a result, (2) reduces to

$$(m - \tilde{m})2^q \equiv 0 \mod 2^r. \quad (3)$$

Dividing both sides by $2^q$ yields

$$m - \tilde{m} \equiv 0 \mod 2^p. \quad (4)$$

In other words, $2^p|(m - \tilde{m})$. Once noticing that $0 \leq m, \tilde{m} \leq 2^p - 1$, we can immediately deduce that $m = \tilde{m}$. Therefore, $x = \tilde{x}$ and $y = \tilde{y}$, such a pair of $\mathcal{X}$ and $\mathcal{Y}$ constitutes a UFCP. This completes the proof of Theorem 1 □

### 2.2. Unique factorization of $2^r$-PSK constellations

By Theorem 1, we can attain the second main result of this paper.

**Theorem 2** *Let $\mathcal{Z}$ denote a $2^r$-PSK constellation, i.e.,*

$$\mathcal{Z} = \left\{ \exp\left( \frac{j2\pi k}{2^r} \right) \right\}_{k=0}^{2^r - 1}. \quad (5)$$

*Then, for any $z \in \mathcal{Z}$, there exists a pair of $x \in \mathcal{X}$ and $y \in \mathcal{Y}$ such that $xy = z$. Furthermore, $x$ and $y$ are uniquely and explicitly determined by $x = \exp\left(j\frac{2\pi m}{2^p}\right)$, $y = \exp\left(j\frac{2\pi n(2^p-1)}{2^r}\right)$, where $n \equiv k(2^p - 1)^{2^{q-1}-1} \mod 2^q$ and $m \equiv \frac{k-(2^p-1)n}{2^q} \mod 2^p$ for $0 \leq m \leq 2^p - 1$ and $0 \leq n \leq 2^q - 1$.* ■

*Proof*: Let $x = \exp\left(j\frac{2\pi m}{2^p}\right), y = \exp\left(j\frac{2\pi n(2^p-1)}{2^r}\right)$ and $z = \exp\left(j\frac{2\pi k}{2^r}\right)$. Then, equation $xy = z$ is equivalent to

$$m2^q + n(2^p - 1) \equiv k \mod 2^r. \quad (6)$$

Since $2^q|2^r$, we have

$$n(2^p - 1) \equiv k \mod 2^q. \quad (7)$$

With the help of the Euler theorem in [28], we can attain

$$(2^p - 1)^{2^{q-1}} \equiv 1 \mod 2^q, \quad (8)$$

Now, combining (7) with (8) results in

$$n \equiv k(2^p - 1)^{2^{q-1}-1} \mod 2^q. \quad (9)$$

There is only one solution to (9), such that $0 \leq n \leq 2^q - 1$. In other words, the solution to $n$ is unique. On the other hand, from (7), we know that $2^q|\left(k - n(2^p - 1)\right)$. Then, according to (6) and noticing that $2^q|2^r$, we can arrive at

$$m \equiv \frac{k - (2^p - 1)n}{2^q} \mod 2^p. \quad (10)$$

Hence, $m$ can also be uniquely determined for $0 \leq m \leq 2^p - 1$. This complete the proof of Theorem 2. □

Theorem 2 tells us that any $2^r$-PSK symbol can be uniquely factorized into the product of a $2^p$-PSK symbol and a $2^q$-PSK symbol with $r = p + q$. In the ensuing sections, we will show how this unique factorization property can be used in the relaying network to allow the relay to transmit its own information along with the source node in the symbol level.

