SUM RATE MAXIMIZATION AND ENERGY HARVESTING FOR TWO-WAY AF RELAY SYSTEMS WITH IMPERFECT CSI

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ABSTRACT

This paper studies the beamforming design of two-way relay systems for simultaneous information and power transfer with consideration of imperfect channel state information (CSI). To this end, we seek the robust beamforming design to maximize the weighted sum rate under the power constraint at relay nodes and the energy harvesting constraints at the two source nodes. Due to the infinitely many nonconvex constraints, the robust beamforming design problem is challenging. Therefore, we firstly propose an approximate to the signal to interference plus noise ratio (SINR). Then an iterative algorithm is developed based on the *semidefinite relaxation* (SDR), *S-procedure* and *successive convex approximation* (SCA) techniques. Furthermore, the effectiveness of the proposed algorithm is validated by numerical experiments.

Index Terms— Two-way relay, robust beamforming, semidefinite relaxation, S-procedure, successive convex approximation

1. INTRODUCTION

Energy harvesting has been considered as a promising technique in wireless communications [1–6], as it can scavenge energy from radio signals to prolong the life time of battery-powered systems. Recently, the works in [1] and [2] discussed the tradeoff between information and power transfer in flat-fading channels and frequency-selective fading channels, respectively. The analysis has been extended to MIMO broadcasting channels [3], two-hop MIMO relay channels [4] and two-user MISO interference channels [5]. The above works assume that perfect channel state information (CSI) is available at the transmitters. However, CSI error is inevitable in practice due to, e.g., the channel estimation error and time-varying nature of wireless channels. In view of this, [6] investigated the robust beamforming design of MIMO broadcasting channels with CSI errors for energy harvesting.

In this paper, we consider the robust beamforming design in an amplify-and-forward (AF) based two-way relay system for simultaneous information and power transfer, wherein the two source nodes exchange their information via multiple relays and harvest energy from radio signals. Specifically, a bounded CSI error model is assumed in this paper, and mathematically the robust relay beamforming design is formulated to maximize the weighted sum rate of the two-way relay system subject to the maximal power limit of relay nodes and minimal energy harvesting requirement of each source node. The design is challenging due to the coupled and nonconvex structure of the two-phase transmission and the existence of the CSI error. Therefore, as a key to this paper, the idea of *approximation* is utilized to fully explore the underlying structure of the problem.

Indeed, the two-way relay scenario has been considered in [7], where the robust power minimization problem without energy harvesting requirements is considered, and in [8], where the rate region is characterized by the rate profile approach. Moreover, it has been shown that a series of rate-based utility maximization problems can be transformed into difference-of-convex programmings, and the successive convex approximation (SCA) algorithm [9, 10] can thus be utilized to obtain a Karush-Kuhn-Tucker (KKT) solution efficiently [11–13]. However, to the best of our knowledge, the robust sum rate maximization problem has not been discussed before in two-way relay systems.

The rest of this paper is organized as follows. In Section 2, the system model and the problem formulation are presented. Afterwards, the signal to interference plus noise ratio (SINR) is approximated and the resulting robust beamforming design is then solved by employing the semidefinite relaxation (SDR), S-procedure and SCA techniques in Section 3. The simulation results are presented in Section 4.

2. SYSTEM MODEL AND PROBLEM FORMULATION

Consider a two-way AF relay system [8], where two source nodes S_1 and S_2 exchange information via K relay nodes and harvest energy also from these relay nodes. Each node is equipped with a single antenna and operates in half-duplex mode. Hence, two consecutive phases are involved to complete a round of information exchange.

In the 1st phase, S_1 and S_2 send their independent information to the K relay nodes, simultaneously. So, the received signal at the relay nodes can be written in a vector form as

$$\mathbf{y}_R = \sqrt{P_1}\mathbf{h}_1 x_1 + \sqrt{P_2}\mathbf{h}_2 x_2 + \mathbf{n}_R, \tag{1}$$

where x_i denotes the transmit symbol of S_i with $\mathrm{E}(|x_i|^2) = 1$, P_i is the transmit power of S_i, $\mathbf{n}_R \sim \mathcal{CN}(0, \sigma_0^2\mathbf{I}_K)$ is the complex Gaussian noise at relay nodes, and $\mathbf{h}_i = [h_{i1}, \ldots, h_{iK}]^T$ represents the flat-fading channel between S_i and all relay nodes for $i = 1, 2$.

