AUTOCORRELATION-BASED ADAPTATION RULE FOR FEEDBACK EQUALIZATION IN WIDEBAND FULL-DUPLEX AMPLIFY-AND-FORWARD MIMO RELAYS

Emilio Antonio-Rodríguez^{‡*} *Roberto López-Valcarce*[‡] *Taneli Riihonen*^{*} *Stefan Werner*^{*} *Risto Wichman*^{*}

[‡] Department of Signal Theory and Communications, University of Vigo, Vigo, Spain * Department of Signal Processing and Acoustics, Aalto University, Helsinki, Finland

ABSTRACT

Simultaneous reception and transmission in the same frequency, so-called full-duplex operation, causes an infinite feedback loop in an amplify-and-forward relay. The unwanted echoes may result in oscillation at the relay, making it unstable, and distorting the spectrum. This paper presents an adaptive MIMO filtering method for full-duplex amplifyand-forward relays that aims at solving the joint problem of self-interference mitigation and equalization of the sourcerelay channel. The scheme exploits the knowledge of the autocorrelation of the transmitted signal as the only side information while allowing the relay, in the best case, to implement precoding as if there was not any self-interference or frequency selectivity in the source-relay channel. Finally, the proposed adaptation algorithm is investigated by determining its stationary points and by performing simulations in a MIMO-OFDM framework.

Index Terms— Full-duplex relaying, loopback selfinterference, MIMO, adaptive filtering, equalization.

1. INTRODUCTION

Full-duplex (FD) multiple-input multiple-output (MIMO) relays [1, 2] are subject to coupling between the transmit and receive arrays, a direct consequence of simultaneous transmission and reception in the same frequency to achieve transparent coverage extension for the main transmitter. With spatial multiplexing, the main transmitter typically applies precoding based on the source-destination channel. This demands the use of several antennas in the relay to avoid the key-hole effect [3], and justifies the need for independent precoding in the relay based on the relay-destination channel.

Although physical isolation between transmit and receive antenna arrays is typically ensured in the relay design [4, 5, 6], it may be insufficient with high power levels at the relay output. This may hamper reception at the final destination if the relay self-interference is not properly attenuated. Consequently, relays must include self-interference mitigation techniques [2, 4, 7, 8]. Many of these assume an instantaneous self-interference channel and resort to spatial processing alone for interference cancellation. In practice, however, the analog transmit/receive filters in the relay frontends always introduce a non-negligible delay, which motivates the use of spatio-temporal cancellation techniques.

In FD amplify-and-forward (AF) relay links [2, 8, 9, 10], array coupling turns the relay into an infinite impulse response system, causing an infinite echo train of the useful signal. This is in contrast with regenerative decode-and-forward relays [2, 11, 12, 13], for which the large decoding delay effectively decorrelates useful and interfering signals.

MIMO relays usually incorporate a precoding stage in order to improve reception quality at the final destination. Normally, these precoders are designed under self-interferencefree assumptions, resulting in a serious performance loss if interference is not sufficiently mitigated [2]. We propose a spatio-temporal approach to the problem of self-interference mitigation at FD AF MIMO relays. Our approach can deal with frequency-selective channels by exploiting the knowledge of the autocorrelation of the useful signal from the main transmitter, thus providing the relay protocol with an interference-free signal. This extends previous work on SISO and MISO relays [14, 15] to the more general MIMO case.

Notation: Linear time-invariant (LTI) systems are denoted by their z-transform, i.e., $H(z) = \sum_k H[k] z^{-k}$, and its paraconjugate is $\tilde{H}(z) \triangleq H^H(1/z^*)$. The delay operator is also denoted by z^{-1} , i.e., $z^{-k} \boldsymbol{x}(n) = \boldsymbol{x}(n-k)$; thus we write the output of an LTI system as $\boldsymbol{y}(n) = \boldsymbol{H}(z) \boldsymbol{x}(n)$. The entries of an $M \times N$ matrix **A** are denoted by $a_{ij}, i = 1, \dots, M$, $j = 1, \dots, N$.

2. SYSTEM MODEL

Fig. 1 represents the considered single-frequency MIMO relay network, where the relay operates in FD mode. Conse-

Work supported by Fundación Pedro Barrié de la Maza under the program *Becas de Postgrado en el Extranjero 2012*, the Spanish Government, ERDF funds (TEC2010-21245-C02-02/TCM DYNACS, CONSOLIDER-INGENIO 2010 CSD2008-00010 COMONSENS), the Galician Regional Government (CN 2012/260 AtlantTIC, 10TIC013CT ESCOLMA), the Centre of Excellence in Smart Radios and Wireless Research (SMARAD) and the Academy of Finland.



