LOW-COMPLEXITY KALMAN FILTER-BASED CARRIER FREQUENCY OFFSET ESTIMATION AND TRACKING FOR OFDM SYSTEMS

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***Abstract*—In this paper, an iterative blind estimator for fractional carrier frequency offset (CFO) in orthogonal frequency division multiplexing (OFDM) systems is proposed. The estimator utilizes the null subcarriers transmitted at the edge of the spectrum and does not require any training. In addition, the proposed estimator does not require any prior knowledge of the frequency response of the channel. The problem is formulated using a state-space model, and an extended Kalman filter (EKF) is employed to estimate the CFO iteratively. Simulation results illustrate the enhanced ability of the proposed algorithm, relative to the existing approaches, to estimate and track the CFO even in the presence of high Doppler.**

***Index Terms*—Orthogonal frequency division multiplexing, carrier frequency offset, Kalman filter.**

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is famous for its robustness against the frequency selectivity of wireless channels. Despite its promises, OFDM is vulnerable to synchronization errors. The presence of carrier frequency offset (CFO) between communicating nodes causes loss in orthogonality among subcarriers. This results in introducing inter-carrier interference (ICI) that severely degrades the system performance [1]. Stimulated by the importance of accurate CFO estimation and compensation, extensive work has been done to solve this problem, e.g., [2]–[9].

Schemes for CFO estimation can be divided into two main categories; data-aided and non data-aided (blind). Data-aided schemes utilize the training symbols embedded in the structure of OFDM symbols. A pilot-aided technique is proposed in [2], where a maximum likelihood coarse estimate is used to initialize a recursive least squares algorithm that estimates and tracks the CFO. Preamble-based techniques are proposed in [3], [4], [8]. Using the preamble, a maximum aposteriori (MAP) estimate is derived in [3], which is then used to compensate for the whole frame, where the CFO is assumed constant. This work is further extended in [4], where this MAP estimate is used to initialize a Kalman filter that iteratively refines and tracks the CFO estimate. In [8], a joint time-domain channel and CFO estimation and tracking algorithm is proposed. A preamble is used to obtain initial estimates of the channel and the CFO. These estimates are refined iteratively via an extended Kalman filter (EKF) implemented in a decision-directed algorithm.

Non data-aided schemes rely mainly on exploiting the intrinsic properties of the OFDM signal structure. Conventional CFO estimators that exploit the redundancy of the cyclic prefix (CP) present in OFDM systems depend on the availability of excess CP [5], i.e, a CP beyond the length of the fading channel. In contrast, Liu and Tureli proposed a blind CFO estimation algorithm that utilizes the null subcarriers in [6]. The estimation problem is formulated and solved via the subspace-based MUltiple SIgnal Classification (MUSIC) algorithm [10]. Another subspace-based algorithm, ESPRIT [11], was also used in [7] to solve this problem.

In this paper, we propose a low-complexity iterative algorithm for CFO estimation and tracking in doubly selective channels. An EKF is used to minimize the mean square error (MSE) between the received signal on the null subcarriers caused by the ICI due to CFO, and the ideally received nulls. The algorithm does not require any prior knowledge of the CFO, i.e., it does not need preamble transmission or training. Moreover, channel state information (CSI) is not required since the channel effect is masked by the transmitted nulls. The robustness of the algorithm to the temporal selectivity of the channel is illustrated in the simulations section.

The main difference between the proposed algorithm and [6] is the iterative implementation of the proposed algorithm. That is, the CFO is estimated on an OFDM symbol-by-symbol basis, while [6] requires a large record of received OFDM symbols. We use the performance of the batch estimator in [6] as a benchmark reference for the performance of the proposed estimator in the simulations section. Compared with [8], the proposed algorithm exhibits low computational complexity per iteration ($\mathcal{O}(L^3)$ real multiplications), where L denotes the number of utilized null subcarriers (typically $L = 3$). On the other hand, the algorithm in [8] has high computational complexity ($\mathcal{O}(N^3)$ complex multiplications), where N denotes the discrete Fourier transform (DFT) size (typically $N = 64$). In addition, the proposed algorithm is independent of CSI, while in [8], CSI is required for equalization in every iteration as it is a decision-directed algorithm. The equalization required by [8] elicits another problem, which is the vulnerability to high order constellations, unlike the proposed algorithm which is independent of the constellation order.