In the 2nd phase, each relay amplifies its received signal by w_k, for $k = 1, \ldots, K$. Thus, the transmitted signal vector of the K relay nodes can be expressed as

$$\mathbf{x}_R = \mathbf{w} \circ \mathbf{y}_R = \sum_{i=1}^{2} \sqrt{P_i}\,\mathrm{diag}(\mathbf{w})\mathbf{h}_i x_i + \mathrm{diag}(\mathbf{w})\mathbf{n}_R, \tag{2}$$

where $\circ$ is the Hadamard product operator, and $\mathbf{w} = [w_1, \ldots, w_K]^T$. It can be shown that the total transmit power of the relay nodes is

$$p_R = \sum_{i=1}^{2} P_i \mathbf{h}_i^H\,\mathrm{diag}(\mathbf{w})^H\,\mathrm{diag}(\mathbf{w})\mathbf{h}_i + \sigma_0^2\mathbf{w}^H\mathbf{w}. \tag{3}$$

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Then, the K relay nodes broadcast the amplified signal to S_1 and S_2, simultaneously. The received signal at S_i can thus be expressed as

$$\begin{aligned} y_i = \mathbf{h}_i^T \mathbf{x}_R + n_i = &\overbrace{\sqrt{P_i}\mathbf{h}_i^T \operatorname{diag}(\mathbf{w})\mathbf{h}_i x_i}^{\text{self-interference}} \\ &+ \sqrt{P_j}\mathbf{h}_i^T \operatorname{diag}(\mathbf{w})\mathbf{h}_j x_j + \mathbf{h}_i^T \operatorname{diag}(\mathbf{w}) n_R + n_i \end{aligned} \quad (4)$$

where $n_i \sim \mathcal{CN}(0, \sigma_i^2)$ is the additive Gaussian noise at S_i.

Upon receiving the superimposed signal from the multiple relays, the two source nodes will perform the information detection (ID) and energy harvesting (EH) operation. More specifically, when performing ID, S_i subtracts the self-interference term from y_i, and then detects its desired signal x_j. Hence, the resulting SINR of the link from S_j to S_i can be expressed as

$$\mathrm{SINR}_i = \frac{P_j |\mathbf{h}_i^T \operatorname{diag}(\mathbf{w})\mathbf{h}_j|^2}{\sigma_0^2 |\mathbf{h}_i^T \operatorname{diag}(\mathbf{w})|^2 + \sigma_i^2}. \quad (5)$$

The Shannon information rate is $R_i = \frac{1}{2}\log_2(1+\mathrm{SINR}_i)$ in bps/Hz, where the factor 1/2 is due to equal-duration time slots for relaying. Accordingly, the weighted sum rate of the system is given by

$$R_{\text{sum}} = \theta R_1 + (1-\theta) R_2, \quad (6)$$

where $\theta \in (0,1)$ is the weight for the link from S_2 to S_1. When performing EH, the harvested energy at S_i can be written as [3]

$$\xi_i = \eta(P_i|\mathbf{h}_i^T \operatorname{diag}(\mathbf{w})\mathbf{h}_i|^2 + P_j|\mathbf{h}_i^T \operatorname{diag}(\mathbf{w})\mathbf{h}_j|^2 + \sigma_0^2|\mathbf{h}_i^T \operatorname{diag}(\mathbf{w})|^2). \quad (7)$$

where the constant η accounts for the energy conversion loss in the transducer. Without loss of generality, we assume that $\eta = 1$.

Noticed that, only the ideal case that the source nodes could execute ID and EH simultaneously is considered in this paper. Moreover, due to the limited power of relay nodes, there is a tradeoff between the information and power transfer. Thus, we consider the optimal beamforming design to maximize the weighted sum rate R_{sum} under the power constraint at relay nodes and the energy harvesting constraints at the source nodes. Mathematically, the problem can be formulated as

$$\begin{aligned} (\mathrm{P}_0) \quad &\max_{\mathbf{w}, R_1, R_2} \ \theta R_1 + (1-\theta) R_2 & (8a) \\ \text{s. t.} \ &\sum_{i=1}^{2} P_i \mathbf{h}_i^H \operatorname{diag}(\mathbf{w}^H) \operatorname{diag}(\mathbf{w})\mathbf{h}_i + \sigma_0^2 \mathbf{w}^H \mathbf{w} \le P_{\max}, & (8b) \\ &P_i|\mathbf{h}_i^T \operatorname{diag}(\mathbf{w})\mathbf{h}_i|^2 + P_j|\mathbf{h}_i^T \operatorname{diag}(\mathbf{w})\mathbf{h}_j|^2 \\ &+ \sigma_0^2|\mathbf{h}_i^T \operatorname{diag}(\mathbf{w})|^2 \ge E_i, & (8c) \\ &\frac{1}{2}\log_2\left(1 + \frac{P_j|\mathbf{h}_i^T \operatorname{diag}(\mathbf{w})\mathbf{h}_j|^2}{\sigma_0^2|\mathbf{h}_i^T \operatorname{diag}(\mathbf{w})|^2 + \sigma_i^2}\right) = R_i, & (8d) \\ &\forall i, j \in \{1,2\}, i \ne j. \end{aligned}$$

