WORST-CASE BASED ROBUST DISTRIBUTED BEAMFORMING FOR RELAY NETWORKS

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ABSTRACT

A robust distributed beamforming technique for half-duplex relay network is developed. With the mismatched channel state information (CSI) being available at the destination, the received minimum quality-of-service (QoS) is maximized subject to the constraints that the maximum of the individual relay transmitted powers is limited. This distributed beamforming problem is shown to be a quasi-convex problem and can be solved using second-order-cone programming (SOCP) along with a bisection search method. Simulation results demonstrate that the proposed method is robust to the imperfect knowledge of the CSI and guarantees no relay power outage.

1. INTRODUCTION

Relay networks have recently attracted much interest in the literature as they not only can exploit cooperative spatial diversity of different users in the network, but also can extend the coverage of wireless communication systems [1]-[3]. These advantages are introduced by the scheme that different users in relay networks share their communication resources to help each other in data transmission.

Different relaying strategies have been proposed to achieve cooperative diversity. Amplify-and-forward (AF), decode-and-forward (DF) and compress-and-forward (CF) relaying protocols have been widely used in relay networks [1]-[3]. Due to its simplicity, the AF relaying protocol has become one of the most popular relaying strategies. In the AF protocol, the relay nodes forward properly scaled and phase-shifted versions of their received signals to the receiver.

Recently, several distributed beamforming approaches [4]-[7] have been developed for the relay networks. In [4] and [5], a relay network with one pair of source and destination is considered. In [6] and [7], peer to peer distributed beamforming techniques were proposed for multiple-pair of sources and destinations. Perfect channel state information (CSI) is assumed to be known in [4] and [6], and second order statistics of the CSI are assumed to be available in [5] and [7]. While the performance of the approaches assuming perfectly known CSI is better than the approaches with known statistics of the CSI, the former approaches suffer from the performance degradation introduced by CSI estimation errors in practical applications. Several robust distributed beamforming approaches have recently been proposed to relieve the performance degradation [8]-[9]. In [8], the estimated CSI from the relays to the destination is assumed to have errors, while the CSI from the source to the relays is assumed to be perfectly known. In [9], the correlation matrices of the channel vectors are assumed to have independent estimation.

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In this paper, we propose a worst-case based robust distributed beamforming approach for relay networks with one source, one destination and multiple relays. We assume that both the channel vectors of the source-to-relays and relays-to-destination have estimation errors. The quality-of-service (QoS) at the destination is maximized while the individual relay transmitted power is constrained. The resultant problem is shown to be a quasi-convex problem that the optimal solution can be found by using a bisection search method and second-order-cone programming (SOCP). Simulation results show that the proposed beamforming approach can guarantee no power outage.

Relation to prior work: Both this paper and the works in [4]-[9] consider the distributed beamforming method for relay networks. Different from [4]-[7], this paper proposes a roubst distributed beamforming approach for the case that the CSI is imperfectly known. Although robust distributed beamforming methods were developed in [8] and [9], this paper proposes a quite different method. The method in [8] considers the case that only the CSI of relays-to-destination is assumed to have estimation errors and the approach in [9] is developed for the case that the coherent matrices of the channel vectors have estimation errors. In addition, semi-definite programming constraints are considered in [9], which has a much higher computation complexity than SOCP.

Notations: Throughout this paper, bold upper and lower case letters denote matrices and vectors, respectively. $(\cdot)_{ij}$ denotes the (i, j)th element of a matrix. The ith element of a vector $\mathbf{x}$ is denoted by the corresponding lower case letter with a subscript, i.e. x_i. $|\cdot|$ and $\|\cdot\|$ denote the absolute value of a scalar and the Euclidean norm of a vector, respectively. $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^H$ stand for the conjugate, transpose and Hermitian transpose, respectively. $\mathrm{E}\{\cdot\}$ is the statistical expectation.