II. System Model

In this section, we present a matrix-vector model for a discrete-time baseband-equivalent OFDM system that takes into account the CFO effect. The received time-domain signal due to the mth OFDM symbol is written following [12] as

$$\mathbf{y}(m) = e^{j2\pi\epsilon(m-1)(1+N_g/N)}\mathbf{E}\mathbf{W}_P\mathbf{H}(m)\mathbf{s}(m) + \mathbf{n}(m) \quad (1)$$

where the $P \times 1$ vector $\mathbf{s}(m)$ contains the frequency domain symbols transmitted on the active subcarriers in the mth OFDM symbol. Each OFDM symbol is composed of two different types of subcarriers. The first type is the active subcarriers (data or pilot subcarriers). The second type is null subcarriers that represent the guard band inserted at both edges of the spectrum. The number of active subcarriers is P while the remaining $L = N - P$ subcarriers are nulls. In (1), the $P \times P$ matrix $\mathbf{H}(m)$ contains the frequency response of the channel at the indices of the active subcarriers on its main diagonal elements, and the ICI terms due to the Doppler shift on its off-diagonal elements. Let $\mathcal{A} = \{a_1, a_2, \ldots, a_P\}$ denote the set of active subcarriers indices, while $\mathcal{L} = \{l_1, l_2, \ldots, l_L\}$ is the set of null subcarriers indices. The matrix $\mathbf{W}_p$ is obtained from the $N \times N$ inverse discrete Fourier transform matrix $\mathbf{W}_N$. Thus, $\mathbf{W}_P = [\mathbf{w}_{a_1}, \mathbf{w}_{a_2}, \ldots, \mathbf{w}_{a_P}]$ where $\mathbf{w}_i$ is the ith column of $\mathbf{W}_N$. The time domain received samples are modulated by $\mathbf{E} = \text{diag}\{1, e^{j\frac{2\pi}{N}\epsilon}, \ldots, e^{j\frac{2\pi}{N}(N-1)\epsilon}\}$, where ϵ denotes the normalized CFO, i.e., $\epsilon = NT_s\delta_f$, T_s is the sampling time of the system and δ_f is the CFO in Hertz. The duration of the OFDM symbol is denoted by T, where $T = NT_s$. In addition, N_g denotes the length of CP augmented to the transmitted time domain samples to avoid inter-symbol interference (ISI) between consecutive OFDM symbols, and $\mathbf{n}(m)$ is the time domain additive white Gaussian noise added to the mth received OFDM symbol.

In order to illustrate the effect of CFO, let us consider a perfectly synchronized noiseless system, i.e., $\epsilon = 0$. In this case, the mth received symbol is given by

$$\mathbf{y}(m) = \mathbf{W}_P\mathbf{H}(m)\mathbf{s}(m) \quad (2)$$

Thus, applying DFT demodulation to $\mathbf{y}(m)$, we get

$$\mathbf{W}_P^H\mathbf{y}(m) = \mathbf{H}(m)\mathbf{s}(m) \quad (3)$$

where the superscript H denotes hermitian transpose. In contrast, in the presence of CFO, i.e., $\epsilon \neq 0$, the receiver input is modulated by $\mathbf{E}$, and hence,

$$\mathbf{W}_P^H\mathbf{y}(m) = \mathbf{W}_P^H\mathbf{E}\mathbf{W}_P\mathbf{H}(m)\mathbf{s}(m)e^{j2\pi\epsilon(m-1)(1+\frac{N_g}{N})} \quad (4)$$

Since $\mathbf{W}_P^H\mathbf{E}\mathbf{W}_P \neq \mathbf{I}_P$, where $\mathbf{I}_P$ denotes the $P \times P$ identity matrix, the matrix $\mathbf{E}$ destroys the orthogonality among the subcarriers and introduces ICI. In order to alleviate this ICI, the CFO should be compensated in the time domain before applying the DFT.