It is not surprising that (8c) can be satisfied automatically if E_i is small enough, e.g., 0. In such case, (P_0) degenerates into the sum rate maximization problem of two-way relay systems, which is NP-hard in general and is still open [8, 14]. If the CSI error is taken into account, the problem will be more complicated. For simplicity, we consider the following additive CSI error model:

$$\mathbf{h}_i = \hat{\mathbf{h}}_i + \mathbf{e}_i, \ i = 1, 2, \quad (9)$$

where $\hat{\mathbf{h}}_i$ is the channel estimate and $\mathbf{e}_i$ is the corresponding channel error vector. Moreover, we assume that $\mathbf{e}_i$ satisfies the spherical error model, i.e., $\|\mathbf{e}_i\| \le \delta_i$ for $i = 1, 2$.

To cope with the CSI error, we will rewrite the forwarding power at relay nodes and SINR, as well as the energy at each receiver node in an explicit form of $\mathbf{e}$, defined by $\mathbf{e} = [\mathbf{e}_1^T, \mathbf{e}_2^T]^T$. Denoting $\mathbf{W} = \mathbf{w}\mathbf{w}^H$ with $\operatorname{rank}(\mathbf{W}) = 1$, and substituting (9) into (3) yields

$$p_R = \mathbf{e}^H \mathbf{A}\mathbf{e} + 2\Re\{\mathbf{e}^H \mathbf{a}\} + a, \quad (10)$$

where $\mathbf{Q}_1 = [\mathbf{I}_K, \mathbf{0}_K]$, $\mathbf{Q}_2 = [\mathbf{0}_K, \mathbf{I}_K]$, and

$$\begin{aligned} \mathbf{A} &= P_1 \mathbf{Q}_1^H (\mathbf{I} \circ \mathbf{W})\mathbf{Q}_1 + P_2 \mathbf{Q}_2^H (\mathbf{I} \circ \mathbf{W})\mathbf{Q}_2, \\ \mathbf{a} &= P_1 \mathbf{Q}_1^H (\mathbf{I} \circ \mathbf{W})\hat{\mathbf{h}}_1 + P_2 \mathbf{Q}_2^H (\mathbf{I} \circ \mathbf{W})\hat{\mathbf{h}}_2, \\ a &= P_1 \hat{\mathbf{h}}_1^H (\mathbf{I} \circ \mathbf{W})\hat{\mathbf{h}}_1 + P_2 \hat{\mathbf{h}}_2^H (\mathbf{I} \circ \mathbf{W})\hat{\mathbf{h}}_2 + \sigma_0^2 \operatorname{Tr}(\mathbf{W}). \end{aligned}$$

Here, we remark that the power p_R is affected by the CSI errors, which may violate the power limit if $\mathbf{W}$ is not carefully designed.

Further, we approximate $\mathbf{h}_i^T \operatorname{diag}(\mathbf{w})\mathbf{h}_j$ by

$$\hat{\mathbf{h}}_i^T \operatorname{diag}(\mathbf{w})\hat{\mathbf{h}}_j + \hat{\mathbf{h}}_i^T \operatorname{diag}(\mathbf{w})\mathbf{Q}_j \mathbf{e} + \mathbf{e}^T \mathbf{Q}_i^T \operatorname{diag}(\mathbf{w})\hat{\mathbf{h}}_j. \quad (11)$$

Thus, SINR_i in (5) and ξ_i in (7) can be approximated by

$$\mathrm{SINR}_i \approx \frac{P_j(\mathbf{e}^T \mathbf{C}\mathbf{e}^* + 2\Re\{\mathbf{e}^T \mathbf{c}\} + c)}{\mathbf{e}^T \mathbf{D}_i \mathbf{e}^* + 2\Re\{\mathbf{e}^T \mathbf{d}_i\} + d_i}, \quad (12)$$

$$\xi_i \approx \mathbf{e}^T \mathbf{B}_i \mathbf{e}^* + 2\Re\{\mathbf{e}^T \mathbf{b}_i\} + b_i, \quad (13)$$

respectively, where $\mathbf{F}_{ij} = \operatorname{diag}(\hat{\mathbf{h}}_i)\mathbf{W}\operatorname{diag}(\hat{\mathbf{h}}_j^H)$, and

