ENERGY-EFFICIENT RELAYING USING RATELESS CODES

Jörg Bühler and Sławomir Stańczak

Heinrich-Hertz-Lehrstuhl für Informationstheorie
und theoretische Informationstechnik
Technische Universität Berlin
Einsteinufer 25, D-10587 Berlin, Germany
Email: {joerg.buehler,slawomir.stanczak}@mk.tu-berlin.de

ABSTRACT

In this paper, we study a relaying network employing a rateless coding scheme proposed in the literature. We consider the problem of an energy-efficient operation of the scheme and derive algorithms that maximize the achievable rate for specified energy-per-bit bounds at the relay nodes. For this, we identify the rate function as a standard interference function, which allows to design efficient algorithmic solutions for solving the problem.

***Index Terms*—** Energy efficiency, relaying systems, rateless codes, standard interference functions, fixed-point iteration.

1. INTRODUCTION

In this work, we consider the tradeoff between energy efficiency (EE) and spectral efficiency (SE) [1] in wireless relay networks using rateless codes. Energy efficiency is measured by the quantity of *energy-per-bit (Eb)* and refers to the amount of energy that is required to reliably communicate one bit of information at a certain transmission rate. Clearly, we wish Eb to be as small as possible, while the spectral efficiency (or transmission rate at a given bandwidth) should be as high as possible. Unfortunately, in general, these goals contradict each other—there is a tradeoff between the two quantities, commonly referred to as the EE-SE tradeoff [1]. The issue of energy-efficiency is becoming a more and more important aspect of communication systems, especially in the context of environmental challenges and new types of wireless applications [1]. For many applications, it is important to consider not only the transmission rate, but also the energy expenditure. For example, if the nodes are energy-limited since they are powered by batteries or are recharged in an irregular fashion, it is in general more desirable to limit the energy spent for transmission instead of merely maximizing the transmission rates. Situations of this kind are likely to occur e.g. in sensor networks or when using flexible nodes that do not have access to a constant power supply.

Relation to prior work. The problem of maximizing the transmission rate under given constraints on the energy-per-bit values has been studied in [2] for a system with two relay nodes, the so-called diamond network. There, the theory of standard interference functions [3] [4] was successfully applied in order to develop algorithmic solutions for the optimization problem. In this paper, we demonstrate that the theory of standard interference functions can also serve as a valuable tool for finding energy-aware operating points in relaying systems employing *rateless codes* [5] [6]. This interesting coding technique is subject to increasing attention, especially concerning applications in multicast and relaying systems [7] [8] [9]. Here, we study the rateless relaying scheme proposed in [9] in the situation of energy-per-bit constraints at the relay nodes.

This research is supported in part by the German Federal Ministry of Education and Research (BMBF) under grant 01BU920. Parts of this work have been performed in the framework of the European research project SAPHYRE, which is partly funded by the European Union under its FP7 ICT Objective 1.1 - The Network of the Future.

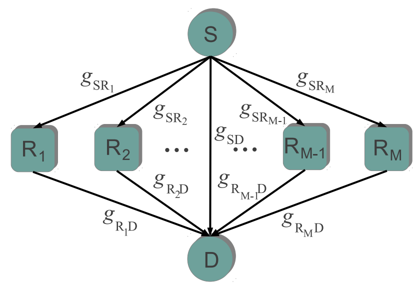

Fig. 1: System model: A message is to be conveyed from sender S to the destination node D via relay nodes R_k.

Organization. The paper is organized as follows: Section 2 introduces the system model and Section 3 the main optimization problem investigated in this paper. Corresponding algorithmic solutions are given in Section 4 and numerically evaluated in Section 5. Finally, Section 6 concludes the paper.

Notation. We use the notation $\mathcal{C}(x) = \log(1+x)$ throughout the paper; logarithms are taken to base e and all data rates expressed in nats. The set of positive reals is denoted by $\mathbb{R}_{++} = \{x \in \mathbb{R} : x > 0\}$. For two vectors $\mathbf{x}, \mathbf{y}$, we denote component-wise inequalities as $\mathbf{x} < \mathbf{q}, \mathbf{x} \le \mathbf{q}$ etc. Moreover, $\mathbf{x}^t$ denotes the transpose of $\mathbf{x}$, and x_k its kth component. The vector with all of its n entries equal to one is denoted by $\mathbf{1}_n$. Similarly, the all zero vector is written as $\mathbf{0}_n$.

