SPACE TIME INTERFERENCE ALIGNMENT SCHEME FOR THE MIMO BC AND IC WITH DELAYED CSIT AND FINITE COHERENCE TIME

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ABSTRACT

Most techniques designed for the the multi-input multiple-output (MIMO) Broadcast Channel (BC) and MIMO Interference Channel (IC) require accurate current and instantaneous channel state information at the transmitter (CSIT). This is not a realistic assumption because of feedback delay. A novel approach by Lee and Heath, space-time interference alignment (STIA), proves that in the underdetermined (overloaded) multi-input single-output (MISO) BC with N_t transmit antennas and $K = N_t + 1$ users N_t (sum) Degrees of Freedom (DoF) are achievable if the feedback delay is not too big, thus disproving the conjecture that any delay in the feedback necessarily causes a DoF loss. However the feedback delay needs to remain less or equal to $\frac{T_c}{N_t+1}$, where T_c is the coherence time. We consider the MIMO BC and show that the use of multi-antenna receivers allows to achieve full (sum) DoF with bigger feedback delay, up to $\frac{T_c}{\frac{N_t}{N_r}+1}$. We also extend this result to the MIMO IC.

Index Terms— Broadcast channel, interference channel, delayed CSIT, interference alignment

1. INTRODUCTION

Interference is a major limitation in wireless networks and the search for efficient ways of transmitting in this context has been productive and diversified [1–3]. Numerous techniques allow the increase of the multiplexing gain. However, most techniques, even promising linear solutions [4,5] rely on perfect current CSIT which is not realistic. Though interesting results have been found concerning imperfect CSIT [6], feedback delay can also be an issue. However, [7] caused a paradigm shift by proposing a scheme (MAT) yielding more than one degree of freedom (DoF) in the MISO BC while relying solely on perfect but completely outdated CSIT.

Since the assumption of totally independent channel variation is overly pessimistic for numerous practical scenarios another scheme was proposed in [8] for the time correlated MISO BC with 2 users. This scheme optimally combines delayed CSIT and current CSIT (both imperfect) but has not been generalized for a larger number of users. Another scheme that simply performs ZF and superposes MAT only during the dead times of ZF has been proposed in [9]. It recovers the results of optimality of [8] for $K = 2$ but is valid for any number of users. It is based on a block fading model but it has been shown that stationary fading can be modeled as a special block fading model in [9, 10].

It was generally believed that any delay in the feedback necessarily causes a DoF loss. However, Lee and Heath in [11] proposed a scheme, hereafter referred to as space-time interference alignment (STIA), that achieves N_t (sum) DoF in the block fading underdetermined MISO BC with N_t transmit antennas and $K = N_t + 1$ users if the feedback delay is small enough ($\leq \frac{T_c}{K}$). We will show that in the case of multi-antenna receivers the full sum DoF can be preserved up to feedback delays of $\frac{T_c}{\frac{N_t}{N_r}+1}$. We will also extend this result to the MIMO Interference Channel (IC).

2. MIMO BC SYSTEM MODEL

We consider a MIMO BC with K users equipped with N_r antennas and a transmitter equipped with N_t antennas. The input-output relationship of the channel at time n for user k is given by

$$\mathbf{y}^{(k)}[n] = \mathbf{H}^{(k)}[n]\mathbf{x}[n] + \mathbf{v}^{(k)}[n] \quad (1)$$

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where $\mathbf{y}^{(k)}[n] = [y_1^{(k)}[n], \cdots, y_{N_r}^{(k)}[n]]^T \in \mathbb{C}^{N_r \times 1}$ is the signal received by user k and more specifically for $i \in [1, N_r]$, $y_i^{(k)}[n]$ is the signal received by the ith antenna of user k. $\mathbf{x}[n] \in \mathbb{C}^{N_t \times 1}$ is the signal sent by the transmitter, $\mathbf{H}^{(k)}[n] = \left[h_{i,j}^{(k)}[n]\right] \in \mathbb{C}^{N_r \times N_t}$ and $\mathbf{v}^{(k)}[n]$ respectively denote the channel matrix and the independent and identically distributed (i.i.d.) Gaussian noise for user k. We consider a block fading model: the channel coefficients are constant for the channel coherence time T_c and change independently between blocks. However the equivalence of this model with the more realistic stationary fading was proven in [10]. T_{fb} is the feedback delay.