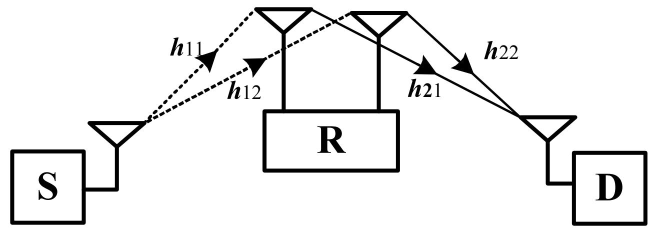


**Fig. 1**. One-way dual-hop relaying networks with uniquely-factorable distributed concatenated Alamouti codes

## 3. DISTRIBUTED CONCATENATED ALAMOUTI CODES

Now, let us consider a one-way dual-hop relay network as depicted in Fig. 1, where the communication between the two single antenna terminal S and D is assisted by a relay node R equipped with two antennas. There are two different phases with each covering consecutive two time slots. The channel gain from the $i$-th phase of each communication to the $j$-th antenna of the relay R is denoted by $h_{ij}$ for $i, j = 1, 2$, which are assumed to be independent and circularly symmetric complex Gaussian distributed with each having zero mean and the variances of which are assumed to be $\Omega_i$, i.e., $\mathrm{E}[|h_{ij}|^2] = \Omega_i$. Let $x_\ell \in \mathcal{X}$, $y_\ell \in \mathcal{Y}, \ell = 1, 2$ be symbols to be transmitted from the source and relay, respectively, which are randomly, independently and equally likely chosen from the UFCP mentioned above.

During the first and second time slots in the first communication phase, source node S transmits its message $x_1$ and $x_2^*$ to R respectively, all along the channel link $\mathbf{h}_1 = [h_{11}, h_{12}]^T$. Therefore, within these two time slots, the relay R receives two signal vectors $\mathbf{r}_\ell$ for $\ell = 1, 2$, given by

$$\mathbf{r}_1 = \sqrt{P_1}\mathbf{h}_1 x_1 + \mathbf{n}_1, \quad (11a)$$
$$\mathbf{r}_2 = \sqrt{P_1}\mathbf{h}_1 x_2^* + \mathbf{n}_2, \quad (11b)$$

where $P_1$ is the transmission power of S in each time slot, $\mathbf{r}_\ell = [r_{\ell 1}, r_{\ell 2}]^T$, $\mathbf{n}_\ell = [n_{\ell 1}, n_{\ell 2}]^T$ denotes complex Gaussian noise vectors with each having zero mean and covariance matrix $\sigma^2\mathbf{I}$.

In the second phase, the received signals $\mathbf{r}_1$ and $\mathbf{r}_2$ are multiplied by $y_1$ and $y_2^*$ , respectively, to generate the composite signal $z_\ell \in \mathcal{Z}$, where $z_\ell = x_\ell y_\ell, \ell = 1, 2$, which are given by

$$\mathbf{s}_1 = \sqrt{P_1}\mathbf{h}_1 z_1 + y_1\mathbf{n}_1, \quad (12a)$$
$$\mathbf{s}_2 = \sqrt{P_1}\mathbf{h}_1 z_2^* + y_2^*\mathbf{n}_2. \quad (12b)$$

Now, the new signal contains information from both the source and relay nodes. We will show that, by the property of the UFCP with PSK constellations and joint signal processing at the relay node, the information from different node can be uniquely recovered. To do that, R first properly combines these four received signals using the Alamouti coding scheme, i.e., $t_1 = s_{11} + s_{22}^*$ and $t_2 = s_{12} - s_{21}^*$, which can be re-expressed in a two by one vector,

$$\mathbf{t} = \sqrt{P_1}\mathbf{H}_1^T\mathbf{z} + \boldsymbol{\mu} \quad (13)$$

with $\mathbf{t} = [t_1, t_2]^T$, $\mathbf{z} = [z_1, z_2]^T$, and

$$\mathbf{H}_1 = \begin{bmatrix} h_{11} & h_{12} \\ h_{12}^* & -h_{11}^* \end{bmatrix}, \quad \boldsymbol{\mu} = \begin{bmatrix} y_1 n_{11} + y_2 n_{22}^* \\ y_1 n_{12} - y_2 n_{21}^* \end{bmatrix}.$$

It is worth noticing that $\boldsymbol{\mu}$ is a two by one complex Gaussian noise vector with zero mean and covariance matrix $2\sigma^2\mathbf{I}$ for given $y_\ell, \ell = 1, 2$.