Fig. 1. System model of a full-duplex amplify-and-forward relay with self-interference mitigation and channel equalization.

quently, self-interference may be present at the relay input, and a mitigation method is needed. The source transmitter (S) has M_t antennas that transmit signal t(n). The destination receiver (D) is equipped with M_d antennas for the reception of the signal d(n). The relay (\mathcal{R}) has N_r receive and N_t transmit antennas, and its input and output signals are denoted by x(n) and r(n), respectively. The different MIMO channels featuring in the setting of Fig. 1 are summarized next:

- Source to relay channel $H_{sr}(z)$, of size $N_r \times M_t$.
- Relay to destination channel $H_{rd}(z)$, of size $M_d \times N_t$.
- Self-interference channel due to FD transmission mode $H_{rr}(z)$, of size $N_r \times N_t$.
- Source to destination channel $H_{sd}(z)$, of size $M_d \times M_t$.

All of these are modeled as time-invariant, causal, finite impulse response (FIR) filters (with $H_{rr}(z)$ strictly causal). The order of $H_{\alpha}(z)$ is denoted by L_{α} .

Since our main goal is to provide equalization at the relay, we focus on the two-hop link $S \to \mathcal{R} \to \mathcal{D}$. As shown in Fig. 1, the signal transmitted by S is given by $t(n) = G_s(z)s(n)$, where s(n) is composed of $M_s \leq M_t$ independent streams. The precoder $G_s(z)$ may be designed using any available information about the $S \to \mathcal{D}$ link. Thus, the power spectral density (psd) of s(n), denoted $R_{ss}(z)$, has rank M_s .

The relay, shown at the right-hand side of Fig. 1, consists of two sections. One consists of the $N_t \times N_r$ precoder $G_r(z)$, whose input is $y(n) \in \mathbb{C}^{N_r}$, and which represents the relay protocol (we restrict ourselves to protocols imposing a linear input-output relation). The $N_r \times N_r$ transfer function of the other section is denoted by $W_r(z)$ and is in charge of selfinterference mitigation. Its design is oriented to providing a signal y(n) at its output such that $y(n) = H_{eq}(z)s(n)$, for some target response $H_{eq}(z)$. If we denote by $H_{xy}(z)$ the transfer function from the received signal x(n) to y(n), then

$$\boldsymbol{H}_{eq}(z) = \boldsymbol{H}_{xy}(z)\boldsymbol{H}_{sr}(z)\boldsymbol{G}_{s}(z). \tag{1}$$

Taking the coupling path $H_{rr}(z)$ into account, one has

$$\boldsymbol{H}_{xy}(z) = \left[\boldsymbol{I} - \boldsymbol{W}_r(z)\boldsymbol{H}_{rr}(z)\boldsymbol{G}_r(z)\right]^{-1}\boldsymbol{W}_r(z) \quad (2)$$

which is an infinite impulse response (IIR) transfer function in general. The coupling path $H_{rr}(z)$ may thus turn $H_{xy}(z)$ (and therefore $H_{eq}(z)$ as well) into a channel with very slowly decaying taps, especially if $H_{rr}(z)$ has large gain. This will degrade performance at the destination \mathcal{D} , since the precoders are usually not designed for operation in this scenario (oftentimes precoding is based on the direct link $H_{rd}(z)$ alone). Consequently, if self-interference is not properly mitigated, the order of the effective channel as seen from \mathcal{D} will be rather high, likely exceeding the equalization capabilities of \mathcal{D} . On that basis, it seems reasonable to design $W_r(z)$ in order to ideally achieve $L_{eq} = 0$ (up to an unavoidable bulk delay), or $L_{eq} \leq \tau$ if we consider a modulation scheme using a cyclic prefix of τ samples.