The performance metric is (sum) DoF (also called multiplexing gain), it is the prelog of the sum rate. Let $R(P)$ be the ergodic (sum) throughput of a MIMO BC with transmit power P then DoF $= \lim_{P \to \infty} \frac{R(P)}{\log_2(P)}$.

3. SPACE-TIME INTERFERENCE ALIGNMENT FOR THE MIMO BC WITH DELAYED ONLY CSIT

Lee and Heath [11] proposed a scheme (STIA) to achieve N_t DoF in the MISO BC with $K = N_t + 1$ users when $\gamma = \frac{T_{fb}}{T_c} \leq \frac{1}{K}$. This result was unexpected as it was previously conjectured that any delay in the feedback caused a DoF loss, however the condition on γ can become problematic with a large number of transmit antennas (or users). We will see how having receivers with multiple antennas allow to preserve the full sum DoF for a wider range of feedback delays.

We consider the MIMO-BC with N_t transmit antennas, N_r receive antennas and K users such that $K = \frac{N_t}{N_r} + 1$, therefore we assume $\frac{N_t}{N_r}$ to be an integer. Concerning the feedback delay, we are looking for the maximum feedback delay for which we still reach full sum DoF. The borderline case for our scheme is $\gamma = \frac{1}{K} = \frac{N_r}{N_t + N_r}$. In this case the current CSI is known at the transmitter only after the first $T_{fb} = \frac{T_c}{K}$ symbol periods.

3.1. STIA-MIMO Scheme for the MIMO BC

The STIA-MIMO scheme we propose allows the transmission of N_t messages to each of the K users in K symbol periods scattered over K coherence blocks. More precisely, we use symbol periods $\{n_1, n_2, \cdots, n_K\}$ respectively in blocks $\{n+1, n+2, \cdots, n+K\}$. This results in a transient regime for the first K blocks after which we have KT_{fb} instances of the scheme in each block assuring the N_t DoF announced in the stationary state. We now focus on one instance of the STIA-MIMO scheme scattered over blocks $n+1$ to $n+K$ for a $n \geq K$ so that we are in steady state. Only the symbol period n_1 in the first block corresponds to the transmitter not having the current CSI.

Messages $\mathbf{s}^{(k)} = \left[s_1^{(k)}, \cdots, s_{N_t}^{(k)}\right]^T \in \mathbb{C}^{N_t \times 1}$ are intended for user $k, k \in [1, K]$. $\mathbf{H}[n] = \left[\mathbf{H}^{(1)^T}[n], \cdots, \mathbf{H}^{(K)^T}[n]\right]^T \in \mathbb{C}^{KN_r \times N_t}$ represents the channel matrix during block n and $\mathbf{y}[n_j] = \left[\mathbf{y}^{(1)^T}[n_j], \cdots, \mathbf{y}^{(K)^T}[n_j]\right]^T \in \mathbb{C}^{KN_r \times 1}$ the concatenation of the received signals at the receivers during symbol period n_j. Since we are interested in the DoF provided by the scheme, we hereafter omit the noise variables to be concise. The transmitter always sends a combination of all symbols at each symbol period, always the same symbols for an instance of the scheme but with time-varying beamforming matrices $\mathbf{V}^{(k)}[n_j] \in \mathbb{C}^{N_t \times N_t}$

$$\mathbf{x}[n_j] = \sum_{k=1}^{K} \mathbf{V}^{(k)}[n_j]\mathbf{s}^{(k)}.$$

During the first symbol period n_1, the transmitter does not have any information on the current channel state, so for $k \in [1, K]$, $\mathbf{V}^{(k)}[n_1] = \mathbf{I}_{N_t}$, the N_t by N_t identity matrix, is as good as any other matrix of full rank. The transmission scheme is summarized as follows