Then, R spends another consecutive two time slots on broadcasting the scaled versions of $t_1$ and $t_2$ through the two antennas to D also using the Alamouti coding scheme, or more specifically, during the following two time slots, it transmits two signal vectors $\mathbf{u}_1 = \beta[t_1, \ -t_2^*]^T$ and $\mathbf{u}_2 = \beta[t_2, \ t_1^*]^T$ to D. The scale $\beta$ is a fixed value and determined to satisfy the average power constraint at the relay, i.e., $\beta^2 = \frac{1}{4\left(P_1\Omega_1+\sigma^2\right)} \approx \frac{1}{4P_1\Omega_1}$ in high SNR regime. Hence, the signal received at D is represented by

$$\mathbf{v} = \sqrt{P_2}h_{21}\mathbf{u}_1 + \sqrt{P_2}h_{22}\mathbf{u}_2 + \boldsymbol{\eta}, \tag{14}$$

where $P_2$ is the transmission power of R in each time slot. The above equation (14) is equivalent to

$$\bar{\mathbf{v}} = \beta\sqrt{P_2}\mathbf{H}_2\mathbf{t} + \bar{\boldsymbol{\eta}}. \tag{15}$$

where $\bar{\mathbf{v}} = [v_1, v_2^*]^T$, $\bar{\boldsymbol{\eta}} = [\eta_1, \eta_2^*]^T$ denotes two by one complex Gaussian noise vector with zero mean and covariance matrix $\sigma^2\mathbf{I}_2$, and $\mathbf{H}_2$ is an Alamouti matrix, i.e.,

$$\mathbf{H}_2 = \begin{bmatrix} h_{21} & h_{22} \\ h_{22}^* & -h_{21}^* \end{bmatrix}.$$

Now, substituting (13) into (15) yields

$$\bar{\mathbf{v}} = \beta\sqrt{P_2P_1}\mathbf{H}_2\mathbf{H}_1^T\mathbf{z} + \xi, \tag{16}$$

where $\xi = [\xi_1, \xi_2]^T$, in which

$$\xi_1 = \beta\sqrt{P_2}\left[h_{21}\left(y_1n_{11} + y_2n_{22}^*\right) + h_{22}\left(y_1n_{12} - y_2n_{21}^*\right)\right] + \eta_1,$$
$$\xi_2 = \beta\sqrt{P_2}\left[h_{22}^*\left(y_1n_{11} + y_2n_{22}^*\right) - h_{21}^*\left(y_1n_{12} - y_2n_{21}^*\right)\right] + \eta_2^*.$$

Now, we can see clearly that by carefully using the Alamouti scheme twice at the relay, the equivalent channel between S and D, i.e., $\mathbf{H}_2\mathbf{H}_1^T$ in (16) is essentially a product of two Alamouti matrices, thereby, still being Alamouti matrices. This is the reason of why our code is called distributed concatenated Alamouti STBC. Furthermore, for the given $h_{ij}$, each noise vector $\boldsymbol{\xi}$ is Gaussian distributed with zero mean and covariance matrix $\sigma^2[1+2\beta^2P_2(|h_{21}|^2+|h_{22}|^2)]\mathbf{I}_2$. Therefore, when channel state information is perfectly available at D, it is known that the optimal detector for estimation of the transmitted signals is the ML detector, aiming to solve the following optimization problem,

$$\hat{\mathbf{z}} \quad = \quad \arg\min_{\mathbf{z}} \|\bar{\mathbf{v}} - \beta\sqrt{P_2P_1}\mathbf{H}_2\mathbf{H}_1^T\mathbf{z}\|^2. \tag{17}$$

Once $\hat{\mathbf{z}}$ has been obtained, then, $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ can be uniquely determined by using Theorem 2. More specifically, let $\hat{z}_\ell = \exp\left(\frac{j2\pi\hat{k}_\ell}{2^r}\right)$. Then, Theorem 2 gives us a unique pair of $\hat{x} \in \mathcal{X}$ and $\hat{y} \in \mathcal{Y}$ satisfying

$$\hat{x}_\ell \quad = \quad \exp\left(j\frac{2\pi\hat{m}_\ell}{2^p}\right),$$
$$\hat{y}_\ell \quad = \quad \exp\left(j\frac{2\pi\hat{n}_\ell(2^p-1)}{2^r}\right),$$

where $\hat{n}_\ell \equiv \hat{k}_\ell(2^p-1)^{2^{q-1}-1} \mod 2^q$ and $\hat{m}_\ell \equiv \frac{\hat{k}_\ell-(2^p-1)\hat{n}_\ell}{2^q} \mod 2^p$ for $0 \le \hat{m}_\ell \le 2^p-1$ and $0 \le \hat{n}_\ell \le 2^q-1$.