2. SIGNAL MODEL

As shown in Fig. 1, a half-duplex relay network with one single-antenna transmitting source, one single-antenna destination and R single-antenna relays is considered. We assume that there is no direct link between the source and destination, and each transmission consists of two stages. In the first stage, the source broadcasts its data to the relays. In the second stage, the signals received at the relays are scaled by complex values and transmitted to the destination. In the first stage, the $R \times 1$ vector of signals received by the relays can be written as

$$\mathbf{r}(n) = \mathbf{f}s(n) + \boldsymbol{\eta}(n) \tag{1}$$

where n is the time index, $\mathbf{f}$ is the $R\times1$ vector of channel coefficients between the source and the relays, $s(n)$ is the signal transmitted by the source, and $\boldsymbol{\eta}(n)$ is the $R \times 1$ vector of the relay noise. In the

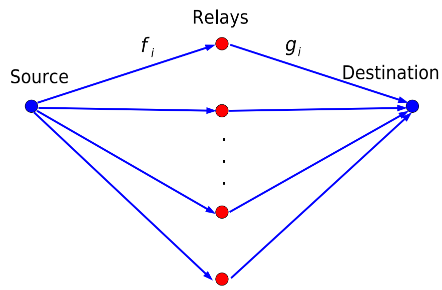

Fig. 1. System model.

second stage, the received signals of the relays are scaled by complex values and transmitted to the destination, which can be written as

$$\mathbf{t}(n) = \mathbf{W}^H \mathbf{r}(n) \tag{2}$$

where $\mathbf{W}$ is a diagonal matrix with w_i, $i = 1, \cdots, R$, on the diagonal, and w_i is the weight coefficient of the ith relay. Let $\mathbf{g}$ denote the vector of channel coefficients between the relays and the destination. Using (1) and (2), the signal received by the destination can be written as

$$\begin{aligned} y(n) &= \mathbf{g}^T \mathbf{t}(n) + \upsilon(n) \\ &= \mathbf{g}^T \mathbf{W}^H \mathbf{f} s(n) + \mathbf{g}^T \mathbf{W}^H \boldsymbol{\eta}(n) + \upsilon(n) \end{aligned} \tag{3}$$

where $\upsilon(n)$ denotes the destination noise. Let

$$y_s(n) \triangleq \mathbf{g}^T \mathbf{W}^H \mathbf{f} s(n) \tag{4}$$
$$y_n(n) \triangleq \mathbf{g}^T \mathbf{W}^H \boldsymbol{\eta}(n) + \upsilon(n) \tag{5}$$

denote the desired signal and noise components at the destination, respectively. Equation (3) can be rewritten as

$$y(n) = y_s(n) + y_n(n) \tag{6}$$

Throughout this paper, it will be assumed that the destination estimates the CSI and computes the relay weights, which are fed back to the relays via low rate feedback channels. Due to many reasons, such as the mobility of the user and the relays, the perfect CSI is difficult to be obtained in practical applications. In this paper, we assume that the CSI available at the destination has uncertainties and can be modeled as

$$\mathbf{f} = \hat{\mathbf{f}} + \boldsymbol{\delta}_f \tag{7}$$
$$\mathbf{g} = \hat{\mathbf{g}} + \boldsymbol{\delta}_g \tag{8}$$

where $\hat{\mathbf{f}}$ and $\hat{\mathbf{g}}$ are the estimated values of $\mathbf{f}$ and $\mathbf{g}$, respectively, and $\boldsymbol{\delta}_f$ and $\boldsymbol{\delta}_g$ denote the estimation error of $\mathbf{f}$ and $\mathbf{g}$, respectively. We also assume that the errors of the channel coefficients can be bounded by some known constants $\varepsilon > 0$ and $\beta > 0$

$$|\delta_{fi}| \le \varepsilon, \qquad |\delta_{gi}| \le \beta, \qquad i = 1, \cdots, R. \tag{9}$$

3. WORST-CASE BASED ROBUST DISTRIBUTED BEAMFORMING

Let us consider the following network beamforming problem that maximizes the destination quality of service (QoS) subject to the individual relay power constraints. The signal-to-noise-ratio (SNR) will be used as a measure of QoS. In the case of perfectly known CSI, the beamforming problem can be written as

$$\max_{\mathbf{W}} \mathrm{SNR} \qquad \text{s.t.} \quad P_i \le P_i^{\max} \quad i = 1, \cdots, R \tag{10}$$

where P_i is the individual relay transmit power and $P_i^{\max}$ is the maximum allowed individual relay transmit power. In the case of imperfectly known CSI, the worst-case based beamforming problem can be written as

$$\max_{\mathbf{W}} \min_{\substack{|\delta_{fi}| \le \varepsilon \\ |\delta_{gi}| \le \beta}} \mathrm{SNR} \ \text{s.t.} \ \max_{|\delta_{fi}| \le \varepsilon} P_i \le P_i^{\max} \quad i = 1, \cdots, R. \tag{11}$$