$$\begin{bmatrix} \mathbf{y}[n_1] \\ \mathbf{y}[n_2] \\ \vdots \\ \mathbf{y}[n_K] \end{bmatrix} = \mathrm{diag}(\mathbf{H}[n+1], \mathbf{H}[n+2], \cdots, \mathbf{H}[n+K])$$
$$* \begin{bmatrix} \mathbf{I}_{N_t} & \cdots & \mathbf{I}_{N_t} \\ \mathbf{V}^{(1)}[n_2] & \cdots & \mathbf{V}^{(K)}[n_2] \\ \vdots & & \vdots \\ \mathbf{V}^{(1)}[n_K] & \cdots & \mathbf{V}^{(K)}[n_K] \end{bmatrix} \begin{bmatrix} \mathbf{s}^{(1)} \\ \mathbf{s}^{(2)} \\ \vdots \\ \mathbf{s}^{(K)} \end{bmatrix} =$$
$$\begin{bmatrix} \mathbf{H}[n+1] & \cdots & \mathbf{H}[n+1] \\ \mathbf{H}[n+2]\mathbf{V}^{(1)}[n_2] & \cdots & \mathbf{H}[n+2]\mathbf{V}^{(K)}[n_2] \\ \vdots & & \vdots \\ \mathbf{H}[n+K]\mathbf{V}^{(1)}[n_K] & \cdots & \mathbf{H}[n+K]\mathbf{V}^{(K)}[n_K] \end{bmatrix} \begin{bmatrix} \mathbf{s}^{(1)} \\ \mathbf{s}^{(2)} \\ \vdots \\ \mathbf{s}^{(K)} \end{bmatrix}$$

The received signal at user i and time n_1 is

$$\mathbf{y}^{(i)}[n_1] = \sum_{k=1}^{K} \mathbf{H}^{(i)}[n+1]\, \mathbf{s}^{(k)} = \mathbf{H}^{(i)}[n+1] \sum_{k=1}^{K} \mathbf{s}^{(k)}. \quad (2)$$

The beamforming matrices are constructed so that the interference alignment is simply done at each receiver by a subtraction of two received signal vectors: $\mathbf{y}^{(i)}[n_j] - \mathbf{y}^{(i)}[n_1], j \in [2, K]$. For user i, at time $n_j, j \in [2, K]$, we have

$$\mathbf{y}^{(i)}[n_j] - \mathbf{y}^{(i)}[n_1] = \sum_{k=1}^{K} \left(\mathbf{H}^{(i)}[n+j]\mathbf{V}^{(k)}[n_j] - \mathbf{H}^{(i)}[n+1] \right) \mathbf{s}^{(k)}$$

so the interferences are aligned if

$$\mathbf{H}^{(i)}[n+j]\mathbf{V}^{(k)}[n_j] - \mathbf{H}^{(i)}[n+1] = \mathbf{0}_{N_t}, \forall i \neq k$$

where $\mathbf{0}_{N_t}$ is the $N_t \times N_t$ null matrix. In other words the beamforming matrices $\mathbf{V}^{(k)}[n_j]$ should transform the channel

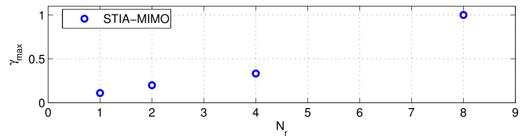

Fig. 1. Maximum value of γ to assure full sum DoF for $N_t = 8$ as a function of N_r.

matrix $\mathbf{H}^{(i)}[n+j]$ in $\mathbf{H}^{(i)}[n+1]$ for $i \neq k$ so that the same interferences are received at any time $n_j, j \in [1, K]$. This is done by defining the beamforming matrix for user k and time n_j as follows

$$\mathbf{V}^{(k)}[n_j] = \begin{bmatrix} \mathbf{H}^{(1)}[n+j] \\ \vdots \\ \mathbf{H}^{(k-1)}[n+j] \\ \mathbf{H}^{(k+1)}[n+j] \\ \vdots \\ \mathbf{H}^{(K)}[n+j] \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{H}^{(1)}[n+1] \\ \vdots \\ \mathbf{H}^{(k-1)}[n+1] \\ \mathbf{H}^{(k+1)}[n+1] \\ \vdots \\ \mathbf{H}^{(K)}[n+1] \end{bmatrix} \quad (3)$$

for $j \in [2, K]$ which assures

$$\begin{bmatrix} \mathbf{y}^{(i)}[n_2] - \mathbf{y}^{(i)}[n_1] \\ \mathbf{y}^{(i)}[n_3] - \mathbf{y}^{(i)}[n_1] \\ \vdots \\ \mathbf{y}^{(i)}[n_K] - \mathbf{y}^{(i)}[n_1] \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{H}^{(i)}[n+2]\mathbf{V}^{(i)}[n_2] - \mathbf{H}^{(i)}[n+1] \\ \mathbf{H}^{(i)}[n+3]\mathbf{V}^{(i)}[n_3] - \mathbf{H}^{(i)}[n+1] \\ \vdots \\ \mathbf{H}^{(i)}[n+K]\mathbf{V}^{(i)}[n_K] - \mathbf{H}^{(i)}[n+1] \end{bmatrix}}_{\mathbf{H}^{(i)}_{eff}} \mathbf{s}^{(i)}$$

and user i can decode $\mathbf{s}^{(i)}$ since the rank of the $N_t \times N_t$ matrix $\mathbf{H}^{(i)}_{eff}$ is almost surely N_t because all channel vectors are independent with a continuous distribution. This scheme allows to transmit a total of $N_t K$ independent data symbols in K channels uses thus yielding N_t sum DoF.