The proposed architecture for the interference mitigation stage $W_r(z)$ is shown in the right-hand side of Fig. 1, and consists of two adaptive elements connected in a feedback configuration: an $N_r \times N_r$ matrix \boldsymbol{B} (feedforward gain), and an $N_r \times N_r$ strictly causal FIR filter $\boldsymbol{A}(z)$ with order L_a : $\boldsymbol{A}(z) = \sum_{k=1}^{L_a} \boldsymbol{A}[k] z^{-k}$. With this notation, the transfer function $W_r(z)$ becomes

$$\boldsymbol{W}_{r}(z) = \left[\boldsymbol{I} - \boldsymbol{B}\boldsymbol{A}(z)\right]^{-1}\boldsymbol{B}.$$
(3)

Substituting (3) in (2) yields

$$\boldsymbol{H}_{xy}(z) = \left[\boldsymbol{I} - \boldsymbol{B} \left[\boldsymbol{A}(z) + \boldsymbol{H}_{rr}(z)\boldsymbol{G}_{r}(z)\right]\right]^{-1} \boldsymbol{B} \quad (4)$$

We see from (4) that the proposed architecture can remove self-interference by setting $A(z) = -H_{rr}(z)G_r(z)$, resulting in $H_{eq}(z) = BH_{sr}(z)G_s(z)$. A(z) has enough degrees of freedom, in addition to removing self-interference, to also equalize the multipath channel $H_{sr}(z)$. That is, one can appropriately choose $H_{xy}(z)$ in an attempt to recover the original signal s(n). This is in the same spirit as the approach presented in [14] for the SISO case with no precoding.

Typically, the relay location is carefully selected during network design, and thus a good quality channel between Sand \mathcal{R} is expected. In addition, most of the times the source is a base station which is able to guarantee a high-power transmission. Thus, for a well-designed system, it is reasonable to assume that $H_{sr}(z)G_s(z)$ is a full-rank channel and that the SNR at the relay input is relatively high, which makes $N_r = M_s$ a reasonable choice, since the use of beamforming techniques at the receive side of \mathcal{R} will not add a significant performance increase in this case. In summary, $W_r(z)$ is a relay equalizer aiming to achieve, in the ideal case, an $H_{eq}(z)$ of order zero (up to an unavoidable delay). However, such design goal for $W_r(z)$ requires the knowledge of both $H_{rr}(z)$ and $H_{sr}(z)$, which may not be available at the relay. Although in principle they could be estimated in an initial training stage, we propose instead a *blind* adaptive scheme that exploits the knowledge of the psd of s(n) as the only side information.

Note that with the proposed architecture shown in Fig. 1, the relay precoder $G_r(z)$ and the cancellation stage $W_r(z)$ are connected in series. Consequently, these two stages can be implemented with different sampling rates, or even in different domains. A joint design in which they are feedbackconnected, i.e., r(n) becomes the input to A(z), seems also possible, and will be the subject of future research.

3. AUTOCORRELATION-BASED INTERFERENCE CANCELLER AND CHANNEL EQUALIZER

3.1. Adaptation rule

We propose an update rule for $W_r(z)$ that does not require the knowledge of $H_{rr}(z)$ and $H_{sr}(z)$ at the relay. Our solution is inspired by the approaches from [14, 15] for FD SISO and MISO relays, where spectrum shaping algorithms were designed to achieve blind estimation of the self-interference and transmitter-to-relay channel. The proposed adaptation rules for B and A(z) are

$$\boldsymbol{B}(n+1) = \boldsymbol{B}(n) + \mu_b \left(\boldsymbol{R}_{\star}[0] - \boldsymbol{y}(n) \boldsymbol{y}^H(n) \right), \quad (5)$$

$$\begin{aligned} \boldsymbol{A}[k](n+1) &= \boldsymbol{A}[k](n) + \\ &+ \mu_a \left(\boldsymbol{R}_{\star}[k] - \boldsymbol{y}(n) \boldsymbol{y}^H(n-k) \right), \quad (6) \end{aligned}$$

for $k = 1, ..., L_a$, where μ_a and μ_b are positive step-sizes. We see from (5)-(6) that the driving terms of the proposed algorithm, $\mathbf{R}_{\star}[k] - \mathbf{y}(n)\mathbf{y}^H(n-k)$, are biased functions of the output autocorrelation, in the same way as in the spectrum shaping algorithms from [14, 15]. In fact, when $N_r =$ $M_s = 1$, the update rules (5)-(6) coincide with those in [14] for FD SISO relays. The bias terms $\mathbf{R}_{\star}[k], k = 0, 1, \ldots, L_a$, are $M_s \times M_s$ matrices which are selected beforehand according to the design criterion for $\mathbf{W}_r(z)$. In our case, we select $\mathbf{R}_{\star}[k] = \mathbf{R}_{ss}[k]$ for $k = 0, 1, \ldots, L_a$, where $\mathbf{R}_{ss}[k] =$ $\mathbb{E}\{s(n)s^H(n-k)\}$ is the autocorrelation sequence of the unprecoded sequence s(n).