The received SNR at the destination is given by

$$\mathrm{SNR} = \frac{\mathrm{E}\{|y_s(n)|^2\}}{\mathrm{E}\{|y_n(n)|^2\}}. \tag{12}$$

Using (4), we can write the power of the signal component at the destination as

$$\begin{aligned} \mathrm{E}\{|y_s(n)|^2\} &= \mathrm{E}\{\mathbf{g}^T \mathbf{W}^H \mathbf{f} s(n) s^*(n) \mathbf{f}^H \mathbf{W} \mathbf{g}^*\} \\ &= P_s \mathbf{g}^T \mathbf{W}^H \mathbf{f} \mathbf{f}^H \mathbf{W} \mathbf{g}^* \end{aligned} \tag{13}$$

where P_s is the transmitted power of the source. Let $\mathbf{w} \triangleq \mathrm{diag}\{\mathbf{W}\}$, where $\mathrm{diag}\{x\}$ denotes the operation that stacks the diagonal of a matrix in a vector if x is a matrix, or generates a matrix that has the elements of x on the diagonal if x is a vector. Equation (13) can be rewritten as

$$\begin{aligned} \mathrm{E}\{|y_s(n)|^2\} &= P_s \mathbf{w}^H \mathrm{diag}\{\mathbf{g}\} \mathbf{f} \mathbf{f}^H \mathrm{diag}\{\mathbf{g}^*\} \mathbf{w} \\ &= P_s \mathbf{w}^H (\mathbf{g} \odot \mathbf{f})(\mathbf{g} \odot \mathbf{f})^H \mathbf{w} \\ &= P_s \mathbf{w}^H \mathbf{h} \mathbf{h}^H \mathbf{w} \end{aligned} \tag{14}$$

where $\mathbf{h} \triangleq \mathbf{g} \odot \mathbf{f}$ can be viewed as the equivalent channel coefficient between the source and the destination, and $\odot$ denotes the element-wise Schur-Hadamard product.

Using (7) and (8), $\mathbf{h}$ can be written as

$$\begin{aligned} \mathbf{h} &= (\hat{\mathbf{f}} + \boldsymbol{\delta}_f) \odot (\hat{\mathbf{g}} + \boldsymbol{\delta}_g) \\ &= \hat{\mathbf{f}} \odot \hat{\mathbf{g}} + \hat{\mathbf{f}} \odot \boldsymbol{\delta}_g + \boldsymbol{\delta}_f \odot \hat{\mathbf{g}} + \boldsymbol{\delta}_f \odot \boldsymbol{\delta}_g \\ &= \hat{\mathbf{h}} + \boldsymbol{\delta}_h \end{aligned} \tag{15}$$

where $\hat{\mathbf{h}} \triangleq \hat{\mathbf{f}} \odot \hat{\mathbf{g}}$ and $\boldsymbol{\delta}_h \triangleq \hat{\mathbf{f}} \odot \boldsymbol{\delta}_g + \boldsymbol{\delta}_f \odot \hat{\mathbf{g}} + \boldsymbol{\delta}_f \odot \boldsymbol{\delta}_g$. Using (9), we can obtain

$$|\delta_{hi}| \le \beta |\hat{f}_i| + \varepsilon |\hat{g}_i| + \varepsilon\beta \triangleq \xi_i, \quad i = 1, \cdots, R. \tag{16}$$

As a result, the norm of the error vector $\boldsymbol{\delta}_h$ can be written as

$$\|\boldsymbol{\delta}_h\| = \Big(\sum_{i=1}^{R} |\delta_{hi}|^2\Big)^{\frac{1}{2}} \le \Big(\sum_{i=1}^{R} \xi_i^2\Big)^{\frac{1}{2}} \triangleq \zeta. \tag{17}$$