Conventional CFO estimation can be performed using the CP by correlating N-spaced samples of the received time-domain vector. Due to the effect of ISI, the first L_{ch} symbols of the CP are discarded where L_{ch} denotes the channel length. The parameter ϵ can then be estimated as

$$\hat{\epsilon}(m) = \frac{1}{2\pi}\angle\left(\frac{1}{N_g - L_{\text{ch}}}\sum_{i=0}^{N_g-L_{\text{ch}}-1} r_{N+i}(m)r_i^*(m)\right) \quad (5)$$

where $*$ denotes the complex conjugate, $\angle$ denotes the angle of a complex variable, and $r_i(m)$ is the ith sample of the received time-domain signal (including the CP) corresponding to the mth OFDM symbol after removing ISI.

III. Proposed Algorithm

In the absence of CFO and noise, the demodulated signal (after applying the DFT) on null subcarrier l_i of the mth OFDM symbol is given by

$$\mathbf{w}_{l_i}^H\mathbf{y}(m) = \mathbf{w}_{l_i}^H\mathbf{W}_P\mathbf{H}(m)\mathbf{s}(m) = 0 \quad (6)$$

However, this is not true in the presence of CFO due to the resulting ICI, where the signal from the neighboring non-zero subcarriers leaks to the null subcarriers. Let the $N \times N$ diagonal matrix $\mathbf{\Phi}$ be defined as [6]

$$\mathbf{\Phi} = \text{diag}\left\{1, e^{j\frac{2\pi}{N}\phi}, \ldots, e^{j\frac{2\pi}{N}(N-1)\phi}\right\} \quad (7)$$

where ϕ is the value used to compensate for the effect of CFO. The matrix $\mathbf{\Phi}$ contains the progressive phase shift caused by the estimated CFO parameter ϕ on its main diagonal. CFO compensation is performed by multiplying the received time-domain OFDM symbol (after cyclic prefix removal) by the inverse of $\mathbf{\Phi}$. Hence, it is obvious that if $\phi = \epsilon$, the received signal on null subcarrier l_i after CFO compensation becomes

$$\mathbf{w}_{l_i}^H\mathbf{\Phi}^{-1}\mathbf{y}(m) = 0. \quad (8)$$

As a measure of the compensation error, we define the MSE in CFO compensation as the power leaking into null subcarriers

$$\text{MSE} = \sum_{i=1}^{L}\mathbb{E}\left\{\mid 0 - \mathbf{w}_{l_i}^H\mathbf{\Phi}^{-1}\mathbf{y}(m)\mid^2\right\} \quad (9)$$

where $\mathbb{E}$ denotes the statistical expectation that is taken over the random variables contained in the vector $\mathbf{y}(m)$. Hence, the parameter ϕ can be estimated by minimizing the power contained in the received signal on each of the null subcarriers. Since the Kalman filter is an MMSE estimator/tracker, this definition of the MSE motivates the use of a Kalman filter to minimize the MSE in (9).

In order to use the Kalman filter, we need a dynamic state model for the CFO. Since it originates from the difference between local oscillators used at both communication ends, and this difference can vary slowly, e.g., due to temperature variations, we model the temporal evolution of the parameter ϕ from the $(m-1)$th OFDM symbol to the mth one as

$$\phi(m) = \phi(m-1) + u(m) \quad (10)$$

where $u(m)$ is a Gaussian zero-mean process with variance σ_u^2. Note that the process noise $u(m)$ allows the Kalman filter to track possible drifts in the CFO.

