SYNCHRONIZATION AND RANGING BY SCHEDULED BROADCASTING

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ABSTRACT

In this paper we introduce a novel method for synchronization and range estimation in wireless networks, that can also be applied to other broadcast-based networks. The method is based on broadcasting messages by the nodes in a single neighborhood, and estimating their time of arrival at every node. Timing errors and pairwise distances are estimated simultaneously. The number of messages needed in our method is linear to the number of nodes, versus quadratic for commonly used techniques. The algorithm is analyzed by simulation, showing equal performance compared to the state of the art at significantly lower complexity in communication.

Index Terms— Timing, Distance measurement

1. INTRODUCTION

The market force behind Location Based Services (LBS) [1], poses an urgency to improve positioning accuracy in mobile devices, especially for indoor environments [2]. Wireless ad hoc networks are going to play a central role in future communications [3, 4]. A reliable localization solution for wireless ad hoc networks is beneficial to many applications and services, such as packet routing, flexible spectrum use, and positioning of sensor networks, first responders, vehicles and so on. Time-based ranging, that is a widely used technique for precise localization, is very challenging due to strict synchronization requirements. Meanwhile, increasing deployment of real-time services on Packet Switching (PS) networks and worldwide replacement of TDM-based synchronous networks [5] pose an urgent requirement of precise synchronization mechanisms for PS networks. Better synchronization also improves the communication performance and saves costs [6]. In this paper we introduce a novel low complexity method called Scheduled Broadcast Synchronization (SBS) for synchronization and ranging in networks with shared medium. The method is based on broadcasting messages by nodes in the neighborhood, and estimating their Time of Arrival (ToA) at every node. It simultaneously estimates pairwise distances, clock offsets and clock rates.

Synchronization and positioning in wireless networks have been extensively studied in recent years [7, 8]. GPS as a legacy solution for synchronization and localization has limited operation in indoor scenarios and requires extra hardware and power. Any solution for mobile ad hoc network should deal with limited cost, power and communication budget, and unavailability of any infrastructure. Existing synchronization and ranging algorithms (e.g. IEEE 1588 [9] and TPSN [10]) mainly rely on Two-Way Messaging (TWM) where the complexity is quadratic to the number of nodes in communication. The method proposed in this paper has linear complexity in communication. This is a significant benefit. The complexity in communication for a TWM synchronizer is $O(n^2)$ versus $O(n)$ for a broadcast synchronizer [11]. Existing synchronization techniques that exploit the broadcast property of the network e.g. Reference Broadcast Synchronization (RBS) protocol [12, 13], and flooding time synchronization protocol [14] do not have sufficient accuracy for range estimation and localization. Recently, broadcast ranging has been studied for underwater acoustic networks [15, 16]. However, these schemes do not use scheduling in MAC layer and do not compensate for clock skews, resulting in large timing and localization errors when used with radio signals. To the best of our knowledge, there is no existing algorithm for cooperative synchronization and ranging in wireless or wired networks with linear complexity in communication.

The rest of this paper is organized as follows. In Section 2 clocks and measurement errors are modeled. Section 3 introduces SBS measurement and algorithm. Section 4 contains simulation results, and Section 5 summarizes the work.

2. CLOCKS, DELAYS AND TIMESTAMPS

Internal clocks of typical mobile devices are driven by crystal oscillators. Assuming a constant oscillator frequency, the linear model for clock readings may be written as

$$c(t) = \theta + \rho\, t + \xi, \tag{1}$$

where θ is the clock offset (w.r.t reference time), ρ is the clock rate, and ξ is the random error term. We model the offset and rate as deterministic constants; and for ideal (reference) clock we have $\theta = 0$ and $\rho = 1$. The error term ξ includes higher order oscillator instabilities and time quantization error.