Substituting (15) into (14), we have

$$\mathrm{E}\{|y_s(n)|^2\} = P_s \mathbf{w}^H (\hat{\mathbf{h}} + \boldsymbol{\delta}_h)(\hat{\mathbf{h}} + \boldsymbol{\delta}_h)^H \mathbf{w} \tag{18}$$

Using (5), we can write the power of the noise component at the destination as

$$\begin{aligned} \mathrm{E}\{|y_n(n)|^2\} &= \mathrm{E}\{\mathbf{g}^T \mathbf{W}^H \boldsymbol{\eta}(n) \boldsymbol{\eta}(n)^H \mathbf{W} \mathbf{g}^*\} + \mathrm{E}\{|\upsilon(n)|^2\} \\ &= \sigma_\eta^2 \mathbf{w}^H \mathrm{diag}\{\mathbf{g}\} \mathrm{diag}\{\mathbf{g}^*\} \mathbf{w} + \sigma_\upsilon^2 \\ &= \sigma_\eta^2 \mathbf{w}^H \mathbf{G} \mathbf{w} + \sigma_\upsilon^2 \end{aligned} \tag{19}$$

where we have assumed that the noises at the relays are spatially white and the relay noise and the destination noise are statistically independent, σ_η^2 and σ_υ^2 are noise powers at the relay and the destination, respectively, and $\mathbf{G} \triangleq \mathrm{diag}\{\mathbf{g}\}\mathrm{diag}\{\mathbf{g}^*\}$ is a diagonal matrix with $(\mathbf{G})_{ii} = |g_i|^2 = |\hat{g}_i + \delta_{gi}|^2$.

By making use of (1) and (2), the transmitted power of the ith relay can be written as

$$\begin{aligned} P_i &= \mathrm{E}\{|t_i(n)|^2\} \\ &= \mathrm{E}\{|w_i(f_i s(n) + \eta_i(n))|^2\} \\ &= |w_i|^2 (P_s |f_i|^2 + \sigma_\eta^2) \end{aligned} \tag{20}$$

where we have assumed that the transmitted signal and the relay noise are statistically independent.

Substituting (7) into (20), we can obtain

$$P_i = |w_i|^2 (P_s |\hat{f}_i + \delta_{fi}|^2 + \sigma_\eta^2)\,. \tag{21}$$

Using (18), (19) and (21), problem (11) can be rewritten as

$$\begin{aligned} &\max_{\mathbf{w}} \min_{\substack{\|\boldsymbol{\delta}_h\| \le \zeta \\ |\delta_{gi}| \le \beta}} \frac{P_s \mathbf{w}^H (\hat{\mathbf{h}} + \boldsymbol{\delta}_h)(\hat{\mathbf{h}} + \boldsymbol{\delta}_h)^H \mathbf{w}}{\sigma_\eta^2 \mathbf{w}^H \mathbf{G} \mathbf{w} + \sigma_\upsilon^2} \\ &\text{s.t.} \max_{|\delta_{fi}| \le \varepsilon} |w_i|^2 (P_s |\hat{f}_i + \delta_{fi}|^2 + \sigma_\eta^2) \le P_i^{\max} \; i = 1, \cdots, R\,. \end{aligned} \tag{22}$$

We can see that the operation of taking the square root of the objective function of (22) does not change the solution set. As a result, problem (22) can be rewritten as

$$\begin{aligned} &\max_{\mathbf{w}} \min_{\substack{\|\boldsymbol{\delta}_h\| \le \zeta \\ |\delta_{gi}| \le \beta}} \frac{\sqrt{P_s} |\mathbf{w}^H (\hat{\mathbf{h}} + \boldsymbol{\delta}_h)|}{(\sigma_\eta^2 \mathbf{w}^H \mathbf{G} \mathbf{w} + \sigma_\upsilon^2)^{\frac{1}{2}}} \\ &\text{s.t.} \max_{|\delta_{fi}| \le \varepsilon} |w_i|^2 (P_s |\hat{f}_i + \delta_{fi}|^2 + \sigma_\eta^2) \le P_i^{\max} \; i = 1, \cdots, R\,. \end{aligned} \tag{23}$$