3.2. Stationary points

In what follows, we show that the proposed adaptation rule leads to the desired equalization of self-interference and multipath channel once the algorithm has converged, if the bias terms $\mathbf{R}_{\star}[k]$ are selected as in Sec. 3.1. Upon convergence, any stationary point of the algorithm, denoted by \mathbf{B}_{\star} , $\mathbf{A}_{\star}(z)$, will make the mean of the driving terms of (5)-(6) be equal to zero. That is, if we define $\mathbf{R}_{yy}[k] = \mathbb{E}\{\mathbf{y}(n)\mathbf{y}^{H}(n-k)\}\$, then at any stationary point it must hold that

$$R_{yy}[k] = R_{ss}[k], \qquad k = 0, 1, \dots, L_a.$$
 (7)

If L_a is sufficiently large, then (7) implies that the psd of $\{y(n)\}$ matches that of $\{s(n)\}$, i.e.,

$$\boldsymbol{R}_{yy}(z) \triangleq \sum_{k=-\infty}^{\infty} \boldsymbol{R}_{yy}[k] z^{-k} = \boldsymbol{R}_{ss}(z).$$
(8)

Thus, in terms of the psd of $\{x(n)\}$, denoted $R_{xx}(z)$,

$$\boldsymbol{H}_{xy}(z)\boldsymbol{R}_{xx}(z)\tilde{\boldsymbol{H}}_{xy}(z) = \boldsymbol{R}_{ss}(z) \tag{9}$$

Consider now the spectral factorizations $\mathbf{R}_{xx}(z) = \mathbf{\Gamma}(z)\mathbf{\tilde{\Gamma}}(z)$ and $\mathbf{R}_{ss}(z) = \mathbf{\Sigma}(z)\mathbf{\tilde{\Sigma}}(z)$, where $\mathbf{\Gamma}(z)$, $\mathbf{\Sigma}(z)$ are minimum phase (i.e. they are causal, all poles lie in |z| < 1, and the rank is constant in - $|z| \ge 1$). Then all solutions $\mathbf{H}_{xy}^{\star}(z)$ to (9) can be parameterized as follows [16]:

$$\boldsymbol{H}_{xy}^{\star}(z) = \boldsymbol{\Sigma}(z)\boldsymbol{V}(z)\boldsymbol{\Gamma}^{-1}(z) \tag{10}$$

with V(z) paraunitary [17], i.e., $V(z)\tilde{V}(z) = I$. Note from (4) that if B_{\star} is full rank and $H_{xy}^{\star}(z)$ is stable, then $H_{xy}^{\star}(z)$ must be also minimum phase (i.e., full rank for all |z| > 1). Therefore, the inverse $[H_{xy}^{\star}(z)]^{-1} = \Gamma(z)V^{-1}(z)\Sigma^{-1}(z)$ is also minimum phase, and hence causal. But since $V^{-1}(z) = \tilde{V}(z)$, it follows that V(z) = V, a constant unitary matrix. Hence $H_{xy}^{\star}(z) = \Sigma(z)V\Gamma^{-1}(z)$ can be seen as a prewhitening filter $\Gamma^{-1}(z)$ followed by an unknown spatial rotation Vand a conformation filter $\Sigma(z)$. From (1), the transfer function from s(n) to y(n) that results is

$$\begin{aligned} \boldsymbol{H}_{eq}^{\star}(z) &= \boldsymbol{H}_{eq}^{\star}(z) \boldsymbol{H}_{sr}(z) \boldsymbol{G}_{s}(z) \\ &= \boldsymbol{\Sigma}(z) \boldsymbol{V} \boldsymbol{\Gamma}^{-1}(z) \boldsymbol{H}_{sr}(z) \boldsymbol{G}_{s}(z). \end{aligned}$$
(11)