## 4. SEP ANALYSIS FOR DISTRIBUTED CONCATENATED ALAMOUTI CODES

The primary goal of this section is to derive the asymptotic formulas of SEP in high SNR region for the distributed concatenated Alamouti coded relaying network using UFCP constellation and the ML receiver proposed in Sections 2 and 3.

For notational simplicity, we assume that $\mathbf{H}_1 = \sqrt{\Omega_1}\tilde{\mathbf{H}}_1$, $\mathbf{H}_2 = \sqrt{\Omega_2}\tilde{\mathbf{H}}_2$, where $\tilde{\mathbf{H}}_1, \tilde{\mathbf{H}}_2$ are normalized zero mean circularly distributed Gaussian random variable (or say Rayleigh fading). Let us recall that, the channel matrices $\mathbf{H}_2\mathbf{H}_1^T$ in (16) are unitary up to some scale, i.e., $(\mathbf{H}_2\mathbf{H}_1^T)^H(\mathbf{H}_2\mathbf{H}_1^T) = \Omega_2\Omega_1(|\tilde{h}_{11}|^2+|\tilde{h}_{12}|^2)(|\tilde{h}_{21}|^2+|\tilde{h}_{22}|^2)\mathbf{I}$, and each noise vector $\boldsymbol{\xi}$ is white Gaussian for the given $h_{ij}$, with the covariance matrix given by $\sigma^2[1+2\beta^2P_2\Omega_2(|\tilde{h}_{21}|^2+|\tilde{h}_{22}|^2]\mathbf{I}$. Notice that the received SNR at the terminal D is

$$\gamma = \frac{\Omega_2\Omega_1(|\tilde{h}_{11}|^2+|\tilde{h}_{12}|^2)(|\tilde{h}_{21}|^2+|\tilde{h}_{22}|^2)\rho}{1+2\beta^2P_2\Omega_2(|\tilde{h}_{21}|^2+|\tilde{h}_{22}|^2)}, \tag{18}$$

where $\rho = \frac{\beta^2P_2P_1}{\sigma^2}$. Hence, the optimal ML detection for (17) is equivalently reduced to a symbol by symbol detection and its arithmetic average SEP for the composite received signal with the given channel realization is

$$\mathrm{P}_{e|\tilde{h}_{11},\tilde{h}_{12},\tilde{h}_{21},\tilde{h}_{22}} = \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} \exp\big(-\frac{\gamma\sin^2\frac{\pi}{M}}{\sin^2\theta}\big)d\theta \tag{19}$$

The total SEP can be calculated by averaging over all the channel realizations, that is $\bar{\mathrm{P}}_e = \mathbb{E}[\mathrm{P}_{e_1|\tilde{h}_{11},\tilde{h}_{12},\tilde{h}_{21},\tilde{h}_{22}}]$, or equivalently

$$\bar{\mathrm{P}}_e = \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} J(\theta)d\theta, \tag{20}$$

where $J(\theta) = \mathbb{E}_{\tilde{h}_{11},\tilde{h}_{12},\tilde{h}_{21},\tilde{h}_{22}}\big[\exp\big(-\frac{\gamma\sin^2\frac{\pi}{M}}{\sin^2\theta}\big)\big]$. Based on the assumption, if we let $t_i = |\tilde{h}_{i1}|^2+|\tilde{h}_{i2}|^2$, then $t_1$ and $t_2$ are independent, with each being $\chi_4^2$-distributed, i.e., the probabilistic density function of $t_i$ is $t_ie^{-t_i}$. Thus, $J(\theta)$ can be simply calculated by first taking the expectation over $t_1$ and then, $t_2$ such that