$$\begin{aligned} \mathbf{C} &= \mathbf{Q}_1^T \mathbf{F}_{22} \mathbf{Q}_1 + \mathbf{Q}_2^T \mathbf{F}_{11} \mathbf{Q}_2 + \mathbf{Q}_1^T \mathbf{F}_{21} \mathbf{Q}_2 + \mathbf{Q}_2^T \mathbf{F}_{12} \mathbf{Q}_1, \\ \mathbf{c} &= \mathbf{Q}_1^T \mathbf{F}_{22} \hat{\mathbf{h}}_1^* + \mathbf{Q}_2^T \mathbf{F}_{11} \hat{\mathbf{h}}_2^*, \\ c &= \mathbf{h}_1^T \mathbf{F}_{22} \hat{\mathbf{h}}_1^*, \\ \mathbf{D}_i &= \sigma_0^2 \mathbf{Q}_i^T (\mathbf{I} \circ \mathbf{W})\mathbf{Q}_i + 4P_i \mathbf{Q}_i^T \mathbf{F}_{ii} \mathbf{Q}_i, \\ \mathbf{d}_i &= \sigma_0^2 \mathbf{Q}_i^T (\mathbf{I} \circ \mathbf{W})\hat{\mathbf{h}}_i^*, \\ d_i &= \sigma_0^2 \hat{\mathbf{h}}_i^T (\mathbf{I} \circ \mathbf{W})\hat{\mathbf{h}}_i^* + \sigma_i^2, \\ \mathbf{B}_i &= P_j \mathbf{C} + \mathbf{D}_i, \\ \mathbf{b}_i &= P_j \mathbf{c} + \mathbf{d}_i + 2P_i \mathbf{Q}_i^T \mathbf{F}_{ii} \hat{\mathbf{h}}_i^*, \\ b_i &= P_j c + d_i + P_i \hat{\mathbf{h}}_i^T \mathbf{F}_{ii} \hat{\mathbf{h}}_i^* - \sigma_i^2. \end{aligned}$$

Consequently, by taking the CSI errors into account, we consider the following problem

$$\begin{aligned} (\mathrm{P}_1) \quad &\max_{\mathbf{W} \succeq \mathbf{0}, R_1, R_2} \ \theta R_1 + (1-\theta) R_2 & (14a) \\ \text{s. t.} \ &\mathbf{e}^H \mathbf{A}\mathbf{e} + 2\Re\{\mathbf{e}^H \mathbf{a}\} + a \le P_{\max}, & (14b) \\ &\mathbf{e}^T \mathbf{B}_i \mathbf{e}^* + 2\Re\{\mathbf{e}^T \mathbf{b}_i\} + b_i \ge E_i, & (14c) \\ &\frac{1}{2}\log_2\left(1 + \frac{P_j(\mathbf{e}^T \mathbf{C}\mathbf{e}^* + 2\Re\{\mathbf{e}^T \mathbf{c}\} + c)}{\mathbf{e}^T \mathbf{D}_i \mathbf{e}^* + 2\Re\{\mathbf{e}^T \mathbf{d}_i\} + d_i}\right) = R_i, & (14d) \\ &\forall \|\mathbf{e}_i\| \le \delta_i, \ i, j \in \{1,2\}, i \ne j, & (14e) \\ &\operatorname{rank}(\mathbf{W}) = 1. & (14f) \end{aligned}$$

3. ROBUST BEAMFORMING DESIGN BY SDR AND SCA

The problem (P_1) is challenging due to infinitely many constraints in (14b) and (14c) and infinitely many nonconvex constraints in (14d) and the rank-1 constraint. To handle this problem, we propose a suboptimal design which leverages the idea of approximation and relaxation. Specifically, the SDR, S-procedure and the SCA techniques alleviate the difficulties substantially, as we will show below.

3.1. Conservative Approximation to (P_1)

Firstly, we note that the numerator and the denominator of the constraint (14d) share a common CSI error. This fact implies that the CSI error resulting in the worst-case numerator may be beneficial to denominator. Yet, it is difficult, if not mathematically intractable, to make full use of this property [7]. In the sequel, we propose a suboptimal approach for solving the robust beamforming design (P_1).