We note that the above analysis remains valid even if noise is present in $\boldsymbol{x}(n)$. In that case, however, it is difficult to gain more insight into the nature of stationary points due to the nonlinear nature of the equations involved. However, as discussed in Sec. 2, it is reasonable to assume a high SNR at the relay, so in the following we neglect the presence of noise. Furthermore, for a typical good base station-to-relay connection (possibly with line of sight), we can reasonably assume that $H_{sr}(z)G_s(z)$ is minimum phase. In that case, $\Gamma(z) = H_{sr}(z)G_s(z)\Sigma(z)$, and consequently $H_{eq}^{\star}(z) =$ $\Sigma(z)V\Sigma^{-1}(z)$. If in addition the entries of s(n) are mutually uncorrelated and with the same individual power spectra, one has $\Sigma(z) = \sigma(z)I$, so that $H_{eq}^{\star}(z) = V$. In that case the adaptive processor not only has cancelled the self-interference path, but it has also restored the original signal s(n) up to an unknown unitary factor V. Although we cannot avoid this uncertainty, due to the fact that only second-order statistical information is being exploited, it is considered that V is not harmful towards the performance of $G_r(z)$, even in the case of an AF protocol. On that basis, \mathcal{D} will contain, at least, a basic channel equalization stage able to eliminate such unitary uncertainty without any performance loss.



Fig. 2. SNR_{out} as function of κ and SNR_{in}.

4. SIMULATION RESULTS

We consider the following scenario: s(n) consists of $M_s = N_r = 2$ independent streams of an OFDM modulated system with $N_{sub} = 8192$ subcarriers and $N_{pre} = 1/4N_{sub}$ cyclic prefix length (in samples). Additionally, the power of s(n)is normalized, i.e., $\mathbf{R}_{ss}[0] = \mathbf{I}$. The sampling frequency at $\mathbf{W}_r(z)$ is $f_s = M f_{symb}(N_{sub} + N_{pre})$, with f_{symb} being the OFDM symbol rate, while the oversampling factor is M = 2. The combined channel response $\mathbf{H}_{sr}(z)\mathbf{G}_s(z)$ used for simulations has order 2 and is given by

$$\begin{bmatrix} e^{j0.23\pi} + 0.4e^{j0.57\pi}z^{-1} & 0.2e^{j\pi} + 0.15e^{j1.89\pi}z^{-1} \\ 0.3e^{j1.12\pi} + 0.1z^{-2} & 0.9e^{j0.92\pi} \end{bmatrix}.$$

In a similar way, $H_{rr}(z)G_r(z) = 1/\sqrt{\kappa}H_{RR}z^{-4}$, where H_{RR} is taken from [11, eq. (21b)] and κ is a parameter that controls the signal-to-interference ratio between x(n) and $H_{rr}(z)r(n)$. Temporally and spatially white Gaussian noise is added at the receive array input, with power σ_i^2 at the *i*-th antenna. The input SNR, defined as $\frac{\mathbb{E}\{|x_i(n)|^2\}}{\sigma_i^2}$, is the same at both antennas and denoted SNR_{in}. Step-sizes are chosen empirically as $\mu_a = 1.5 \times 10^{-4}$ and $\mu_b = 1.5 \times 10^{-3}$, while $L_a = 4$. Due to the sharp spectrum of the source signal, a leakage factor of 2^{-22} is used to avoid a possible parameter drift [18]. Fig. 2 shows the SNR at the two outputs $y_1(n)$ and $y_2(n)$ of the adaptive canceller $W_r(z)$, SNR_{out}, as a function of κ and SNR_{in}. We see that the proposed mitigation method performs approximately the same for the range of κ tested, resulting in SNR_{out}/SNR_{in} > 1 for all cases.

Fig. 3 shows the mean coefficient trajectories of b_{11} (upper plot) and $a_{11}[4]$ (lower plot) for different values of κ and an SNR of 10 dB. It is observed that convergence slows down as the signal-to-self-interference ratio decreases.

For $\kappa = -5$ dB and SNR_{in} = 10 dB, Fig. 4 shows the psd of each component upon convergence, at both input



Fig. 3. Trajectories of $|b_{11}|$ and $|a_{11}[4]|$.



Fig. 4. Restoration of reference psd by the adaptive canceller.

 $(r_{pre_1}(e^{j\omega}), r_{pre_2}(e^{j\omega}))$ and output $(r_{y_1}(e^{j\omega}), r_{y_2}(e^{j\omega}))$ of the adaptive canceller. The reference psd $r_{ss}(e^{j\omega})$ is also shown. The algorithm properly equalizes the signal and mitigates self-interference while improving the overall SNR. The fact that the SNR may be lower for edge subcarriers than for those near the center is due to the sharp psd of s(n). This effect sets a trade-off between parameter drift and residual distortion, and is less pronounced for smoother reference psds.

5. CONCLUSIONS

A blind adaptive cancellation method for MIMO amplifyand-forward full-duplex relays has been presented, which is able of self-interference mitigation and channel equalization by means of spectrum restoration. Additionally, our method can track temporal variations of the self-interference channel and introduces minimal delay into the system.

