SNR-PER-UNIT-POWER OPTIMIZATION IN RELAY NETWORKS

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ABSTRACT

In this paper, we adopt a novel efficiency measure, namely, the received signal to noise ratio (SNR) per unit power, in relay network design. First, limitations of conventional efficiency measures, spectral efficiency and energy efficiency, are discussed to motivate the SNR-per-unit-power (SNR-PUP) measure. Then for a single-relay network which uses amplify-and-forward (AF) protocol, we find the optimal relay power that maximizes the SNR-PUP for a given transmitter power. The average relay power, the SNR-PUP, and the outage probability of the proposed design are investigated analytically and numerically, and are compared with the conventional design where the relay power is fixed. We also consider a general multi-relay network and use gradient-ascent method for the SNR-PUP maximization. Our results show that with the same average relay transmit power, the proposed design is superior not only in the SNR-PUP but also in the outage probability for both single and multi-relay networks.

Index Terms— Relay network, SNR-per-unit-power, outage probability, efficiency.

1. INTRODUCTION

Wireless communication networks are increasingly facing higher demands for bit-rate and reliability. There are numerous published results on the global performance optimization such as SNR maximization, throughput maximization, and error rate minimization for fixed transmit power, e.g., [1–3]. As the popularity of wireless users and wireless traffic rapidly multiplies, the increase in energy consumption is dramatic, which leads to the increase of greenhouse gas emission that causes severe environmental depredation. As result, in recent years, green communication designs attracted significant attention [4].

Popular efficiency measures include spectral efficiency and energy efficiency [3, 5]. There is a significant volume of literature addressing these two efficiency measures for various network configurations, e.g., [6–8]. Spectral efficiency is defined as the achievable transmission bit-rate and is maximized to guarantee the highest amount of information flow for fixed transmit power, but it does not consider how efficient the transmit power is used to achieve the maximum. Energy efficiency is defined as the number of bits transmitted per unit energy or power [5], and it is a natural efficiency measure. But for most communication systems, energy efficiency is maximized when the transmit power approaches 0, i.e., when the system works in low SNR regime. To see this, we consider the simple point-to-point single-antenna system with transmit power P, unit-variance noise, and channel gain λ. The energy efficiency of the system is $[\log(1+\lambda P)]/P$, which takes its maximum λ when $P \to 0$. Hence, the energy-efficiency-optimal scheme will trap the system in low SNR regime, where the actual bit-rate and reliability can be low.

1.1. Efficiency Measure Using SNR-Per-Unit-Power

The aforementioned limitations of spectral efficiency and energy efficiency measures inspire us to study SNR-PUP as a new criterion to evaluate the network efficiency. SNR-PUP is defined as

$$\eta \triangleq \frac{\mathrm{SNR}}{P_{\mathrm{total}}} \tag{1}$$

where SNR is the end-to-end received SNR of the network and P_{total} is the total power consumed in the network. It represents the achievable received SNR per unit total transmit power. If the noise has unit-variance, P_{total} is also the transmit SNR. In this sense, η represents the received SNR the system gains per unit transmit SNR, which is dimensionless.

Compared with spectral efficiency, SNR-PUP is more natural as it shows the performance per unit power. Compared with energy efficiency, SNR-PUP does not trap the network in low power regime. To see this, we revisit the same point-to-point single-antenna system with transmit power P_{total}, unit-variance noise, and channel gain λ. The SNR-PUP of the system is $\eta = (\lambda P_{\mathrm{total}})/P_{\mathrm{total}} = \lambda$, which is independent of the transmit power. For a point-to-point direct communication system without relaying (e.g., multi-antenna system), the SNR-PUP can be shown directly to be equal to the array gain of the system, independent of the transmit power. Thus, maximization of SNR-PUP is trivial. For wireless relay networks, however, the maximization can be involved, as will be seen later in this paper.

1.2. Summary of Contribution and Paper Organization

SNR-PUP as an efficiency measure was first proposed in one of the authors work [9] and was also used in [10, 11]. While it was employed as a measure for performance in [9–11], the property of SNR-PUP and optimal designs using this measure have yet to be investigated.