By replacing the left-hand side (LHS) of constraint (14d) with its lower bound, the problem (P_1) can thus be conservatively (hence, safely) approximated by the following problem:

$$(\mathrm{P}_2) \max_{\mathbf{W}\succeq\mathbf{0},\, R_1,R_2} \;\theta R_1+(1-\theta)R_2 \tag{15a}$$

$$\text{s.t.} \max_{\|\mathbf{e}_1\|\le\delta_1,\|\mathbf{e}_2\|\le\delta_2} \mathbf{e}^H\mathbf{A}\mathbf{e}+2\Re\{\mathbf{e}^H\mathbf{a}\}+a\le P_{\max}, \tag{15b}$$

$$\min_{\|\mathbf{e}_1\|\le\delta_1,\|\mathbf{e}_2\|\le\delta_2} \mathbf{e}^T\mathbf{B}_i\mathbf{e}^*+2\Re\{\mathbf{e}^T\mathbf{b}_i\}+b_i\ge E_i, \tag{15c}$$

$$\frac{\min_{\|\mathbf{e}_1\|\le\delta_1,\|\mathbf{e}_2\|\le\delta_2} \mathbf{e}^T\mathbf{C}\mathbf{e}^*+2\Re\{\mathbf{e}^T\mathbf{c}\}+c}{\max_{\|\mathbf{e}_1\|\le\delta_1,\|\mathbf{e}_2\|\le\delta_2} \mathbf{e}^T\mathbf{D}_i\mathbf{e}^*+2\Re\{\mathbf{e}^T\mathbf{d}_i\}+d_i}\ge\frac{2^{2R_i}-1}{P_j}, \tag{15d}$$

$$\operatorname{rank}(\mathbf{W})=1,\ \forall i,j\in\{1,2\}, i\neq j. \tag{15e}$$

3.2. SDR and S-procedure

The rank-1 constraint makes the problem highly complicated, and a general treatment is the so-called SDR technique [15], which simply drops the rank-1 restriction on $\mathbf{W}$. Furthermore, by introducing slack variables $\{t_1, t_{21}, t_{22}\}$ to (15d), (P_2) can be relaxed into

$$(\mathrm{P}_3) \max_{\mathbf{W}\succeq\mathbf{0},t_1,\{R_i,t_{2i}\}} \;\theta R_1+(1-\theta)R_2 \tag{16a}$$

$$\text{s.t.} \max_{\|\mathbf{e}_1\|\le\delta_1,\|\mathbf{e}_2\|\le\delta_2} \mathbf{e}^H\mathbf{A}\mathbf{e}+2\Re\{\mathbf{e}^H\mathbf{a}\}+a\le P_{\max}, \tag{16b}$$

$$\min_{\|\mathbf{e}_1\|\le\delta_1,\|\mathbf{e}_2\|\le\delta_2} \mathbf{e}^T\mathbf{B}_i\mathbf{e}^*+2\Re\{\mathbf{e}^T\mathbf{b}_i\}+b_i\ge E_i \tag{16c}$$

$$\min_{\|\mathbf{e}_1\|\le\delta_1,\|\mathbf{e}_2\|\le\delta_2} \mathbf{e}^T\mathbf{C}\mathbf{e}^*+2\Re\{\mathbf{e}^T\mathbf{c}\}+c\ge t_1, \tag{16d}$$

$$\max_{\|\mathbf{e}_1\|\le\delta_1,\|\mathbf{e}_2\|\le\delta_2} \mathbf{e}^T\mathbf{D}_i\mathbf{e}^*+2\Re\{\mathbf{e}^T\mathbf{d}_i\}+d_i\le t_{2i}, \tag{16e}$$

$$\frac{t_1}{t_{2i}}\ge\frac{2^{2R_i}-1}{P_j},\quad \forall i,j\in\{1,2\}, i\neq j. \tag{16f}$$

It is not difficult to verify that the inequalities in (16d)-(16f) hold with equalities at the optimal solution, or else one can always increase $\{t_1, R_1, R_2\}$ and reduce $\{t_{2i}\}$ such that the objective value can be increased without any violation of other constraints. Now, the difficulties of (P3) lie in two aspects: one is the infinite number of constraints in (16b)-(16e), which are convex, but still computationally intractable, and the other is the nonconvex constraint (16f).

The first difficulty can be coped with the well-known S-procedure, which states that:

Lemma 1 [16, S-procedure] *Define $f_j(\mathbf{x})\triangleq\mathbf{x}^H\mathbf{A}_j\mathbf{x}+2\Re(\mathbf{b}_j^H\mathbf{x})+c_j$, where $\mathbf{A}_j\in\mathbb{C}^{n\times n}$ is Hermitian, $\mathbf{b}_j\in\mathbb{C}^n$, $c_j\in\mathbb{R}$, and $\mathbf{x}\in\mathbb{C}^n$, $j=0,1,2$. Then the following two conditions are equivalent:*
1) $f_0(\mathbf{x})\ge 0$ for every $\mathbf{x}\in\mathbb{C}^n$ such that $f_1(\mathbf{x})\ge 0$ and $f_2(\mathbf{x})\ge 0$;
2) There exist $\lambda_1,\lambda_2\ge 0$ such that

$$\begin{bmatrix}\mathbf{A}_0 & \mathbf{b}_0\\ \mathbf{b}_0^H & c_0\end{bmatrix}\succeq\lambda_1\begin{bmatrix}\mathbf{A}_1 & \mathbf{b}_1\\ \mathbf{b}_1^H & c_1\end{bmatrix}+\lambda_2\begin{bmatrix}\mathbf{A}_2 & \mathbf{b}_2\\ \mathbf{b}_2^H & c_2\end{bmatrix}. \tag{17}$$