Internal delays in mobile devices can be significantly larger than the required precision of time synchronization. We shall decompose the various delays of transmission in Physical layer (PHY) from *node i* to *node j* as (i) *Transmit delay* ε_t^i; (ii) *Propagation delay* $\tau_{i,j}$; and (iii) *Reception delay* ε_r^j [10, 12]. Transmit and Reception delays are times spent in the circuitry of transceiver front-end, that are generally deterministic. Delays in digital circuitry are not constant but depend on the hardware clock rate. We assume that the whole delay in transceiver front-end is determined by the sampling clock, i.e. $\varepsilon_t^i = \varepsilon_{t0}^i/\rho^i$ and $\varepsilon_r^j = \varepsilon_{r0}^j/\rho^j$, where ε_{t0}^i and ε_{r0}^j are nominal values. It is reasonable to assume that these nominal values are known from design stage or calibration. We model propagation delay ($\tau_{i,j}$) as a deterministic constant. The assumption holds if the system (nodes and channel) is stationary over measurement duration (~ 1 ms). If T_i and $T_{i,j}$ are the actual receive and transmit time instances for a message from node i to j, we have

$$T_{i,j} = T_i + \tau_{i,j}, \tag{2}$$

where τ_{ij} is the propagation delay between nodes i, j. The times T_i and $T_{i,j}$ are actual (reference) times, not clock readings. Clock reading for the time of an event that is controlled by the same clock (e.g transmission in PHY layer) has no random quantization error. In this case we ignore the error term ξ in (1), assuming higher-order oscillator instabilities are negligible. The clock reading at *node i* for transmission time is

$$c_i(T_i) = \rho^i \left(T_i - \varepsilon_t^i\right) + \theta^i = \rho^i T_i - \varepsilon_{t0}^i + \theta^i, \tag{3}$$

where any delay between timestamping and actual transmission time is modeled by ε_{t0}^i. Clock reading at *node j* for receive time of that message can be written as

$$c_j(T_{i,j}) = \rho^j T_{i,j} + \varepsilon_{r0}^j + \theta^j + \xi. \tag{4}$$

Timestamping is the process of estimating the time of an event according to the local clock. We define $\widehat{T}_{i,j}^k$ as the timestamp of a message transmitted by *node i* and timestamped at *node j* for measurement index k. So for $i = j$ we have transmit timestamps and for $i \neq j$ we have receive timestamps. When ε_{t0}^i and ε_{r0}^i are known for every node, the algorithm can compensate for them in advance. Therefore, through the rest of this work we will use a simplified model for timestamps that can be written as

$$\widehat{T}_{i,j}^k = \begin{cases} c_i(T_i^k) = \rho^i(T_i^k) + \theta^i & \text{if} \quad i = j \\ c_j(T_{i,j}^k) + \zeta_j = \rho^j(T_{i,j}^k) + \theta^j + \zeta_j & \text{otherwise,} \end{cases} \tag{5}$$

where ζ is the timing (measurement) error of the receiver. It contains thermal noise, multipath error, quantization error and other sources of uncertainty in the system. In this paper we assume a Gaussian model for the error, i.e. $\zeta \sim \mathcal{N}\left(0,\ \sigma^2\right)$. In wireless networks, the assumption is valid for timing estimation (with sub-sample accuracy) if Line-Of-Sight (LOS) path is detectable [17–19].

3. SBS ALGORITHM

In a fully connected broadcast-enabled network (single neighborhood), every message can be received by everyone. Scheduled Broadcast Synchronization (SBS) method consists of a measurement procedure and an estimation algorithm for broadcasting networks. In this procedure, each node broadcasts a message in its turn, and others receive and estimate its ToA. The method employs PHY layer timestamps (5) for transmission and receive times. To ensure that everyone participates in the measurement in a predetermined order, i.e. without competition for channel access, we schedule the nodes in advance. Measurements are continued for at least two complete rounds over all the nodes. Two different ways to design such a schedule are (i) *Synchronous scheme*, scheduled by network time (coarse synchronization) or PHY clock to determine transmission time; and (ii) *Asynchronous scheme*, every node counts the number of messages to determine its own turn. SBS method proposed in this paper is based on asynchronous scheduling. The (modified) MAC layer of each node counts the number of received messages and starts transmission in its own turn. The asynchronous transmission principle of SBS is illustrated in Fig. 1, and can be summarized as follows.