In this paper, for relay networks, we adopt SNR-PUP as the efficiency measure and propose the optimal relay power control which adapts to the channel coefficients. Properties of the proposed scheme are analyzed and compared with the traditional scheme where the relay power is fixed. For single relay networks, the SNR-PUP maximization is solved analytically. For multi-relay networks with a sum power constraint at the relays, a gradient-ascent based numerical method is proposed and the solution is proved to converge to the global optimum. We show in this paper that compared with fixed relay power scheme, the proposed scheme is superior not only in the SNR-PUP but also in the outage probability. This implies that SNR-PUP is a promising measure for network efficiency.

The remaining of this paper is organized as follows. Section 2 considers a single-relay network where the solution to SNR-PUP maximization is presented in closed-form and it is compared with fixed relay power scheme. Section 3 is on a multi-relay network, where the SNR-PUP-maximization problem is simplified to a one-dimensional problem and solved numerically. Section 4 concludes the paper.

2. SNR-PER-UNIT-POWER OPTIMIZATION IN SINGLE-RELAY NETWORK

We start with a single-relay network with flat-fading channels. Denote the channel from the transmitter to the relay as f and the channel from the relay to the receiver as g. Assume that f and g are independent circularly symmetric complex Gaussian with zero-mean and unit-variance, so the channel magnitudes follow Rayleigh distribution. The channels are assumed to be known at the relay. No direct link is considered. Let the transmit power of the transmitter be P_0 and the transmit power of the relay be P. The total transmit power in the network is thus $P_{\rm total} = P + P_0$. Assume that all noises experienced during transmission are independent and identically distributed (i.i.d.) Gaussian with zero-mean and unit-variance.

We employ the standard amplify-and-forward (AF) protocol with variable relay power gain. The received SNR of this network has been shown to be [1]

$$\mathrm{SNR} = \frac{|fg|^2 P P_0}{1+|f|^2P_0+|g|^2P} \tag{2}$$

$$\approx \frac{|fg|^2 P P_0}{|f|^2P_0+|g|^2P}. \tag{3}$$

The approximation in (3) is widely used in relay network literature and has shown to be tight in the high SNR regime [12]. From (2) and the definition of η in (1), the SNR-PUP of the network is thus

$$\eta = \frac{|fg|^2 P P_0}{(1+|f|^2P_0+|g|^2P)(P+P_0)}. \tag{4}$$

2.1. Optimal Solution of the Relay Power

Our problem is to find the relay transmit power P such that the SNR-PUP is maximized, i.e.,

$$\arg\max_{0\le P<\infty} \frac{|fg|^2 P P_0}{(1+|f|^2P_0+|g|^2P)(P+P_0)}. \tag{5}$$

In this problem formulation, we actually assume that the relay has an unlimited power constraint. This will surely cause practical issues in SNR or spectral efficiency maximization, as the solution is $P \to \infty$. In this paper, however, due to the nature of the adopted SNR-PUP measure, we can see from later derivations that the problem does not occur.

Differentiating η with respect to P and equaling it to zero, the optimal relay power, denoted as $P_{\rm opt}$, can be shown as:

$$P_{\rm opt} = \frac{\sqrt{P_0(1+|f|^2P_0)}}{|g|}. \tag{6}$$

At high SNR ($P_0 \gg 1$), this solution can be approximated as

$$P_{\rm opt} \approx P_{\rm approx} = \frac{|f|}{|g|}P_0. \tag{7}$$

The same result can be obtained if the approximate SNR formula in (3) is used in the SNR-PUP-maximization. Note that the solutions in (6) and (7) are channel dependent.

2.2. Performance of SNR-PUP-Maximizing Scheme

Now we analyze the performance of the proposed SNR-PUP-maximizing solution, including the average relay transmit power, the average SNR-PUP, and the outage probability of the network. Approximations in (3) and (7) are used to allow a tractable analysis. The following results are obtained.