2. SYSTEM MODEL

The system model and the mode of operation is mostly the same as described in [9]; we now give our notation and briefly state the most important facts and assumptions. For further details and a more elaborate discussion, we refer the reader to [9].

In this paper, we study a relaying system consisting of a transmitter (source node S) which intends to convey a message to a destination node D. All nodes are restricted to be in half-duplex mode, and the message is to be transported with the help of a number of M relaying stations, labeled as R_k for $k \in \{1, \ldots, M\}$ (cf. Figure 1). The channels between all nodes are given by frequency-flat additive white Gaussian noise (AWGN) channels which are assumed to be constant during the transmission of at least one message. For nodes $P, Q \in \{\mathrm{S}, \mathrm{D}, \mathrm{R}_1, \ldots, \mathrm{R}_M\}$, $g_{PQ} \in \mathbb{C}$ denotes the signal-to-noise ratio at node Q when node P transmits with unit power. We assume that the values g_{PQ} are known at all nodes (as opposed to the assumption in [9]).[1] Without loss of generality, we assume an ordering of the relay nodes such that $g_{\mathrm{SR}_k} \geq g_{\mathrm{SR}_{k+1}}$ for $k \in \{1, \ldots, M-1\}$. The channels between S and D and from the relays to the D are assumed to be orthogonal, which is achieved in [9] by employing relay-specific spreading sequences. The power constraint at the source node S is denoted by $p^{\mathrm{S}} > 0$, while the power constraints at the relay nodes are collected in the vector $\mathbf{p} \in \mathbb{R}^M_{++}$, where the mth component p_m specifies the power constraint at relay node R_m. The vector of all powers (both at the source and the relays) is written as $\mathbf{P} = \left(p^{\mathrm{S}}, p_1, \ldots, p_M\right)^t \in \mathbb{R}^{M+1}_{++}$.

The general transmission scheme is the *synchronous operation mode* as described in [9] (we do not consider the asynchronous mode here) and works as follows: The source encodes the message using a rateless code using a length-k source message vector and keeps on transmitting until the destination node D has decoded the message. In the so-called *listening phase*, each relay R_m listens to the source transmission until it has successfully decoded the message. Thereupon, the relay node enters the *collaboration phase* and

- re-encodes the message using its individual rateless code and transmits it employing its individual spreading sequence with power $r_m p_m$, where $r_m \in [0, 1]$.
- re-encodes the message using the same rateless code as the source and transmits it employing the source spreading sequence with power $(1 - r_m)p_m$.

In order to combine the source and relay signals that have been transmitted using the same spreading sequence, the destination node D uses a Rake receiver; this is called *energy combining*. The combination of data obtained from different spreading sequences / rateless codes is referred to as *information combining*. The scheme in [9] uses both energy combining and information combining; hence, it is called *mixed combining* and steered by the vector of power ratios $\mathbf{r} = (r_1, \ldots, r_M)^t \in [0, 1]^M$. We assume the mixed combining vector $\mathbf{r}$ to be arbitrary, but fixed in this work.

The transmission is terminated once the destination node D has decoded the message. Let n be the total number of time slots passed when the destination decodes the message, and let $N \leq M$ be the number of relays that have decoded the message during this time. By n_m, we denote the number of time slots that have passed when relay node R_m decodes the message. Then the time fraction during which relay R_m listens to the source transmission is given by $\lambda_m = \frac{n_m}{n}$, whereas the transmission rate is $R = \frac{k}{n}$. Writing [2]

$$J_i = \mathcal{C}\left(p_{\mathrm{S}} g_{\mathrm{SD}} + \sum_{j=1}^{i} r_j p_j g_{\mathrm{R}_j\mathrm{D}}\right) + \sum_{j=1}^{i} \mathcal{C}\left((1 - r_j) p_j g_{\mathrm{R}_j\mathrm{D}}\right) \quad (1)$$