Algorithm 1 SBS Measurement Procedure

1: Plan the schedule of all the nodes in advance.
2: First node starts procedure by sending a message.
3: Each node counts received messages to find its own turn and then broadcasts a message (after delay T_{Di}).
4: Every node timestamps outgoing messages at PHY.
5: Each node receives all messages and timestamps them in PHY by timing estimation.
6: Every node repeats transmission after a wait time T_{Li}.
7: The delays T_{Di} and T_{Li} are deterministic and controlled by each node's sampling clock.
8: Finally, every node broadcasts its timestamps.

Without loss of generality we can assume that *node 1* is the reference node ($\theta^1 = 0, \rho^1 = 1$), and the transmission time of the *node 1* is the beginning of the schedule ($T_1^1 = 0$). We assume that the delays T_{Di} and T_{Li} are controlled by the clock of each node, i.e. $T_{Di} \approx \widetilde{T}_{Di}/\rho^i$ and $T_{Li} \approx \widetilde{T}_{Li}/\rho^i$, where $\widetilde{T}_{Di}$ and $\widetilde{T}_{Li}$ are the nominal values. It is not necessary for $\widetilde{T}_{Di}$ and $\widetilde{T}_{Li}$ to be constant or measured by nodes, as they can be extracted from timestamps, $\widetilde{T}_{Di} = \widehat{T}_{i,i}^k - \widehat{T}_{i-1,i}^k$ and $\widetilde{T}_{Li} = \widehat{T}_{i,i}^k - \widehat{T}_{i-1,i}^k$. The requirement is that T_{Li} should be long enough to avoid collision ($T_{Li} > \sum T_{Di}$). The actual transmit times (for $i > 1$) can be written as (see Fig. 1)

$$\begin{aligned} T_i^k &= T_{i-1}^1 + \tau_{i-1,i} + \widetilde{T}_{Di}/\rho^i + (k-1)\widetilde{T}_{Li}/\rho^i \\ &= (k-1)\widetilde{T}_{Li}/\rho^i + \sum_{n=2}^{i} [\tau_{n-1,n} + \widetilde{T}_{Di}/\rho^n]. \end{aligned} \tag{6}$$

From (5) and (2), transmit and receive timestamps are

$$\begin{aligned} \widehat{T}_{i,i}^k &= \rho^i T_i^k + \theta_i \\ \widehat{T}_{i,j}^k &= \rho^j (T_i^k + \tau_{i,j}) + \theta_j + \zeta_j \end{aligned} \tag{7}$$

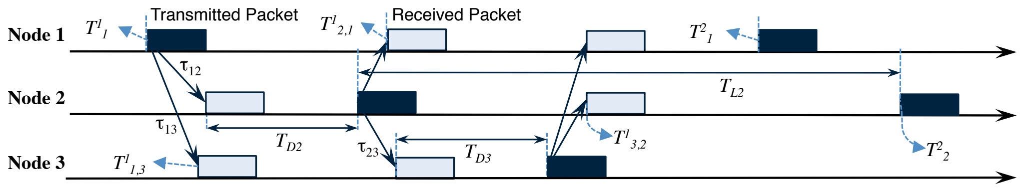

Fig. 1: SBS measurement scheme. Each node broadcasts a message in its pre-scheduled turn and others receiver it. T_i^k and $T_{i,j}^k$ are actual transmit and receive times (not timestamps) of a message from *node i* to *node j* at measurement index k.

The clock rate of *node i* can be directly estimated from the time difference of messages received from *node i.* At least two rounds of measurements should be done. The difference of receive timestamps of *node j* for messages from *node i* is

$$\hat{T}_{i,j}^k - \hat{T}_{i,j}^{k-1} = \rho^j \left(T_i^k - T_i^{k-1}\right) + \zeta_j \approx \rho^j \left(\widetilde{T}_{Li}/\rho^i\right) + \zeta. \quad (8)$$

The approximation is justified by $T_D \ll T_L$. Using two rounds of measurements, all relative clock rates are given by

$$\rho^{ij} = \rho^i/\rho^j = T_{Li}/(\hat{T}_{i,j}^k - \hat{T}_{i,j}^{k-1}), \quad (9)$$

where $j=1$ gives all clock rates (ρ^i) w.r.t to reference clock *node 1*. A better estimator can be formulated using several rounds of measurements. A straight-forward solution is Least Squares (LS) line fitting.