Lemma 1. *When $P_0 \gg 1$, with the relay power design in (7) and using the SNR approximation in (3), the average relay power is $P_{\rm ave} = (\pi/2)P_0$, the average SNR-PUP of the network is $\eta_{\rm ave} = 3\pi/8 - 1$, and the outage probability with SNR threshold $\gamma_{\rm th}$ is*

$$O = 1 - e^{-\frac{\gamma_{\rm th}}{P_0}} \int_0^{+\infty} e^{-\frac{\gamma_{\rm th}^3}{P_0^3 u^2} - \frac{\gamma_{\rm th}^2}{P_0^2 u} - u} du = \frac{\gamma_{\rm th}}{P_0} + \mathcal{O}\left(\frac{1}{P_0^2}\right). \tag{8}$$

Proof. Note that $|f|$ and $|g|$ are Rayleigh with the following probability density function: $f(x) = 2xe^{-x^2}, x \ge 0$. With the design in (7), the average relay power can be derived as

$$P_{\rm ave} = P_0 \int_0^{+\infty}\int_0^{+\infty} \frac{x}{y} 4xy \cdot e^{-(x^2+y^2)} dxdy = \frac{\pi}{2}P_0.$$

Inserting (7) into (3) and (4), the SNR and the SNR-PUP of the proposed scheme can be calculated as

$$\mathrm{SNR} \approx \frac{|f|^2|g|P_0}{|f|+|g|}, \quad \eta = \frac{|fg|^2}{(|f|+|g|)^2}.$$

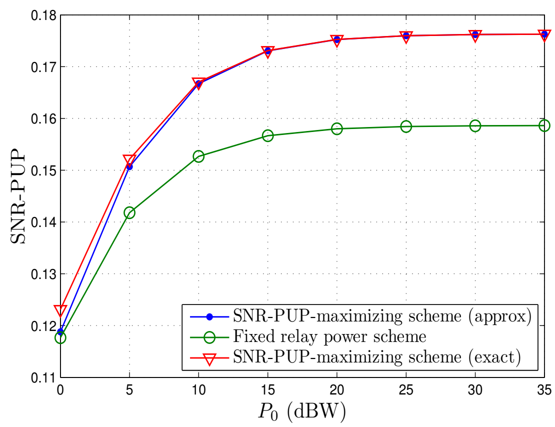

Fig. 1: Average SNR-PUP in single-relay network.

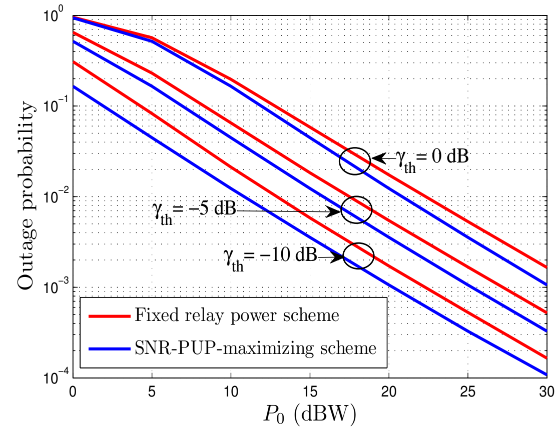

Fig. 2: Outage probability in single-relay network.

The average SNR-PUP can then be calculated as

$$\begin{aligned}\eta_{\text{ave}} &= \int_0^{+\infty}\int_0^{+\infty}\frac{x^2y^2}{(x+y)^2}4xye^{-(x^2+y^2)}dxdy\\ &= \frac{3}{8}\pi - 1 \approx 0.18.\end{aligned}$$

As for the outage probability, we have

$$\begin{aligned}O &= \mathbb{P}\left(\frac{|f|^2|g|P_0}{|f|+|g|}\le\gamma_{\text{th}}\right)\\ &= \int_0^{+\infty}\mathbb{P}(P_0x^2Y-\gamma_{\text{th}}Y\le\gamma_{\text{th}}x|X=x)f_X(x)dx\\ &= 1-e^{-\frac{\gamma_{\text{th}}}{P_0}}\int_0^{+\infty}e^{-\frac{\gamma_{\text{th}}^3}{P_0^3u^2}-\frac{\gamma_{\text{th}}^2}{P_0^2u}-u}du.\\ &= \frac{\gamma_{\text{th}}}{P_0}+\mathcal{O}\left(\frac{1}{P_0^2}\right).\end{aligned}$$

The proof is complete.

□

2.3. Comparison with Fixed Relay Power Design

To evaluate the performance of the proposed design, we compare it with the traditional design where the relay power is fixed. For fairness, we set the average relay power used in the two cases to be the same. Thus, for the fixed relay power scheme, the relay power is fixed as P_{ave} for each transmission. Note that this is also the optimal solution if we want to maximize the received SNR of the network. With calculations similar to those in Section 2.2, the following results can be derived.