$$J(\theta) = \mathbb{E}_{t_2}\left[\mathbb{E}_{t_1}\left[\exp\left(-\frac{\tau(\theta)\rho t_1t_2}{1+2\beta^2P_2\Omega_2t_2}\right)\right]\right] \tag{21a}$$
$$= \mathbb{E}_{t_2}\left[\left(1+\frac{\tau(\theta)\rho t_2}{1+2\beta^2P_2\Omega_2t_2}\right)^{-2}\right] \tag{21b}$$
$$= \int_0^\infty\left(1+\frac{\tau(\theta)\rho t_2}{1+2\beta^2P_2\Omega_2t_2}\right)^{-2}t_2e^{-t_2}dt_2 \tag{21c}$$

where $\tau(\theta) = \frac{\Omega_2\Omega_1\sin^2\frac{\pi}{M}}{\sin^2\theta}$. To further simplify (21c), let $a = 2\beta^2P_2\Omega_2, b = 2\beta^2P_2\Omega_2+\tau(\theta)\rho$ and rewrite it as

$$J(\theta) = \frac{a^2}{b^2}\int_0^\infty\frac{(t_2+a^{-1})^2}{(t_2+b^{-1})^2}\times t_2e^{-t_2}dt_2. \tag{22}$$

Following the strategy similar to that in [24], and noticing that $\beta^2 \approx \frac{1}{4P_1\Omega_1}$, we can attain the following asymptotic formula for $J(\theta)$:

$$J(\theta) \quad = \quad \frac{a^2+2a-1-\mathrm{E}+\ln(\tau(\theta)\rho)}{\tau^2(\theta)\rho^2}+O(\rho^{-3}),$$
$$= \quad \mathcal{K}_1(\theta)\rho^{-2}\ln\rho+\mathcal{K}_2(\theta)\rho^{-2}+O\bigg(\frac{\ln\rho}{\rho^3}\bigg), \tag{23}$$

where

$$\mathcal{K}_1(\theta) = \tau^{-2}(\theta) = \frac{\sin^4\theta}{\Omega_2^2\Omega_1^2\sin^4\frac{\pi}{M}},$$

and

$$\begin{aligned}\mathcal{K}_2(\theta) &= \tau^{-2}(\theta)(a^2+2a-1-\mathrm{E}+\ln\tau(\theta))\\ &= \mathcal{K}_1(\theta)\big[\frac{P_2\Omega_2(P_2\Omega_2+4P_1\Omega_1)}{4P_1^2\Omega_1^2}-1-\mathrm{E}\\ &\qquad +\ln\Omega_2+\ln\Omega_1+2\ln\sin\frac{\pi}{M}-2\ln\sin\theta\big],\end{aligned}$$

where E is the Euler constant. Now, substituting (23) into (20) yields Theorem 1 below.

**Theorem 3** *The average SEP for the one way relaying network with the distributed concatenated Alamouti code has the following asymptotic formula:*

$$\bar{\mathrm{P}}_e = C_1\rho^{-2}\ln\rho + C_2\rho^{-2} + O\left(\frac{\ln\rho}{\rho^3}\right), \quad (24)$$

*where*

$$C_1 = \frac{1}{\pi\Omega_2^2\Omega_1^2\sin^4\frac{\pi}{M}}\left[\frac{3(M-1)\pi}{8M}-\frac{1}{4}\sin\frac{2(M-1)\pi}{M} + \frac{1}{32}\sin\frac{4(M-1)\pi}{M}\right],$$

$$C_2 = C_1\left[\frac{P_2\Omega_2(P_2\Omega_2+4P_1\Omega_1)}{4P_1^2\Omega_1^2}-1-\mathrm{E}+\ln\Omega_2+\ln\Omega_1 +2\ln\sin\frac{\pi}{M}\right]-\frac{2}{\pi\Omega_2^2\Omega_1^2\sin^4\frac{\pi}{M}}\underbrace{\int_0^{\frac{(M-1)\pi}{M}}\sin^4\theta\ln\sin\theta d\theta}_{\mathrm{T}_1}.$$