for the *collaborative capacity* of the relays $\mathrm{R}_1, \ldots, \mathrm{R}_i$, the transmission rate R_N for N decoding relays must satisfy the $N + 1$ constraints (cf. [9])

$$R_N \leq \lambda_1 C_0 + \sum_{m=1}^{N-1} (\lambda_{m+1} - \lambda_m) J_m + (1 - \lambda_N) J_N, \quad (2)$$

$$R_N \leq \lambda_m C_m \text{ for all } m \in \{1, \ldots, N\}. \quad (3)$$

Here, we used the short-hand notation $C_0 = \mathcal{C}(p_{\mathrm{S}} g_{\mathrm{SD}})$ and $C_m = \mathcal{C}(p_{\mathrm{S}} g_{\mathrm{SR}_m})$ for $m \in \{1, \ldots, N\}$ for the capacities of the links originating from the source node S. The optimal rate is obtained for equality in (2)-(3), so that by solving for R_N, we obtain

$$R_N(\mathbf{P}) = \frac{J_N}{1 - \frac{C_0}{C_1} + \sum_{m=1}^{N-1} \left(\frac{1}{C_m} - \frac{1}{C_{m+1}}\right) J_m + \frac{J_N}{C_N}}. \quad (4)$$

The optimal number of decoding relays is given by $N(\mathbf{P}) = \max\{m \in \{1, \ldots, M\} : \lambda_m \leq 1\}$, resulting in the optimal rate $R(\mathbf{P}) = R_{N(\mathbf{P})}(\mathbf{P})$. This rate, according to [9], "is an important measure as, in principle, the application of rateless codes provides the possibility of self-adaptation of the actual code rate arbitrarily close to" $R(\mathbf{P})$. In this paper, we take this theoretically achievable rate as the basis for our energy efficiency analysis. Since this rate can be realized using only the first $N(\mathbf{p})$ relay nodes, the power allocation $s(\mathbf{p}) := (p_1, \ldots, p_N(\mathbf{p}), 0, \ldots, 0)$ achieves the same performance as $\mathbf{p}$.

3. ENERGY EFFICIENCY / PROBLEM FORMULATION

We now introduce the quantities and problems related to energy-efficiency that are central in this paper. Here, we consider energy expenditure at the relay nodes only and assume that the source node transmit power p^{S} is fixed. This assumption is reasonable, for example, in the situation where the source node S is connected to a power supply, whereas the relay nodes are cheap nodes that are deployed in an ad-hoc manner and have limited energy available. Moreover, if the source energy usage also has to be taken into account, the problem left after solving the relay power problem is one-dimensional and can be solved by simple line search or bisection methods. Accordingly, in the following, all quantities will be written as a function of the relay powers $\mathbf{p}$ only instead of $\mathbf{P}$.

For the power allocation $\mathbf{p}$ at the relays, the quantities

$$E_{\mathrm{R}_m}(\mathbf{p}) = \frac{(1 - \lambda_m(\mathbf{p})) p_m}{R(\mathbf{p})} \text{ for } m \in \{1, \ldots, N(\mathbf{p})\}. \quad (5)$$

measure the energy spent for each transmitted bit at each relay node R_m.

If both energy efficiency and spectral efficiency are of interest (as discussed in the introduction), it is reasonable to consider the following problem: given upper bounds $\gamma_{\mathrm{R}_m} > 0, m \in \{1, \ldots, N\}$, on the energy-per-bit at the relay nodes R_m, maximize the transmission rate subject to these constraints on the energy usage. Mathematically, the problem is stated as follows:

$$\textbf{Problem 1}: \underset{\mathbf{p} \in \mathbb{R}^M_{++}}{\text{maximize}} \quad R(\mathbf{p})$$

$$\text{subject to} \quad E_{\mathrm{R}_m}(\mathbf{p}) \leq \gamma_{\mathrm{R}_m}, m \in \{1, \ldots, N(\mathbf{p})\}.$$

In the next section, we propose an algorithmic solution for this problem.

[1] The extension to the case of fading channels without channel state information at the transmitters is possible and subject to ongoing work.

[2] We do not always explicitly state the dependence on the power allocation, and use expressions such as J_i and $J_i(\mathbf{P})$ interchangeably.