The average SNR-PUP for fixed relay power scheme is

$$\eta_{\text{ave\_fix}} = \frac{2\pi}{\pi+2}\cdot\frac{\pi(\pi+\log\frac{16}{\pi^4})-4}{(\pi-2)^3}\approx 0.16.$$

For the outage probability with SNR threshold γ_{th}, we use the cumulative distribution function of the SNR derived in [13, 14]; the outage probability can be derived to be

$$O_{\text{fix}}=1-\frac{e^{-\frac{\pi+2}{\pi P_0}\gamma_{\text{th}}}}{P_0}\sqrt{\frac{8\gamma_{\text{th}}}{\pi}(1+\gamma_{\text{th}})}K_1\left(\frac{1}{P_0}\sqrt{\frac{8\gamma_{\text{th}}}{\pi}(1+\gamma_{\text{th}})}\right), \tag{9}$$

where $K_1(\cdot)$ is the first order modified Bessel function of second kind. By using series expansion, (9) can be written as

$$\begin{aligned}O_{\text{fix}} &= 1-e^{-\frac{\pi+2}{\pi P_0}\gamma_{\text{th}}}\left[1+\mathcal{O}\left(\frac{\ln P_0}{P_0^2}\right)\right]\\ &= \frac{\pi+2}{\pi}\frac{\gamma_{\text{th}}}{P_0}+\mathcal{O}\left(\frac{\ln P_0}{P_0^2}\right).\end{aligned} \tag{10}$$

We first compare the two schemes analytically. From the derived values of η_{ave} and $\eta_{\text{ave\_fix}}$ and the outage probability results (8) and (10), we conclude that SNR-PUP-maximizing scheme is superior in SNR-PUP by 12% and superior in outage probability by 2.14 dB.

Then, we compare the two schemes by simulation. We first show the SNR-PUP in Fig. 1 for the two schemes, denoted as "SNR-PUP-maximizing scheme (approx)" and "Fixed relay power scheme". We can see that the simulation results are in accord with our analytical derivations, where the SNR-PUPs are 0.18 and 0.16, respectively. We also simulate the SNR-PUP for the design using (6), where no approximation is used, denoted as "SNR-PUP-maximizing scheme (exact)". The figure shows that (7) is a good approximation when P_0 is greater than 10 dB.

In Fig. 2, we show the outage probability of the network with thresholds -10 dB, -5 dB, and 0 dB. It can be seen that the SNR-PUP-maximizing scheme is superior by about 2 dB, which is again consistent with our analysis for large P_0. We have also analyzed and simulated the average SNR of the network. The two schemes have comparable performance with the SNR-PUP-maximizing scheme inferior by 2%. The results are not shown due to the space.

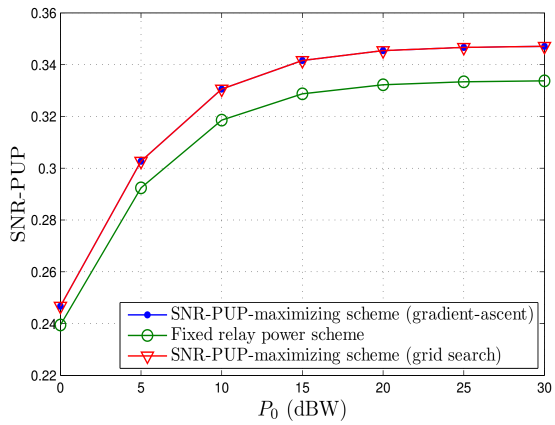

Fig. 3: Average SNR-PUP in two-relay network.

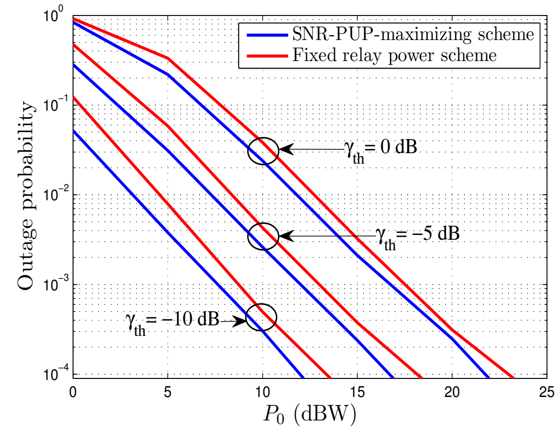

Fig. 4: Outage probability in two-relay network.