To estimate ranges and clock offsets we subtract receive and transmit timestamps in (7). After re-arranging we have

$$\hat{T}_{i,j}^k - \hat{T}_{i,i}^k - (\rho^j - \rho^i)T_i^k - \zeta_j = \rho^j \tau_{i,j} + \theta_j - \theta_i, \quad (10)$$

where all unknown parameters are on the RHS of the equation, clock rates are known from (9). Propagation time is much shorter than internal delays ($\tau_{i,j} \ll T_D$); so we may simplify the equation (6) as

$$T_i^k \approx (k-1)T_L/\rho^i + T_D \sum_{n=2}^{i} 1/\rho^n, \;\; i > 1. \quad (11)$$

The approximation is valid because the term T_i^k is multiplied by the *difference of clock rates* to compensate for clock skew. Hence, its contribution is small compared to propagation delays. We substitute (11) in (10); then present the LHS of (10) for every i, j, k in a vector form as $\boldsymbol{y} + \boldsymbol{\zeta}$. The vector $\boldsymbol{y}^{q\times1}$ is the measurement vector; and $\boldsymbol{\zeta}^{q\times1}$ is the noise vector. Each entry in $\boldsymbol{y}$ represents a timing measurement between a pair of nodes in one direction, ordered lexicographically by indices (i, j, k). The RHS of (10) for every i, j, k can be represented in a vector form $\boldsymbol{Ax}$, where $\boldsymbol{x} = [\tau_1, \ldots, \tau_m, \theta_2, \ldots, \theta_n]^{\mathrm{T}}$ is the $p \times 1$ column vector of unknown parameters (θ_1 is known). The parameter τ_e is the propagation delay for edge e, ordered lexicographically. It can be shown that, the number of parameters is smaller than the number of measurements, $p < q$, for $n \geq 2$. Matrix $\boldsymbol{A}^{q\times p}$ is the coefficients matrix (populated primarily with zeros) with non-zero elements of ρ^j,+1,−1 as seen in the RHS of (10). Unknown parameters in $\boldsymbol{x}$ can be estimated by solving

$$\boldsymbol{A}^{q\times p}\boldsymbol{x}^{p\times1} - \boldsymbol{y}^{q\times1} = \boldsymbol{\zeta}^{q\times1}. \quad (12)$$

Assuming $\boldsymbol{A}$ has full column rank, the standard LS solution for the problem is given by $\hat{\boldsymbol{x}} = (\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T\boldsymbol{y}$.

A formal model for measurement setup is required in order to describe the coefficients matrix $\boldsymbol{A}$. We model measurement setup with a digraph, where each directed edge represents a message transmitted between two nodes. In order to consider multiple measurements between a pair of nodes, the digraph model is a directed multigraph, i.e. edges can appear more than once. For a digraph on n vertices and m edges, the *Incidence matrix* $\boldsymbol{B}^{n\times m}$ represents the relation between vertices and edges, such that $b_{ij} = -1$ if edge e_j leaves *node i*; $b_{ij} = 1$ if it enters *node i*; and 0 otherwise. The order of the edge labels e_j is the same order as for the measurements vector $\boldsymbol{y}$ in (12). The row rank of Incidence matrix of a connected graph is $n-1$ [20, Chapter 12]. A *Reduced Incidence matrix* $\widetilde{\boldsymbol{B}}$ is obtained by removing one row of the Incidence matrix, i.e. first row by defining *node 1* as the reference node, so it has has *full row rank*. The point formation of the nodes can be modeled with a complete simple graph. For a network with a digraph of m arcs and corresponding simple graph of ℓ edges, we define *Arrangement matrix* $\boldsymbol{C}^{m\times\ell}$ to represent the relation between the edges of the digraph (measurements) and the simple graph (ranges), with entries $c_{ij} = 1$ if simple edge e_i corresponds to directed edge e_j; and 0 otherwise. The edges are ordered lexicographically conforming with the Incidence matrix. For a complete digraph, the Arrangement matrix $\boldsymbol{C}^{m\times q}$ has *full row rank*. The *Modified Arrangement matrix* $\widetilde{C}$ in (13) is obtained by replacing ones by clock rates in the Arrangement matrix, such that $\widetilde{c}_{ij} = \rho_v$ for every directed edge $e_j = (u, v)$. The coefficients matrix $\boldsymbol{A}$ in (12) can be constructed as