4. OPTIMIZATION FOR RELAY EB CONSTRAINTS

We first study a version of Problem 1 in which we relax the optimization constraints at the relay nodes. For this relaxed problem, we give a fixed-point iteration algorithm and prove that, under feasible constraints, it converges to the unique optimal solution of the relaxed problem (which also constitutes a suboptimal solution of Problem 1). As will be described in detail below, the crucial insight here is that R satisfies the properties of a standard interference function [3] [4].

Motivated by the solution for the relaxed problem, we argue that a slight modification of the fixed-point iteration results in an algorithmic solution of Problem 1. Although numerical studies suggest convergence to a fixed point as well, proving convergence remains an open problem. To formulate the relaxed problem, we let

$$\widehat{E}_{\mathrm{R}_m}(\mathbf{p}) = \frac{p_m}{R(\mathbf{p})} \text{ for } m \in \{1, \ldots, M\} \tag{6}$$

and define the relaxed problem as follows:

$$\textbf{Problem 1a}: \underset{\mathbf{p}\in\mathbb{R}^M_{++}}{\text{maximize}} \quad R(\mathbf{p})$$
$$\text{subject to} \quad \widehat{E}_{\mathrm{R}_m}(\mathbf{p}) \leq \gamma_{\mathrm{R}_m}, m \in \{1, \ldots, M\}.$$

The algorithmic solution to Problem 1a is motivated by an analogy to the problem of optimizing under SINR constraints [3] [4], which can be loosely described as follows: SINR expressions are functions of power and given by the ratio of received signal power to interference; if the interference satisfies the standard interference function properties, a simple fixed-point algorithm convergences to a unique fixed point. As a fraction of powers and rate expressions, the constraints in Problem 1a have a similar structure as the SINR constraints. This leads to the idea of using a fixed-point iteration approach for the problem at hand as well. It is worth mentioning that even though R has no direct interpretation of interference here, we can still exploit its structural properties. To be precise, we make use of the following interesting property of the rate function R:

Proposition 1. *The function R is a standard interference function in the relay powers $\mathbf{p}$, i.e., has the following properties:*

1. $R(\mathbf{p}) > 0$ *for all* $\mathbf{p} > \mathbf{0}$ *(positivity)*
2. $\alpha R(\mathbf{p}) > R(\alpha\mathbf{p})$ *for* $\alpha > 1$ *(scalability)*
3. $\mathbf{p} \leq \mathbf{q} \Rightarrow R(\mathbf{p}) \leq R(\mathbf{q})$ *(monotonicity)*

Proof. Monotonicity holds since increasing the power constraint at one relay can only shorten the time the destination node D requires to decode the message. The positivity property is also obvious. For scalability, note that $\alpha J_i(\mathbf{p}) > J_i(\alpha\mathbf{p})$ can easily be derived using $\alpha\mathcal{C}(X) > \mathcal{C}(\alpha x)$ for $\alpha > 1, x > 0$ (cf. [2]). As a consequence, writing $\Delta_m = \frac{1}{C_m} - \frac{1}{C_{m+1}}$, we obtain for $N \in \{1, \ldots, M\}$ and any fixed $\alpha > 1$ and $\mathbf{p}$:

$$\begin{aligned}
\alpha R_N(\mathbf{p}) &= \frac{\alpha J_N(\mathbf{p})}{1 - \frac{C_0}{C_1} + \sum_{m=1}^{N-1} \Delta_m J_m(\mathbf{p}) + \frac{J_N(\mathbf{p})}{C_N}} \\
&> \frac{J_N(\alpha\mathbf{p})}{1 - \frac{C_0}{C_1} + \sum_{m=1}^{N-1} \Delta_m J_m(\mathbf{p}) + \frac{J_N(\mathbf{p})}{C_N}} \\
&\geq \frac{J_N(\alpha\mathbf{p})}{1 - \frac{C_0}{C_1} + \sum_{m=1}^{N-1} \Delta_m J_m(\alpha\mathbf{p}) + \frac{J_N(\alpha\mathbf{p})}{C_N}} \\
&= R_N(\alpha\mathbf{p}).
\end{aligned} \tag{7}$$