By applying S-procedure to (16b)-(16e), (P_3) can be alternatively expressed as

$$(\mathrm{P}_4) \max_{\mathbf{W}\succeq\mathbf{0},\boldsymbol{\lambda}\succeq\mathbf{0}.t_1,\{R_i,t_{2i}\}} \;\theta R_1+(1-\theta)R_2 \tag{18a}$$

$$\text{s.t.} \begin{bmatrix}-\mathbf{A}+\lambda_1\widetilde{\mathbf{Q}}_1+\lambda_2\widetilde{\mathbf{Q}}_2 & -\mathbf{a}\\ -\mathbf{a} & -a+P_{\max}-\lambda_1\delta_1^2-\lambda_2\delta_2^2\end{bmatrix}\succeq\mathbf{0}, \tag{18b}$$

$$\begin{bmatrix}\mathbf{B}_i+\lambda_{3i}\widetilde{\mathbf{Q}}_1+\lambda_{4i}\widetilde{\mathbf{Q}}_2 & \mathbf{b}_i\\ \mathbf{b}_i^H & b_i-E_i-\lambda_{3i}\delta_1^2-\lambda_{4i}\delta_2^2\end{bmatrix}\succeq\mathbf{0}, \tag{18c}$$

$$\begin{bmatrix}\mathbf{C}+\lambda_5\widetilde{\mathbf{Q}}_1+\lambda_6\widetilde{\mathbf{Q}}_2 & \mathbf{c}\\ \mathbf{c}^H & c-t_1-\lambda_5\delta_1^2-\lambda_6\delta_2^2\end{bmatrix}\succeq\mathbf{0}, \tag{18d}$$

$$\begin{bmatrix}-\mathbf{D}_i+\lambda_{7i}\widetilde{\mathbf{Q}}_1+\lambda_{8i}\widetilde{\mathbf{Q}}_2 & -\mathbf{d}_i\\ -\mathbf{d}_i^H & -d_i+t_{2i}-\lambda_{7i}\delta_1^2-\lambda_{8i}\delta_2^2\end{bmatrix}\succeq\mathbf{0}, \tag{18e}$$

$$\frac{t_1}{t_{2i}}\ge\frac{2^{2R_i}-1}{P_j},\quad \forall i,j\in\{1,2\}, i\neq j. \tag{18f}$$

where $\boldsymbol{\lambda}\triangleq[\lambda_1,\lambda_2,\lambda_{3i},\lambda_{4i},\lambda_5,\lambda_6,\lambda_{7i},\lambda_{8i}]$, $\widetilde{\mathbf{Q}}_1\triangleq\mathbf{Q}_1^T\mathbf{Q}_1$, $\widetilde{\mathbf{Q}}_2\triangleq\mathbf{Q}_2^T\mathbf{Q}_2$. It can be seen that the constraints (18b)-(18e) are now linear, thus convex, and the only remaining difficulty lies in the (18f).

3.3. Successive Convex Approximation to (P_4)

Motivated by [9, 11, 12], we will utilize the SCA algorithm to tackle the nonconvex constraints (18f). In order to further reveal the nonconvex nature of (18f), let us introduce some slack variables s, u_i, v_i, and rewrite (P_4) as

$$(\mathrm{P}_5) \max_{\substack{\mathbf{W}\succeq\mathbf{0},\boldsymbol{\lambda}\succeq\mathbf{0},s,t_1,\\ \{R_i,u_i,v_i,t_{2i}\}}} \;\theta R_1+(1-\theta)R_2 \tag{19a}$$

$$\text{s.t.}\ (18\text{b})-(18\text{e}), \tag{19b}$$

$$t_1\ge e^s, \tag{19c}$$

$$t_{2i}\le e^{u_i}, \tag{19d}$$

$$2^{2R_i}-1\le e^{v_i}, \tag{19e}$$

$$u_i+v_i-s\le\log(P_j),\ \forall i,j\in\{1,2\},\ i\neq j, \tag{19f}$$

where the equivalence between (P_4) and (P_5) is established based on the fact that the inequality constraints of (19c)-(19f) hold with equalities at the optimal solution. Here, it can be clearly seen that (19d) and (19e) are of the form of difference of convex functions [12], which result in the non-convexity issue. Here, a good observation is that the exponential function admits a linear and locally tight lower bound by the first-order Taylor approximation, i.e, for all $\bar{x}\in\mathbb{R}$, $e^{\bar{x}}(x-\bar{x}+1)\le e^x$. Hence, (19d) and (19e) can be guaranteed by