3. SNR-PER-UNIT-POWER OPTIMIZATION IN MULTIPLE-RELAY NETWORK

In this section, we consider a general network with R relays. Denote the channel from the transmitter to the ith relay as f_i and the channel from the ith relay to the receiver as g_i. Assume the same channel statistics for all channels. Let the transmit power of the transmitter be P_0, the transmit power of the ith relay be P_i and the total power consumed by all relays is denoted as P, i.e., $P = \sum_{i=1}^{R} P_i$. All noises are assumed to follow i.i.d. Gaussian distribution with zero-mean and unit-variance. The two-step relay beamforming scheme with perfect phase synchronization is used [1,2]. The received SNR of the network is [1]

$$\mathrm{SNR} = \frac{\left|\sum_{i=1}^{R} \frac{f_i\sqrt{P_0 P_i}}{\sqrt{f_i^2 P_0+1}} g_i\right|^2}{\sum_{i=1}^{R} \frac{P_i}{f_i^2 P_0+1} g_i^2 + 1}.$$

The SNR-PUP-maximizing problem can be modeled as

$$\arg \max_{P=\sum_{i=1}^{R} P_i} \frac{\mathrm{SNR}}{P_0 + P}. \tag{11}$$

We first rewrite (11) as

$$\arg\max_{P} \frac{1}{P_0 + P}\left(\max_{\sum_{i=1}^{R} P_i = P} \mathrm{SNR}\right). \tag{12}$$

The inner SNR-maximization for a given P has been solved in [1] where the optimal power for the ith relay is

$$P_i = \frac{\left(\frac{|f_i|^2|g_i|^2 P_0 P(|f_i|^2 P_0+1)}{|f_i|^2 P_0+|g_i|^2 P+1}\right)^2}{\sum_{i=1}^{R}\left(\frac{|f_i|^2|g_i|^2 P_0 P(|f_i|^2 P_0+1)}{|f_i|^2 P_0+|g_i|^2 P+1}\right)^2} P. \tag{13}$$

and the optimal received SNR is

$$\max_{\sum_{i=1}^{R} P_i = P} \mathrm{SNR} = \sum_{i=1}^{R} \frac{|f_i|^2|g_i|^2 P_0 P}{|f_i|^2 P_0 + |g_i|^2 P + 1}. \tag{14}$$

By using (14) in (12), the SNR-PUP-maximizing problem reduces to the following one-dimensional problem:

$$\arg\max_{P} \sum_{i=1}^{R} \frac{|f_i|^2|g_i|^2 P_0 P}{(|f_i|^2 P_0 + |g_i|^2 P + 1)(P + P_0)}. \tag{15}$$

Lemma 2. *The objective function in (15) has unique maximum.*

Proof. This lemma can be proved by showing that the objective function in (15) is semi-strictly quasi-concave [15] [16]. We omit the details here due to the limited space. □

In what follows, we show simulated SNR-PUP and outage probability of a network for the SNR-PUP-maximizing scheme and fixed relay power scheme. The average relay powers are set to be the same for fair comparison. With the result in Lemma 2, we can use gradient-ascent method to find the SNR-PUP-maximizing solution. To justify the optimality of gradient-ascent method, grid search is also used in simulations as a performance benchmark. Fig. 3 shows the average SNR-PUP for a two-relay network. For the SNR-PUP-maximizing scheme, the gradient-ascent algorithm and grid search achieve the same performance. This implies that gradient-ascent method finds the global optimum. Compared with the fixed relay power scheme, SNR-PUP-maximizing scheme is superior in SNR-PUP by about 5%. In Fig. 4, we show the outage probabilities of the network with thresholds -10 dB, -5 dB and 0 dB. We can see that the proposed SNR-PUP-maximizing scheme is superior by 1.5 dB in high SNR range.

4. CONCLUSIONS

In this paper, we studied SNR-PUP as a new efficiency measure for relay network design. In both single and multi-relay networks with sum power constraint, our results indicate that SNR-PUP is an appropriate measure of efficiency for all SNR range and SNR-PUP-maximizing design is superior in both SNR-PUP and outage probability to fixed relay power design.

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