Scalability of R can then be shown using continuity of $\alpha R(\mathbf{p}) - R(\alpha\mathbf{p})$ in α. □

Algorithm 1a (choose ϵ small, e.g. $\epsilon = 10^{-6}$)

$\mathbf{q} = 10^{-6} \cdot \mathbf{1}_M$ ▷ initial value is arbitrary
repeat
  $\mathbf{p} \leftarrow \mathbf{q}$
  $\mathbf{q} \leftarrow (\gamma_{\mathrm{R}_1} R(\mathbf{p}), \ldots, \gamma_{\mathrm{R}_M} R(\mathbf{p}))^t$
until $\max_{1\leq k\leq M} |q_k - p_k| < \epsilon \cdot \max_{1\leq l\leq M} p_l$
return $s(\mathbf{q})$

We now show how these properties can be applied in order to solve Problem 1a.

The first important insight is that each solution $\mathbf{p}$ of Problem 1a must be located on the boundary of the constraint set in the following sense: For each optimal power allocation $\mathbf{p}$, there is a power allocation $\widehat{\mathbf{p}}$ such that $R(\widehat{\mathbf{p}}) = R(\mathbf{p})$ and

$$\widehat{E}_{\mathrm{R}_m}(\widehat{\mathbf{p}}) = \gamma_{\mathrm{R}_m} \text{ for all } m \in \{1, \ldots, M\}. \tag{8}$$

This can easily be seen by the following argument: assume a solution $\mathbf{p}$ of Problem 1a such that one of the constraints is not active, i.e. is satisfied with a *strict* inequality. Due to monotonicity of R (property 3. of a standard interference function), we can then immediately find a solution $\mathbf{p}' \geq \mathbf{p}$ with $R(\mathbf{p}') \geq R(\mathbf{p})$ by increasing the corresponding power component until the constraint holds with equality. Note that we invoke essentially the same argument as applied for solving a power minimization problem under QoS constraints [4].

Secondly, we can rewrite (8) as a fixed-point equation

$$\widehat{\mathbf{p}} = (\gamma_{\mathrm{R}_1} R(\widehat{\mathbf{p}}), \ldots, \gamma_{\mathrm{R}_M} R(\widehat{\mathbf{p}}))^t. \tag{9}$$

Now R is as standard interference function. Hence, it follows from the theory of standard interference functions (see e.g. [3] or [4] for details), that if a solution of the fixed-point equation exists (i.e., if Problem 1a is feasible), then it is unique, and the standard *fixed-point iteration* converges to the solution of (9). This leads to Algorithm 1a. We summarize the above considerations in the following Proposition:

Proposition 2. *If Problem 1a is feasible for the Eb constraints $\gamma_{R_m} > 0, m \in \{1, \ldots, N\}$, then Algorithm 1a converges to the optimal solution of Problem 1a, which is also a feasible (but generally suboptimal) power allocation for Problem 1.*

As mentioned above already, the operation and derivation of Algorithm 1a also motivates an algorithmic approach for Problem 1. We elaborate on this in the following. For this, note that $\lambda_m(\mathbf{p}) = \frac{R(\mathbf{p})}{C_m}$, from which

$$\frac{E_{\mathrm{R}_m}(\mathbf{p})}{p_m} = \frac{(1 - \lambda_m(\mathbf{p}))}{R(\mathbf{p})} = \frac{1}{R(\mathbf{p})} - \frac{1}{C_m}, \tag{10}$$

which is monotonically decreasing in each component of $\mathbf{p}$ by monotonicity of R. Hence, applying the same line of argument as for the relaxed problem given above, it follows that $\mathbf{p}$ is a solution of Problem 1 if and only if there exists a power allocation $\widehat{\mathbf{p}}$ that satisfies the fixed-point equation

$$\widehat{\mathbf{p}} = \left(\frac{\gamma_{\mathrm{R}_1} R(\widehat{\mathbf{p}})}{1 - \lambda_1(\widehat{\mathbf{p}})}, \ldots, \frac{\gamma_{\mathrm{R}_{N(\widehat{\mathbf{p}})}} R(\widehat{\mathbf{p}})}{1 - \lambda_{N(\widehat{\mathbf{p}})}(\widehat{\mathbf{p}})}, \widehat{p}_{N(\widehat{\mathbf{p}})+1}, \ldots, \widehat{p}_M \right)^t. \tag{11}$$