To simplify the problem in (23), we first find the optimal objective values of the following three sub-problems:

$$\min_{\boldsymbol{\delta}_h} |\mathbf{w}^H (\hat{\mathbf{h}} + \boldsymbol{\delta}_h)| \quad \text{s.t. } \|\boldsymbol{\delta}_h\| \le \zeta \tag{24}$$

$$\max_{\delta_{gi}} (\sigma_\eta^2 \mathbf{w}^H \mathbf{G} \mathbf{w} + \sigma_\upsilon^2)^{\frac{1}{2}} \quad \text{s.t. } |\delta_{gi}| \le \beta \tag{25}$$

$$\max_{\delta_{fi}} |w_i|^2 (P_s |\hat{f}_i + \delta_{fi}|^2 + \sigma_\eta^2) \quad \text{s.t. } |\delta_{fi}| \le \varepsilon\,. \tag{26}$$

Using triangle and Cauchy-Schwarz inequalities along with $\|\boldsymbol{\delta}_h\| \le \zeta$, we have

$$\begin{aligned} |\mathbf{w}^H (\hat{\mathbf{h}} + \boldsymbol{\delta}_h)| &\ge |\mathbf{w}^H \hat{\mathbf{h}}| - |\mathbf{w}^H \boldsymbol{\delta}_h| \\ &\ge |\mathbf{w}^H \hat{\mathbf{h}}| - \|\mathbf{w}\| \cdot \|\boldsymbol{\delta}_h\| \\ &\ge |\mathbf{w}^H \hat{\mathbf{h}}| - \zeta \|\mathbf{w}\| \end{aligned} \tag{27}$$

where we have assumed that $|\mathbf{w}^H \hat{\mathbf{h}}| > \zeta \|\mathbf{w}\|$. It can be shown that the equalities hold true when

$$\boldsymbol{\delta}_h = -\zeta \frac{\mathbf{w}}{\|\mathbf{w}\|} e^{j\phi} \tag{28}$$

where $\phi \triangleq \mathrm{angle}(\mathbf{w}^H \hat{\mathbf{h}})$. Combining (27) and (28), we can obtain the optimal objective value of (24)

$$\min_{\|\boldsymbol{\delta}_h\| \le \zeta} |\mathbf{w}^H (\hat{\mathbf{h}} + \boldsymbol{\delta}_h)| = |\mathbf{w}^H \hat{\mathbf{h}}| - \zeta \|\mathbf{w}\|\,. \tag{29}$$

Applying triangle inequality and the constraint $|\delta_{gi}| \le \beta$, we have

$$\begin{aligned} \sigma_\eta^2 \mathbf{w}^H \mathbf{G} \mathbf{w} &= \sigma_\eta^2 \sum_{i=1}^{R} |w_i|^2 |\hat{g}_i + \delta_{gi}|^2 \\ &\le \sigma_\eta^2 \sum_{i=1}^{R} |w_i|^2 (|\hat{g}_i| + |\delta_{gi}|)^2 \\ &= \sigma_\eta^2 \sum_{i=1}^{R} |w_i|^2 (|\hat{g}_i|^2 + 2|\hat{g}_i|\,|\delta_{gi}| + |\delta_{gi}|^2) \\ &\le \sigma_\eta^2 \sum_{i=1}^{R} |w_i|^2 (|\hat{g}_i|^2 + 2\beta|\hat{g}_i| + \beta^2) \\ &= \mathbf{w}^H \mathbf{D} \mathbf{w} \end{aligned} \tag{30}$$

where $\mathbf{D}$ is a diagonal matrix with $\mathbf{D}_{ii} \triangleq \sigma_\eta^2 (|\hat{g}_i|^2 + 2\beta|\hat{g}_i| + \beta^2)$. It can be verified that the equalities hold true when

$$\delta_{gi} = \beta \frac{\hat{g}_i}{|\hat{g}_i|}, \quad i = 1, \cdots, R\,. \tag{31}$$

Combining (30) and (31), the maximum of the objective function in (25) can be written as

$$\max_{\delta_{gi} \le \beta} (\sigma_\eta^2 \mathbf{w}^H \mathbf{G} \mathbf{w} + \sigma_\upsilon^2)^{\frac{1}{2}} = (\mathbf{w}^H \mathbf{D} \mathbf{w} + \sigma_\upsilon^2)^{\frac{1}{2}}\,. \tag{32}$$