$$\boldsymbol{A}^{q\times p} = [\widetilde{\boldsymbol{C}}^{\mathrm{T}} \; \widetilde{\boldsymbol{B}}^{\mathrm{T}}]. \quad (13)$$

In a fully connected network every node can communicate with others in both directions. The measurement setup is a complete digraph; and both $\widetilde{\boldsymbol{B}}$ and $\widetilde{\boldsymbol{C}}$ have full row rank. Moreover, it can be shown that the rows of $\widetilde{\boldsymbol{B}}$ are linearly independent from the rows of $\widetilde{C}$. Hence, the matrix $\boldsymbol{A}^{q\times p}$ in (12) has full column rank; and all parameters can be uniquely estimated. The number of edges for a complete simple graph, i.e. pairwise ranges, is $m = n(n-1)/2$. Hence, the number of parameters in vector $\boldsymbol{x}$ is $p = m + n - 1 = (n-1)(n+2)/2$. For one round of measurements over a Hamiltonian path, the number of edges in the complete digraph model is $2m = n(n-1)$. This gives the number of measurements q in (12).

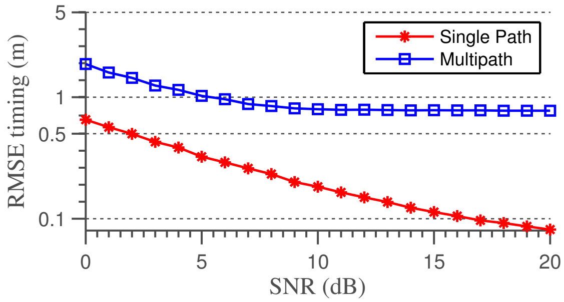

Fig. 2: ToA estimation error of FFT-WLS algorithm for two channel models in IEEE 802.11 OFDM (BW: 20 MHz). Multipath is modeled with no fading, as clusters of paths with log-distance loss model. The plot reflects ranging error for *ideally synchronized clocks*, that is not a contribution of this paper. The result of this simulation is used as measurement error reference in SBS simulations.

4. SIMULATION RESULTS

We simulated IEEE 802.11 OFDM transceiver and a multipath channel model, in order to model ToA estimation error. ToA estimation is done using the FFT-WLS algorithm [19]. Fig. 2 shows the timing error of FFT-WLS algorithm multiplied by the speed of light to represent distance. The timing error reflected in Fig. 2, that largely depends on the density and power of multipath components, is used to model the measurement error for the simulation of the SBS algorithm. The measurement error is modeled as a Gaussian random variable with the variance coming form timing estimation algorithm, see Fig. 2. The error is reciprocal for each pair, i.e. $\zeta_{i\to j} = \zeta_{j\to i}$ because it mainly depends on the channel. In our simulation the values of clock offset and clock rate for different nodes are randomly drawn from continuous uniform distributions, and they are constant for each node during the measurement period. That is $\theta_i \sim \mathcal{U}\,(-25\,\mu s,\ +25\,\mu s)$ and $\rho_i \sim \mathcal{U}\,(1-25\,\text{ppm},\ 1+25\,\text{ppm})$, conforming to the IEEE Std. 802.11-2007 [21]. Fig. 3 shows the ranging error of SBS algorithm (synchronization error multiplied by the speed of light) versus ToA estimation error, and it is compared with Two-Way Ranging (TWR) error. The results are for 5 and 10 nodes distributed uniformly in the area of 10×10 meters. The internal turnaround delay of nodes is $T_D = 100\,\mu s$ and the delay between measurement rounds is $T_L = 10$ ms. It can be observed that the synchronization and ranging performance directly depends on the performance of timing estimation (detection). Fig. 4 shows the performance of clock skew compensation. When clock skew is not compensated for, the error grows linearly with measurement delay. The algorithm performs very well in compensating for clock skew, such that there is no additional error caused by longer measurement delays and increased number of nodes. We assumed that local oscillators are stable, but in practice uncertainties in clock model may limit the total duration of measurement.