Even without proof of uniqueness of a solution of (11) or of convergence of the fixed-point iteration to it, we can still apply it in

Algorithm 1 (choose ϵ small, e.g. $\epsilon = 10^{-6}$ and $maxit \in \mathbb{N}$, e.g. $maxit = 1000$)

$\mathbf{q} = 10^{-6} \cdot \mathbf{1}_M$ ▷ initial value is arbitrary
$n = 0$ ▷ counts number of iterations
repeat
  $n \leftarrow n + 1$
  $\mathbf{p} \leftarrow \mathbf{q}$
  $\mathbf{q} \leftarrow \left(\frac{\gamma_{\mathrm{R}_1} R(\mathbf{p})}{1-\lambda_1(\mathbf{p})}, \dots, \frac{\gamma_{\mathrm{R}_{N(\mathbf{p})}} R(\mathbf{p})}{1-\lambda_{N(\mathbf{p})}(\mathbf{p})}, p_{N(\mathbf{p})+1}, \dots, p_M \right)^t$
until $\max_{1\le k\le M} |q_k - p_k| < \epsilon \cdot \max_{1\le l\le M} p_l$ or $n \geq maxit$
if $n < maxit$ **then**
  return $s(\mathbf{q})$
else
  return “no convergence“
end if

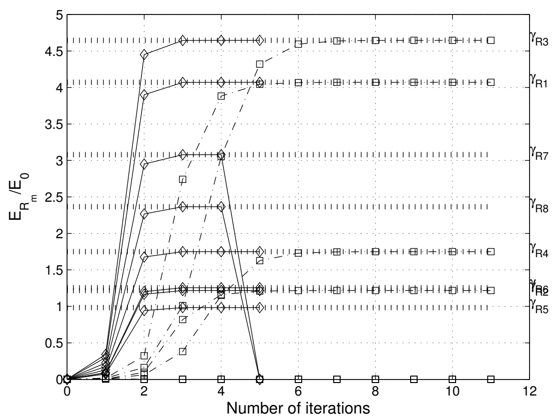

Fig. 2: Convergence of Algorithm 1a (solid line / diamonds) and Algorithm 1 (dashed line / squares) to the required Eb values.

order to try to find a solution of Problem 1. For this, we adjust the fixed-point iteration step according to (11) (and add a maximum iteration counter to ensure termination), resulting in Algorithm 1. Note that numerical evidence points to the fact that Algorithm 1 generally convergences to a solution of Problem 1. Even if this was not the case, we can still apply the following reasonable procedure: Apply both Algorithm 1 and Algorithm 1a.

- If Algorithm 1 converges, compare the two outcomes and use the power allocation outcome with higher rate.
- If Algorithm 1 does not convergence, use the power allocation found by Algorithm 1a as a suboptimal solution.

5. NUMERICAL RESULTS

In this section, we provide some numerical results demonstrating the algorithms presented above.

For the purpose of normalization, we choose $p^S = 1$ and $g_{\mathrm{SD}} = 1$, and the other SNR values g_{PQ} are obtained using the following simple path loss model: For transmit power P_{t}, the received power at distance d from the transmitter is given by $P_{\mathrm{r}} = \left(\frac{d_0}{d}\right)^\beta P_{\mathrm{t}}$. We use $M = 8$ relay nodes, which are arranged as depicted in Figure 1, where the distance between S and D is $5000\,\mathrm{m}$, whereas the two outermost relay nodes are separated by $500\,\mathrm{m}$. For ease of display, we normalize all rate and energy-per-bit values to the corresponding values of the direct link between S and D, which we denote by $R_0 = \mathcal{C}(1)$ and $E_0 = \frac{1}{R_0}$, respectively. The Eb requirements γ_{R_m} were chosen arbitrarily out of the interval $[0, 5 \cdot E_0]$; the used values are displayed next to the right border in Figure 2. Note that for Algorithm 1a, the relays R_7 and R_8 are “turned off” in the final step as a result of applying the function s. Finally, we use $\mathbf{r} = \frac{1}{2}\mathbf{1}_8$ as the combining vector.