Applying triangle inequality and $|\delta_{fi}| \le \varepsilon$ in (26), we have

$$\begin{aligned} P_s |\hat{f}_i + \delta_{fi}|^2 + \sigma_\eta^2 &\le P_s (|\hat{f}_i| + |\delta_{fi}|)^2 + \sigma_\eta^2 \\ &= P_s (|\hat{f}_i|^2 + 2|\hat{f}_i|\,|\delta_{fi}| + |\delta_{fi}|^2) + \sigma_\eta^2 \\ &\le P_s (|\hat{f}_i|^2 + 2\varepsilon|\hat{f}_i| + \varepsilon^2) + \sigma_\eta^2 \\ &= \alpha^2 \end{aligned} \tag{33}$$

where $\alpha \triangleq (P_s(|\hat{f}_i|^2 + 2\varepsilon|\hat{f}_i| + \varepsilon^2) + \sigma_\eta^2)^{\frac{1}{2}}$. The equalities in (33) hold true when

$$\delta_{fi} = \varepsilon \frac{\hat{f}_i}{|\hat{f}_i|}, \quad i = 1, \cdots, R\,. \tag{34}$$

Making use of (33) and (34), the maximum of the objective function in (26) can be written as

$$\max_{\delta_{fi} \le \varepsilon} |w_i|^2 (P_s |\hat{f}_i + \delta_{fi}|^2 + \sigma_\eta^2) = |w_i|^2 \alpha^2\,. \tag{35}$$

Using (29), (32) and (35), we can get a sub-optimal solution to problem (23) by solving

$$\begin{aligned} &\max_{\mathbf{w}} \frac{\sqrt{P_s}(|\mathbf{w}^H \hat{\mathbf{h}}| - \zeta \|\mathbf{w}\|)}{(\mathbf{w}^H \mathbf{D} \mathbf{w} + \sigma_\upsilon^2)^{\frac{1}{2}}} \\ &\text{s.t. } |w_i|^2 \alpha^2 \le P_i^{\max}, \; i = 1, \cdots, R\,. \end{aligned} \tag{36}$$

The optimal objective value of problem (36) is a lower bound to that of problem (23) and they are equal to each other when the optimal solution to (36) satisfies $-\zeta \frac{w_i e^{j\phi}}{\|\mathbf{w}\|} = \varepsilon\beta \frac{\hat{f}_i \hat{g}_i}{|\hat{f}_i|\,|\hat{g}_i|}$, $i = 1, \cdots, R$. Introducing an auxiliary variable τ and noticing that $\mathbf{w}$ can be rotated with arbitrary phase without affecting the SNR, we can rewrite (36) as

$$\begin{aligned} &\max_{\mathbf{w}, \tau} \quad \tau \\ &\text{s.t.} \quad \mathbf{w}^H \hat{\mathbf{h}} \ge \frac{\tau}{\sqrt{P_s}} (\mathbf{w}^H \mathbf{D} \mathbf{w} + \sigma_\upsilon^2)^{\frac{1}{2}} + \zeta \|\mathbf{w}\| \\ &\qquad |w_i|^2 \alpha^2 \le P_i^{\max}, \; i = 1, \cdots, R\,. \end{aligned} \tag{37}$$

The problem in (37) is a quasi-convex problem [10] and can be solved using SOCP by introducing two auxiliary variables. In particular, for any given τ, we check the feasibility of the following convex problem

$$\begin{aligned} &\text{find} \quad \mathbf{w} \\ &\text{s.t.} \quad \mathbf{w}^H\hat{\mathbf{h}} \geq \frac{\tau}{\sqrt{P_s}}(\mathbf{w}^H\mathbf{D}\mathbf{w}+\sigma_v^2)^{\frac{1}{2}}+\zeta\|\mathbf{w}\| \\ &\qquad |w_i|^2\alpha^2 \leq P_i^{\max},\ i=1,\cdots,R. \end{aligned} \tag{38}$$

Let $\tau_{\max}$ denote the optimal value in (37). If the feasibility problem in (38) is feasible, then we have $\tau \leq \tau_{\max}$. It can be proven by the contradict method. Assuming that $\tau > \tau_{\max}$ and problem (38) is feasible, we can see that it contradicts the assumption that $\tau_{\max}$ is the optimal value of problem (37). On the contrary, if the feasibility problem (38) is infeasible, then we can conclude that $\tau > \tau_{\max}$.