6. REFERENCES

- D. W. Bliss, P. A. Parker, and A. R. Margetts, "Simultaneous transmission and reception for improved wireless network performance," in *Proc. IEEE 14th Workshop on Statistical Signal Processing*, Aug. 2007.
- [2] T. Riihonen, S. Werner, and R. Wichman, "Mitigation of loopback self-interference in full-duplex MIMO relays," *IEEE Trans. Signal Process.*, vol. 59, no. 12, pp. 5983– 5993, Dec. 2011.
- [3] P. Almers, F. Tufvesson, and A. F. Molisch, "Keyhole effect in MIMO wireless channels: Measurements and theory," *IEEE Transactions on Wireless Communications*, vol. 5, no. 12, pp. 3596–3604, Dec. 2006.
- [4] T. Riihonen, A. Balakrishnan, K. Haneda, S. Wyne, S. Werner, and R. Wichman, "Optimal eigenbeamforming for suppressing self-interference in full-duplex MIMO relays," in *Proc. 45th Annual Conference on Information Sciences and Systems*, Mar. 2011.
- [5] P. Persson, M. Coldrey, A. Wolfgang, and P. Bohlin, "Design and evaluation of a 2 x 2 MIMO repeater," in *Proc. 3rd European Conference on Antennas and Propagation*, Mar. 2009.
- [6] K. Haneda, E. Kahra, S. Wyne, C. Icheln, and P. Vainikainen, "Measurement of loop-back interference channels for outdoor-to-indoor full-duplex radio relays," in *Proc. 4th European Conference on Antennas* and *Propagation*, Apr. 2010.
- [7] P. Larsson and M. Prytz, "MIMO on-frequency repeater with self-interference cancellation and mitigation," in *Proc. IEEE 69th Vehicular Technology Conference*, Apr. 2009.
- [8] P. Lioliou, M. Viberg, M. Coldrey, and F. Athley, "Selfinterference suppression in full-duplex MIMO relays," in *Proc. 44th Asilomar Conference on Signals, Systems,* and Computers, Nov. 2010, pp. 658–662.
- [9] Y. Y. Kang and J. H. Cho, "Capacity of MIMO wireless channel with full-duplex amplify-and-forward relay," in

Proc. IEEE 20th International Symposium on Personal, Indoor and Mobile Radio Communications, Sep. 2009.

- [10] B. Chun and Y. H. Lee, "A spatial self-interference nullification method for full duplex amplify-and-forward MIMO relays," in *Proc. IEEE Wireless Communications* and Networking Conference, Apr. 2010.
- [11] T. Riihonen, S. Werner, and R. Wichman, "Transmit power optimization for multiantenna decode-andforward relays with loopback self-interference from full-duplex operation," in *Proc. 45th Asilomar Conference on Signals, Systems, and Computers*, Nov. 2011.
- [12] B. P. Day, A. R. Margetts, D. W. Bliss, and P. Schniter, "Full-duplex MIMO relaying: Achievable rates under limited dynamic range," *IEEE J. Selected Areas in Communications*, vol. 30, no. 8, pp. 1541–1553, Sep. 2012.
- [13] J.-H. Lee and O.-S. Shin, "Full-duplex relay based on block diagonalisation in multiple-input multiple-output relay systems," *IET Communications*, vol. 4, no. 15, pp. 1817–1826, Oct. 2010.
- [14] R. López-Valcarce, E. Antonio-Rodríguez, C. Mosquera, and F. Pérez-González, "An adaptive feedback canceller for full-duplex relays based on spectrum shaping," *IEEE J. Selected Areas in Communications*, vol. 30, no. 8, pp. 1566–1577, Sep. 2012.
- [15] E. Antonio-Rodríguez and R. López-Valcarce, "Adaptive self-interference suppression for full-duplex relays with multiple receive antennas," in *Proc. 13th IEEE International Workshop on Signal Processing Advances in Wireless Communications*, Jun. 2012.
- [16] L. Crone, "Second order adjoint matrix equations," *Linear Algebra Applicat.*, vol. 39, pp. 61–71, 1981.
- [17] P. P. Vaidyanathan, *Multirate systems and filter banks*, Prentice Hall, Englewood Cliffs, NJ, 1993.
- [18] J. R. Treichler, C. R. Johnson Jr, and M. G. Larimore, *Theory and Design of Adaptive Filters*, Prentice Hall, Englewood Cliffs, NJ, 2001.