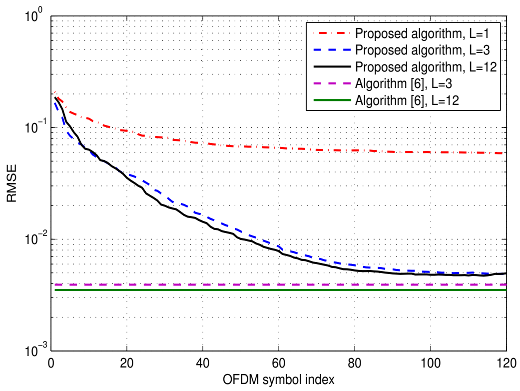

Fig. 1: RMSE of CFO estimate at $E_b/N_0 = 20$dB and $f_dT = 0.025$ for different values of L.

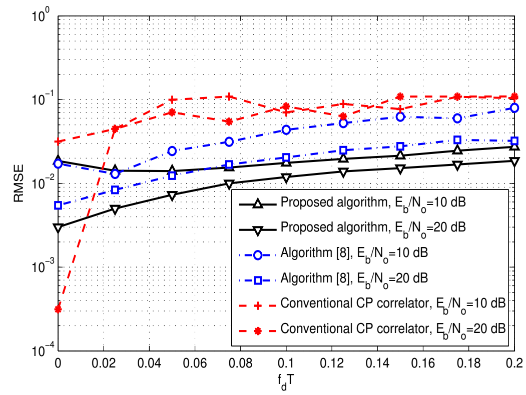

Fig. 2: RMSE of CFO estimate versus normalized Doppler at $E_b/N_0 = 20$dB

Since the Kalman filter minimizes the uncertainty due to the process and measurement noises, we can minimize the MSE in (9) by defining the measurement equation as

$$\mathbf{z}(m) = \mathbf{f}(\phi(m)) + \boldsymbol{\nu}(m) \tag{11}$$

where $\mathbf{z}(m) = \mathbf{0}_{2L}$ which denotes a vector of $2L$ zeros, $\boldsymbol{\nu}(m)$ is a zero-mean Gaussian noise with covariance matrix $\sigma_\nu^2 \mathbf{I}_{2L}$, and the $2L \times 1$ vector $\mathbf{f}(\phi(m))$ is given by

$$\mathbf{f}(\phi(m)) = [\mathbf{f}_1(\phi(m))^T \ \mathbf{f}_2(\phi(m))^T \ \ldots \ \mathbf{f}_L(\phi(m))^T]^T \tag{12}$$

where superscript T denotes transpose. The first and second components of the 2×1 vector $\mathbf{f}_i(\phi(m))$ are, respectively, the real and the imaginary parts of the received signal on null subcarrier l_i (after CFO compensation), i.e., the real and the imaginary parts of $\mathbf{w}_{l_i}^H \boldsymbol{\Phi}^{-1}\mathbf{y}(m)$. Thus, substituting by the elements of $\mathbf{w}_{l_i}$ and $\boldsymbol{\Phi}(\phi(m))$, we can write

$$f_{i_1}(\phi(m)) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \Re\Big(y_k(m)e^{-j\frac{2\pi}{N}l_i k}\Big) \cos\left(\frac{2\pi}{N}\phi(m)k\right) + \Im\Big(y_k(m)e^{-j\frac{2\pi}{N}l_i k}\Big) \sin\left(\frac{2\pi}{N}\phi(m)k\right) \tag{13}$$

$$f_{i_2}(\phi(m)) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \Im\Big(y_k(m)e^{-j\frac{2\pi}{N}l_i k}\Big) \cos\left(\frac{2\pi}{N}\phi(m)k\right) - \Re\Big(y_k(m)e^{-j\frac{2\pi}{N}l_i k}\Big) \sin\left(\frac{2\pi}{N}\phi(m)k\right) \tag{14}$$

where $\Re$ and $\Im$ denote the real and imaginary parts respectively. We can see from (13) and (14) that the measurement equation is nonlinear in the state variable. An approximate solution can be derived through linearization of $\mathbf{f}(\phi(m))$ and subsequent application of the linear Kalman filter. This results in the EKF which we use to adaptively track the CFO.