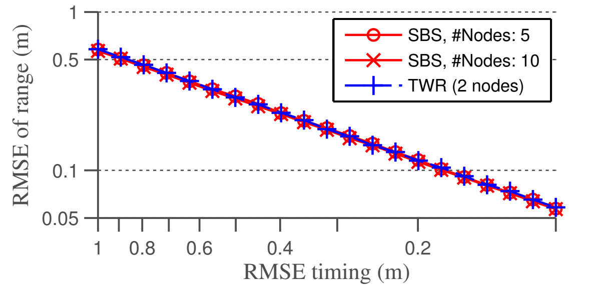

Fig. 3: SBS ranging error (RMS of Mean Absolute Error) versus measurement (symbol timing) error. The error is similar to Two-Way Ranging (TWR) error, and does not depend to the number of nodes. Two rounds of measurements. # Parameters (τ, θ): 15, 55, # Messages in SBS: 10, 20, # Messages in TWR: 40, 180, for 5 and 10 nodes respectively.

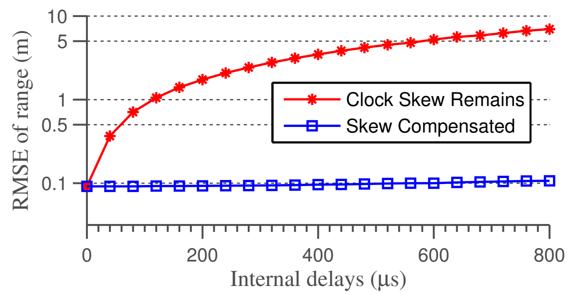

Fig. 4: SBS ranging error versus single measurement duration. The algorithm performs very well in compensating for clock skew. When clock skew is not compensated for, the error significantly grows with measurement delay. # Nodes: 10, # Messages: 20, # Parameters (τ, θ, ρ): 65, Measurement RMSE: 0.18 m, Clock skew: ±25 ppm.

5. CONCLUSIONS

In this paper a novel algorithm was proposed for low complexity synchronization and ranging in wireless ad hoc networks and other broadcast-based networks. The algorithm simultaneously estimates clock parameters and propagation delays with high precision. In comparison to typical synchronization methods, its complexity in communication is $O(n)$ versus $O(n^2)$. Furthermore, it has a much shorter measurement duration, with a simple control mechanism in MAC layer. Short measurement duration is important in maintaining clock stability, stationarity of the system, low probability of interference, and low network traffic. We simulated the algorithm using an implementation of IEEE 802.11 OFDM PHY [21]. However, the SBS method is applicable to other networks as long as symbol timing requirements are met. Next steps include dealing with non-idealities of the system, extending the algorithm for the case of partially connected networks (multiple neighborhoods), and applying it on empirical measurements.

6. REFERENCES

[1] J. t. Sythoff and J. Morrison, *Location-Based Services Market Forecast, 2011-2015.* Pyramid Research, May 2011.

[2] T. S. Perry, "Navigating the great indoors, the smartphone industry is gearing up to get you around when out of sight of gps satellites," *IEEE Spectrum*, vol. 49, no. 11, pp. 20–20, Nov 2012.

[3] M. Conti and S. Giordano, "Multihop ad hoc networking: The reality," *IEEE Communications Magazine*, vol. 45, no. 4, pp. 88–95, 2007.