$$t_{2i}\le e^{\bar{u}_i}(u_i-\bar{u}_i+1), \tag{20a}$$

$$2^{2R_i}\le e^{\bar{v}_i}(v_i-\bar{v}_i+1)+1, \tag{20b}$$

for $i=1,2$ with any given $\bar{u}_i$ and $\bar{v}_i$. Noticed that the approximations in (20) are tight at $u_i=\bar{u}_i$ and $v_i=\bar{v}_i$, respectively.

Based on the above approximate constraints and the idea of SCA, the problem (P_5), hence, (P_3) can be iteratively solved through the following convex problem:

$$(\mathrm{P}_6)\ \{\mathbf{W}[n],u_i[n],v_i[n]\}=\arg\max_{\substack{\mathbf{W}\succeq\mathbf{0},\boldsymbol{\lambda}\succeq\mathbf{0},s,t_1,\\ \{R_i,u_i,v_i,t_{2i}\}}} \;\theta R_1+(1-\theta)R_2 \tag{21a}$$

$$\text{s.t.}\ (18\text{b})-(18\text{e}),\,(19\text{c}),(19\text{f}), \tag{21b}$$

$$t_{2i}\le e^{u_i[n-1]}(u_i-u_i[n-1]+1), \tag{21c}$$

$$2^{2R_i}\le e^{v_i[n-1]}(v_i-v_i[n-1]+1)+1, \tag{21d}$$

where n is the iteration index. Here, one can see that the iterates $R_{\text{sum}}[n]$ is nondecreasing. Accordingly, the SCA based algorithm is summarized in **Algorithm 1**.

Algorithm 1 SCA based algorithm for solving (P_3)

1: **Initialize** $\{u_i[0], v_i[0]\}_{i=1}^2$; Set $n=0$;
2: **repeat**
3: Update $\{\mathbf{W}[n], u_i[n], v_i[n]\}_{i=1}^2$ by solving (P_6);
4: $n := n+1$;
5: **until** a stopping criterion is met.

3.3.1. Design Feasibility and SCA Initialization

It is readily shown that the design feasibility of (P_3) can be checked by solving the following problem:

$$(\mathrm{P}_7) \quad \text{find } \mathbf{W}, \quad \text{s.t. (16b), (16c) and } \mathbf{W} \succeq \mathbf{0}. \tag{22}$$

Besides, a feasible $\mathbf{W}^*$ to (P_7) can be used to initialize Algorithm 1 by solving the following two problems

$$t_3 = \min_{\|\mathbf{e}_1\| \le \delta_1, \|\mathbf{e}_2\| \le \delta_2} \mathbf{e}^T \mathbf{C} \mathbf{e}^* + 2\Re\{\mathbf{e}^T \mathbf{c}\} + c, \tag{23a}$$

$$t_{4i} = \max_{\|\mathbf{e}_1\| \le \delta_1, \|\mathbf{e}_2\| \le \delta_2} \mathbf{e}^T \mathbf{D}_i \mathbf{e}^* + 2\Re\{\mathbf{e}^T \mathbf{d}_i\} + d_i, \tag{23b}$$

which can be solved by using S-procedure. Then, from the activeness of (19d)-(19f), $u_i[0]$ and $v_i[0]$ are obtained by

$$u_i[0] = \log(t_{4i}), \quad v_i[0] = \log(P_j t_3 / t_{4i}). \tag{24}$$

3.3.2. Convergence Analysis

Due to the limited forwarding power at relay nodes, the set of all feasible $(\mathbf{W}, R_1, R_2)$ is compact. And using a similar argument as in [12, Theorem 1], we can draw the following proposition:

Proposition 1 *The sequence $\{\theta R_1[n] + (1-\theta) R_2[n]\}$ generated by the proposed Algorithm 1 converges; and every limit point is a KKT point of (P_3).*

Proof: Due to the limited space, we omit the detailed proof here. ■

From Proposition 1, we adopt the following stopping criteria: the algorithm continues until $R_{\text{sum}}[n]/R_{\text{sum}}[n-1] \le 1 + \varepsilon$ is satisfied, where ε is a predetermined parameter of rate precision.