Let $\hat{\phi}(m|m-1)$ denote the one-step predicted state estimate at time m given knowledge of the process prior to step m. Also, let $\hat{\phi}(m|m)$ denote the posterior state estimate at time m given the measurement $\mathbf{z}(m)$. In addition, let $P(m|m-1)$ and $P(m|m)$ denote, respectively, the prior and posterior MSE associated with the corresponding estimates. Proceeding with the EKF derivation, we linearize $\mathbf{f}(\phi(m))$ around $\hat{\phi}(m|m-1)$

$$\mathbf{f}(\phi(m)) \approx \mathbf{f}(\hat{\phi}(m|m-1)) + \left.\frac{\partial \mathbf{f}}{\partial \phi(m)}\right|_{\phi(m)=\hat{\phi}(m|m-1)} \tag{15}$$

Let

$$\mathbf{F}(m) = \left.\frac{\partial \mathbf{f}}{\partial \phi(m)}\right|_{\phi(m)=\hat{\phi}(m|m-1)} \tag{16}$$

Then, the EKF equations are as follows [13]:
Time update equations (TUE):

$$\hat{\phi}(m|m-1) = \hat{\phi}(m-1|m-1) \tag{17}$$

$$P(m|m-1) = P(m-1|m-1) + \sigma_u^2 \tag{18}$$

Measurement update equations (MUE):

$$\hat{\phi}(m|m) = \hat{\phi}(m|m-1) + \mathbf{K}(m)\Big(\mathbf{0}_{2L} - \mathbf{f}(\hat{\phi}(m|m-1))\Big) \tag{19}$$

$$P(m|m) = [1 - \mathbf{K}(m)\mathbf{F}(m)]P(m|m-1) \tag{20}$$

where the innovation covariance matrix $\mathbf{S}(m)$ and the filter gain $\mathbf{K}(m)$ are given respectively by

$$\mathbf{K}(m) = P(m|m-1)\mathbf{F}(m)^T \mathbf{S}(m)^{-1} \tag{21}$$

$$\mathbf{S}(m) = \sigma_\nu^2 \mathbf{I}_{2L} + \mathbf{F}(m)P(m|m-1)\mathbf{F}(m)^T \tag{22}$$

IV. Simulation Results

In this section, the performance of the proposed algorithm is evaluated in terms of the root mean square error (RMSE) in the CFO estimate. We consider a SISO-OFDM system employing 16-quadrature-amplitude-modulation (QAM) with

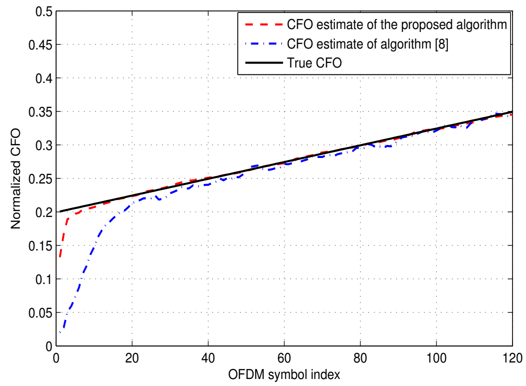

Fig. 3: CFO estimate versus OFDM symbol index at $E_b/N_0 = 20$dB and $f_dT = 0.025$