*where*

$$\begin{aligned}\mathrm{T}_1 = &\left(\frac{3(M-1)\pi}{8M}-\frac{1}{4}\sin\frac{2(M-1)\pi}{M}+\frac{1}{32}\sin\frac{4(M-1)\pi}{M}\right)\times\\ &\ln\sin\frac{(M-1)\pi}{M}+\frac{3}{32}\sin\frac{2(M-1)\pi}{M}\\ &+\frac{3(M-1)\pi}{16M}-\frac{1}{128}\sin\frac{4(M-1)\pi}{M}+\frac{(M-1)\pi}{32M}\\ &-\frac{3}{8}\sum_{k=0}^{\infty}(-1)^k\frac{2^{2k}B_{2k}}{(1+2k)(2k)!}\left(\frac{(M-1)\pi}{M}\right)^{1+2k},\end{aligned}$$

*with $B_n$ being the Bernoulli numbers.* ■

The derivation of $\mathrm{T}_1$ in Theorem 3 is completed by using the integration by parts and the fact [29] that

$$\int\theta^p\cot\theta d\theta = \sum_{k=0}^{\infty}(-1)^k\frac{2^{2k}B_{2k}}{(p+2k)(2k)!}\theta^{p+2k}$$

for $p\geq 1, |\theta|\leq\pi$.

From Theorem 3, we can see that when SNR is large, the SEP for the one-way relaying network with the proposed distributed concatenated Alamouti STBC decays as fast as $\ln\mathtt{SNR}/\mathtt{SNR}^2$, which is the optimal diversity gain function with the ML receiver.

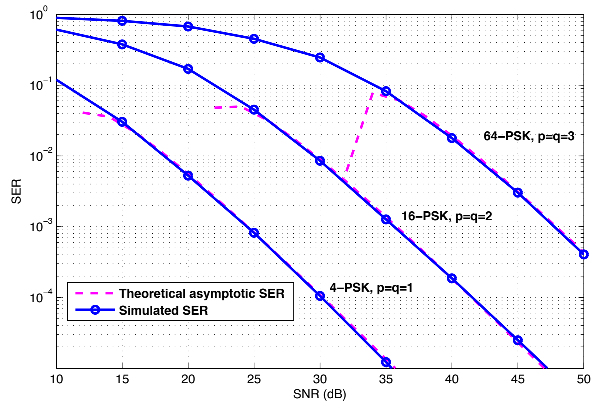


**Fig. 2**. SER performance of the relaying networks with uniquely-factorable distributed concatenated Alamouti codes

## 5. SIMULATIONS

In this section, computer simulations are carried out to verify the error performance of our relying scheme. Throughout the simulation, we assume that, destination node D knows the perfect channel state information, whereas only first and second-order statistics are available at the relay node. Fig. 2 shows the simulated symbol error rate and the dominant theoretical SEP of the proposed relay network against the total SNR of the network with a equal power distribution between source and relay. Without loss of generality, we set $p = q = 1, 2, 3$, which implies that $4, 16, 64$-PSK constellation are received at destination node. It can be observed that asymptotic and simulated error performance curves match very well when SNR is relatively high, which verifies the accuracy of our asymptotic SEP expression given by (24). In addition, the slopes of the SER curves for different PSK constellations are identical in the high SNR regime, which further affirms the conclusion that the full diversity gain function for the network is proportional to $\ln\mathtt{SNR}/\mathtt{SNR}^2$.

## 6. CONCLUSION

In this paper, we have developed a novel UFCP using the PSK constellation. With this unique factorization, we have designed a new distributed concatenated Alamouti code for a relaying network consisting of two single-antenna terminals and one relay having two antennas. This newly-designed code enables the relay to transmit its own information. By taking advantage of the Alamouti coding scheme twice and jointly processing the signals from the two antennas at the relay node, such a code makes the equivalent channel between the source and the destination be a product of the two Alamouti channels, thus, called distributed concatenated Alamouti STBC. In addition, the asymptotic symbol error probability formula has been attained for the maximum likelihood receiver, showing that the optimal diversity gain function is achieved and proportional to $\ln\mathtt{SNR}/\mathtt{SNR}^2$.

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