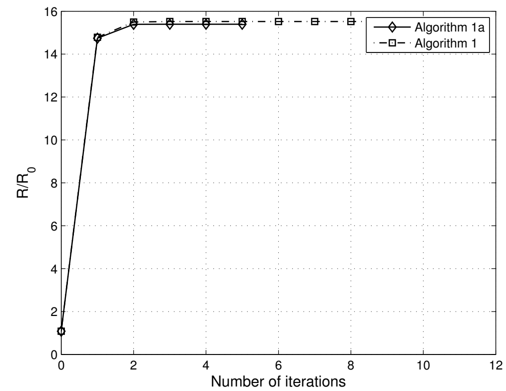

Fig. 3: The rate achieved in each iteration step of Algorithm 1a and Algorithm 1, respectively.

For this setup, Figure 2 displays the Eb values (relative to E_0) $E_{\mathrm{R}_m}(\mathbf{p})/E_0$ and $E_{\mathrm{R}_m}(\mathbf{p})$ at each iteration step of Algorithm 1a and Algorithm 1, respectively. We can see that both algorithms converge very fast to the specified Eb values, i.e. to solutions of the fixed-point equations (9) and (11), respectively.

Figure 3 compares the rate obtained in each iteration step, both for Algorithm 1a and Algorithm 1. It is interesting to see that the performance of Algorithm 1a in terms of rate is almost identical to the performance of Algorithm 1. This supports the potential applicability of using Algorithm 1a as a heuristic suboptimal solution for Problem 1.

6. CONCLUSIONS

In this paper, we studied a relaying network employing the rateless coding scheme proposed in [9]. in maximizing the transmission rate under restrictions on the energy that may be spent per transmitted bit at each of the relay nodes. For a problem with relaxed constraints, we showed that a simple fixed-point iteration converges to the optimal solution. This result was obtained by identifying the rate function as a standard interference function. This approach does not only provide a heuristic (generally suboptimal) solution for the actual problem, but also motivates an algorithmic approach for the solution for it by a slight modification of the fixed-point iteration. While numerical evidence suggests convergence to a unique fixed point for this algorithm as well, we could not prove it yet. This poses an interesting open question for future work.

For future work, we will also consider the case of fading channels, where the transmitters do not have access to channel state information. Other directions for further investigations are the inclusion of energy constraints at the source node and the consideration of hardware energy consumption.

7. REFERENCES

[1] Yan Chen, Shunqing Zhang, Shugong Xu, and G.Y. Li, "Fundamental trade-offs on green wireless networks," *IEEE Communications Magazine*, vol. 49, no. 6, pp. 30–37, June 2011.

[2] J. Bühler and S. Stańczak, "Energy-efficient decode and forward relaying in diamond networks," in *Proc. 50th Annual Allerton Conference on Communication, Control and Computing*, Oct. 2012.

[3] R.D. Yates, "A framework for uplink power control in cellular radio systems," *IEEE Journal on Selected Areas in Communications*, vol. 13, no. 7, pp. 1341–1347, Sep. 1995.

[4] M. Schubert and H. Boche, *Interference Calculus*, Springer Verlag, Berlin Heidelberg, 2012.

[5] M. Luby, "Lt codes," in *Proc. 43rd Ann. IEEE Symp. on Foundations of Computer Science*, 2002, pp. 271–280.

[6] O. Etesami, M. Molkaraie, and A. Shokrollahi, "Raptor codes on symmetric channels," in *Proc. IEEE Int. Symp. Information Theory (ISIT)*, June/July 2004.

[7] J. Castura and Y. Mao, "Rateless coding and relay networks," *IEEE Signal Processing Magazine*, vol. 24, no. 5, pp. 27–35, Sep. 2007.

[8] J. Castura and Y. Mao, "Rateless coding for wireless relay channels," *IEEE Trans. on Wireless Communications*, vol. 6, no. 5, pp. 1638–1642, May 2007.

[9] A. Ravanshid, L. Lampe, and J. B. Huber, "Dynamic decode-and-forward relaying using raptor codes," *IEEE Trans. on Wireless Communications*, vol. 10, no. 5, pp. 1569–1581, May 2011.