However, it is worthy to remark that the optimal solution $\mathbf{W}^*$ obtained by Algorithm 1 is not necessarily rank-1 (due to SDR). If so, a possible suboptimal approach is to apply the Gaussian randomization [15] to obtain an approximate $\mathbf{w}^*$ for (P_2) from $\mathbf{W}^*$.

4. SIMULATION RESULTS

In this section, we present the numerical results of the proposed robust beamforming design. A system with $K = 4$ relay nodes and the power limit $P_{\max} = 1.5$ Watt is considered. The channel vectors are randomly generated following the standard complex Gaussian distribution. The channel realizations used in our simulations are $\hat{\mathbf{h}}_1 = [1.13+0.19j, 0.02+0.27j, 0.23+0.77j, 0.85+0.51j]^T$ and $\hat{\mathbf{h}}_2 = [-0.58+0.45j, 1.44-0.08j, 0.13-0.23j, 1.23+0.24j]^T$, where $j = \sqrt{-1}$. Set $\delta_i = \sqrt{K}\epsilon_i$, and for convenience, let $\epsilon_1 = \epsilon_2 = \epsilon$. In addition, the noise powers are set to be equal ($\sigma_i^2 = 1$ for $i = 0, 1, 2$), while the rate weight θ is equal to $1/2$. The convex optimization problems are solved by `CVX` in the simulations.

Fig. 1 compares the cumulative density functions (CDFs) of the power consumption at relay nodes between nonrobust design and the proposed robust design over 1000 random channel realizations with $P_1 = P_2 = E_1 = E_2 = 1$. One can observe that the required

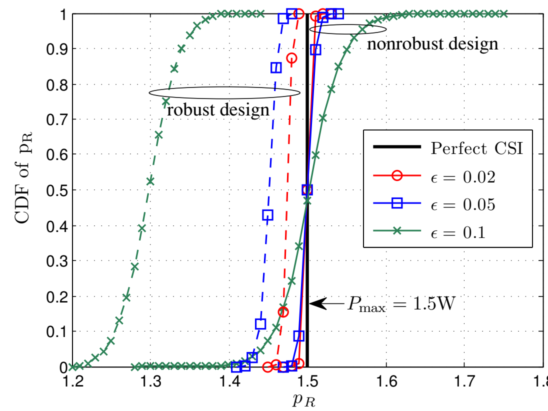

Fig. 1. The CDF of p_R

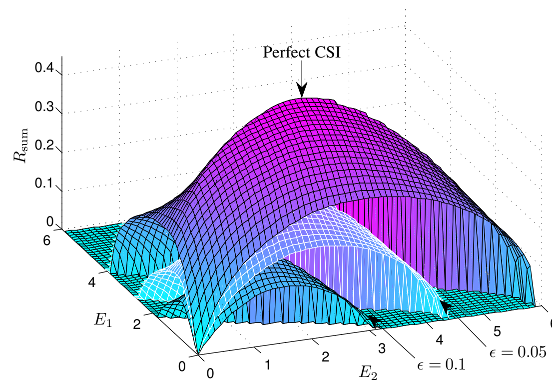

Fig. 2. R_{sum} versus E_1 and E_2

forwarding power will exceed the power limit with the probability of 50%, and the dynamic range get larger with larger ϵ. Meanwhile, the proposed robust design can always satisfy the power constraint.

Fig. 2 shows the sum rate versus EH powers with $E_1 = P_1$ and $E_2 = P_2$. The sum rate increases when E_1 and E_2 increase, but rapidly diminishes to zero if E_1 or E_2 is large. This is due to the fact that the relays' power is used to fulfill the EH requirements, and little left for information transfer. It can also be observed that the sum rate decreases with the increase of ϵ, as expected.

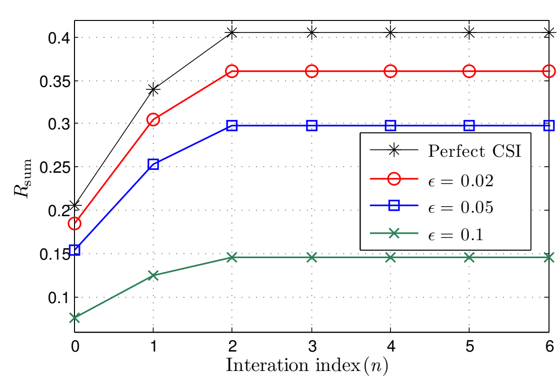

Fig. 3. R_{sum} versus iteration index

Fig. 3 illustrates the convergence behaviors of Algorithm 1 for $P_1 = P_2 = E_1 = E_2 = 1$ with different channel error bounds. It can be seen that R_{sum} increases monotonically, and the proposed algorithm converges in few steps in each case.

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