[4] I. Chlamtac, M. Conti, and J. J.-N. Liu, "Mobile ad hoc networking: imperatives and challenges," *Ad Hoc Networks*, vol. 1, no. 1, pp. 13–64, 2003.

[5] (2012, May) $186 billion to be spent on carrier ethernet equipment over next 5 years. Infonetics Research. Campbell, California, USA. [Online]. Available: http://www.infonetics.com/pr/2012/Carrier-Ethernet-Market-Highlights.asp

[6] C. Lee, J. Kevin Rhee, K. Lee, S.-H. Kim, and S. Lee. (2009, Jul) Broad market potential for time synchronization. San Francisco, CA. [Online]. Available: http://www.ieee802.org/3/time_adhoc/public/jul09/lee_01_0709.pdf

[7] B. Sundararaman, U. Buy, and A. Kshemkalyani, "Clock synchronization for wireless sensor networks: a survey," *Ad Hoc Networks*, vol. 3, no. 3, pp. 281–323, 2005.

[8] H. Liu, H. Darabi, P. Banerjee, and J. Liu, "Survey of wireless indoor positioning techniques and systems," *IEEE Transactions on Systems, Man, and Cybernetics, Part C: Applications and Reviews*, vol. 37, no. 6, pp. 1067 –1080, Nov 2007.

[9] *IEEE Std 1588-2008 (Revision of IEEE Std 1588-2002)*, IEEE Std., 2008.

[10] S. Ganeriwal, R. Kumar, and M. B. Srivastava, "Timing-sync protocol for sensor networks," in *Proceedings of the 1st international conference on Embedded networked sensor systems*, ser. SenSys '03. New York, NY, USA: ACM, 2003, pp. 138–149.

[11] B. Awerbuch, "Complexity of network synchronization," *Journal of the ACM (JACM)*, vol. 32, no. 4, pp. 804–823, 1985.

[12] J. Elson, L. Girod, and D. Estrin, "Fine-grained network time synchronization using reference broadcasts," *SIGOPS Oper. Syst. Rev.*, vol. 36, no. SI, pp. 147–163, Dec 2002.

[13] A. Hu and S. Servetto, "Asymptotically optimal time synchronization in dense sensor networks," in *Proceedings of the 2nd ACM international conference on Wireless sensor networks and applications.* ACM, 2003, pp. 1–10.

[14] M. Maróti, B. Kusy, G. Simon, and Á. Lédeczi, "The flooding time synchronization protocol," in *Proceedings of the 2nd international conference on Embedded networked sensor systems.* ACM, 2004, pp. 39–49.

[15] D. Mirza and C. Schurgers, "Energy-efficient ranging for post-facto self-localization in mobile underwater networks," *IEEE Journal on Selected Areas in Communications*, vol. 26, no. 9, pp. 1697 –1707, Dec 2008.

[16] B. Liu, H. Chen, Z. Zhong, and H. Poor, "Asymmetrical round trip based synchronization-free localization in large-scale underwater sensor networks," *IEEE Transactions on Wireless Communications*, vol. 9, no. 11, pp. 3532 –3542, Nov 2010.

[17] C. Knapp and G. Carter, "The generalized correlation method for estimation of time delay," *IEEE Transactions on Acoustics, Speech and Signal Processing*, vol. 24, no. 4, pp. 320 – 327, Aug 1976.

[18] N. Patwari, J. Ash, S. Kyperountas, I. Hero, A.O., R. Moses, and N. Correal, "Locating the nodes: cooperative localization in wireless sensor networks," *IEEE Signal Processing Magazine*, vol. 22, no. 4, pp. 54 – 69, Jul 2005.

[19] E. T. Abrudan, A. Haghparast, and V. Koivunen, "Time synchronization and ranging in OFDM systems using time-reversal," IEEE Transactions on Instrumentation and Measurement, 2012, to be published.

[20] C. Vasudev, *Graph theory with applications.* New Age International, 2006.

[21] *IEEE Std 802.11-2007 (Revision of IEEE Std 802.11-1999)*, IEEE Std., Jun 2007.