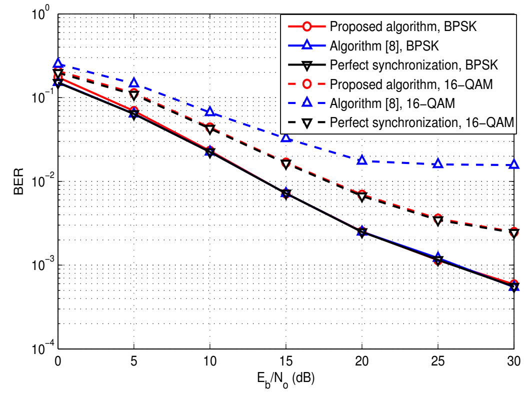

Fig. 4: BER versus E_b/N_0 at $f_dT = 0.025$

$N = 64$, $N_g = N/4$, $N_p = N/16$, and $1/T_s = 20$MHz. The number of null subcarriers is 12. There are 6 consecutive nulls at one spectrum edge, 5 at the other edge, and the DC subcarrier. The transmitted frame consists of 120 OFDM symbols. We consider a Rayleigh channel with 4 taps whose power delay profile is given by $(0, -1.5, -2.5, -3.6)$ dB. The EKF parameters are selected as $\sigma_u^2 = 10^{-8}$ and $\sigma_\nu^2 = 10^{-3}$. Simulation results are averaged over 1000 Monte Carlo runs.

Fig. 1 shows the RMSE of the CFO estimate versus the OFDM symbol index. In each transmitted frame, ϵ is generated randomly in the interval $[-0.5, 0.5]$ to span the whole range of fractional CFO. The filter is initialized by a normalized CFO value of 0 to indicate that we have no prior knowledge of the actual CFO value. The performance of the EKF is investigated for different number of null subcarriers, L. In the curve corresponding to $L = 1$, only the DC subcarrier was used in the estimation process. The next curve shows the performance with $L = 3$, where the considered nulls are the DC and the first null subcarrier on each edge of the spectrum. The third curve depicts the performance of the filter in the case of $L = 12$. We can see from Fig. 1 that selecting $L = 3$ gives a satisfactory performance with much lower complexity than using the whole null band. This is attributed to the fact that the information about the CFO is contained in the amount of ICI that it causes to its neighboring subcarriers. As a result, taking one null subcarrier at each edge of the spectrum captures most of the information about the CFO and additional null subcarriers do not provide significant additional information. Thus, $L = 3$ is used in the remainder of this section. Fig. 1 also shows the RMSE of the batch algorithm [6] that utilizes the available 120 OFDM symbols. We can see that the performance of the EKF is very close to that of the batch estimator.

Next, we investigate the effect of E_b/N_0 and normalized Doppler on the performance of the estimator. We compare the performance of the proposed algorithm with both the conventional cyclic prefix correlator with infinite memory length, and the algorithm presented in [8] whose subsequent simulations are done for a BPSK constellation, unless otherwise stated. Fig. 2 shows the RMSE in the CFO estimate after 100 iterations versus the normalized Doppler at different values of E_b/N_0. The figure shows the robustness of the proposed algorithm to severe temporal variations in the channel in the presence of noise. The curves also show the superiority of our algorithm to [8].

The tracking capability of the Kalman filter is investigated next, where the normalized CFO linearly changes within the transmitted frame starting with a value of 0.2 and ending with 0.35. Fig. 3 shows the estimate produced by the EKF versus OFDM symbol index. We can see from this figure that the estimate closely tracks the true value throughout the transmitted frame.

In Fig. 4 we compute the bit error rate (BER) of a system with a normalized CFO value of 0.2. We consider a perfectly synchronized system, i.e., we use a value of $\phi = 0.2$, as well as CFO-compensated system using the estimated value of the CFO output from the proposed algorithm and algorithm [8]. BPSK and 16-QAM constellations are considered. From the figure, we can notice that the curves corresponding to the BPSK constellation approximately coincide. On the contrary, for the 16-QAM constellation, the curve of the proposed algorithm matches that of the perfectly synchronized system, while algorithm [8] experiences a severe performance degradation.

V. Conclusion

In this paper, a low-complexity fractional CFO estimation and tracking algorithm is proposed. The proposed algorithm is CSI independent as it utilizes the null subcarriers. In addition, it does not require any training. Simulation results show the robustness of the proposed algorithm to noise as well as high Doppler shifts. Moreover, simulations show that the proposed algorithm is capable of accurately tracking the CFO when it changes throughout the transmitted frame.

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