As a result, we can use a bisection search technique to solve such quasi-convex problem by checking the feasibility of the convex problem (38) in each step. Firstly, we choose certain interval $[\tau_l, \tau_u]$ that contains the optimal $\tau_{\max}$. Then we solve the convex feasibility problem (38) at the middle point $\tau = (\tau_l + \tau_u)/2$. If the problem is feasible, then we set the lower bound $\tau_l = \tau$. Otherwise, we assign the upper bound $\tau_u = \tau$. The midpoint of the new interval is used to check the feasibility of problem (38) again. This bisection search will stop until the width of the interval $[\tau_l, \tau_u]$ is small enough. We briefly summarize the bisection search method as following:

1. $\tau := (\tau_l + \tau_u)/2$.
2. Solve the convex feasibility problem (38). If (38) is feasible, then $\tau_l := \tau$, otherwise $\tau_u := \tau$.
3. If $(\tau_u - \tau_l) < \varepsilon_0$ then stop. Otherwise, go to Step 1.

Here, ε_0 is the tolerance of our error in finding $\tau_{\max}$.

4. SIMULATION RESULTS

In our simulations, we consider a relay network with $R = 20$ relays and assume Rayleigh flat-fading channels whose coefficients have unit variance. The relay and destination noise powers are assumed to be equal to each other. The transmitted power of the source is assumed to be 10 dB higher than the noise powers. The values of the norm bounds are assumed to be $\varepsilon = \beta = 0.05$, which is equivalent to the case that the error vectors of the channel coefficients are bounded as $\|\boldsymbol{\delta}_f\| = \|\boldsymbol{\delta}_g\| = \sqrt{R}\varepsilon \approx 0.22$.

Fig. 2 shows the outage probability of the individual relay transmitted power. A power outage occurs when anyone of the relay transmitted powers P_i is larger than the maximum allowed relay transmitted power $P_i^{\max}$. As the perfect CSI is not available in practical applications, the performance of the method of [5] with perfectly known CSI is simulated only as a reference. We can see from Fig. 2 that there is no power outage by using the proposed robust method. While the power outage occurs for all the tested cases when the non-robust method [5] is used. Fig. 3 shows the maximum received SNR at the destination. Compared to the bench method with perfectly known CSI, the proposed method has an SNR degradation of less than 2 dB.

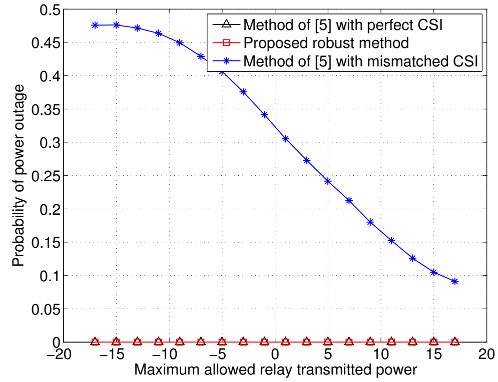

Fig. 2. Outage of individual relay transmitted power.

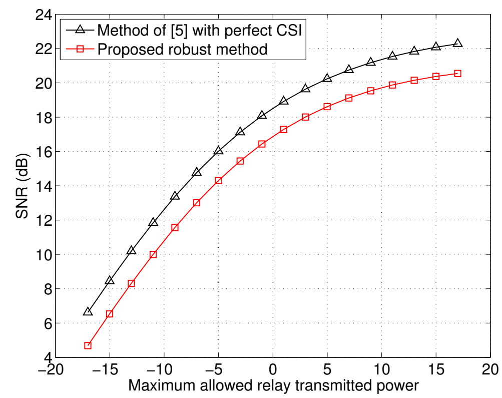

Fig. 3. Maximum received SNR at the destination.

5. CONCLUSIONS

The problem of robust distributed network beamforming has been considered in the case of mismatched source-to-relay and relay-to-destination channels. A novel approach that is based on worst-case optimization has been proposed. In our method, the minimum of the received signal-to-noise-ratio in the uncertain set is maximized subject to the constraints of the maximum individual relay transmitted powers. It has been shown that the problem can be solved using second-order-cone programming along with bisection search method. Our simulation results validate that the proposed robust method guarantees no power outage.

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