3.2. Performances

For $N_r > 1$ we make use of the multiple receive antennas to widen the range of feedback delays for which we can preserve the full sum DoF up to a feedback delay of

$$\gamma_{max} = \frac{1}{\frac{N_t}{N_r} + 1} = \frac{N_r}{N_t + N_r} \quad (4)$$

of the coherence time T_c, which grows with the number of receive antennas N_r. Smaller value of T_{fb} can be dealt with by doing time sharing between the proposed scheme and simple zero-forcing (ZF) since ZF yields full sum DoF with CSIT.

In Fig. 1 we plot the maximum value of the ratio $\frac{T_{fb}}{T_c}$ for which the full sum DoF is preserved with our scheme as a function of N_r for $N_t = 8$ using (4) except for $N_r = N_t$ because in this case full sum DoF is reached with no CSIT at all by doing the ZF on the receiver side. For example for a coherence time of $T_c = 100$ symbol periods with single antenna receivers the full sum DoF can be achieved with delay up to 11 symbol periods whereas receivers with 2 antennas can deal with feedback delay up to 20 symbol periods. From (4) we see that for a large N_t and small values of N_r, γ grows linearly with N_r as a first order approximation.

The theoretical interest of this solution was already stressed in [11] as the STIA scheme (for the MISO BC) outperforms the MAT-ZF association in terms of DoF. But it is also generally true in terms of net DoF, accounting for feedback and overhead, as it is shown in [12] for the MISO BC.

4. MIMO IC

4.1. System Model

We consider a MIMO IC with K transmitter receiver pairs, with N_t transmit antennas per base station and N_r receive antennas per user subject to the constraint $\frac{N_t}{N_r} \in \mathbb{N}$ as we assume $K = \frac{N_t}{N_r} + 1$. The channel matrix between transmitter i and receiver j at time n is $\mathbf{H}^{(i,j)}[n]$. Since in the MIMO BC we always sent a combination of all symbols it can easily be extended to the MIMO IC, the only difference is that at one receiver the signals intended for different receivers are multiplied by different channel matrices whereas in the BC there are all multiplied by the same channel matrix.

4.2. Scheme

As in the BC we want to construct the beamforming matrices so that the interference alignment is done at each receiver by a subtraction of two received signal vectors: $\mathbf{y}^{(i)}[n_j] - \mathbf{y}^{(i)}[n_1], j \in [2, K]$.

This is done by defining the beamforming matrix for user k and time n_j as follows

$$\mathbf{V}^{(k)}[n_j] = \begin{bmatrix} \mathbf{H}^{(k,1)}[n+j] \\ \vdots \\ \mathbf{H}^{(k,k-1)}[n+j] \\ \mathbf{H}^{(k,k+1)}[n+j] \\ \vdots \\ \mathbf{H}^{(k,K)}[n+j] \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{H}^{(k,1)}[n+1] \\ \vdots \\ \mathbf{H}^{(k,k-1)}[n+1] \\ \mathbf{H}^{(k,k+1)}[n+1] \\ \vdots \\ \mathbf{H}^{(k,K)}[n+1] \end{bmatrix} \quad (5)$$

for $j \in [2, K]$ which assures

$$\begin{bmatrix} \mathbf{y}^{(i)}[n_2] - \mathbf{y}^{(i)}[n_1] \\ \mathbf{y}^{(i)}[n_3] - \mathbf{y}^{(i)}[n_1] \\ \vdots \\ \mathbf{y}^{(i)}[n_K] - \mathbf{y}^{(i)}[n_1] \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{H}^{(i,i)}[n+2]\mathbf{V}^{(i)}[n_2] - \mathbf{H}^{(i,i)}[n+1] \\ \mathbf{H}^{(i,i)}[n+3]\mathbf{V}^{(i)}[n_3] - \mathbf{H}^{(i,i)}[n+1] \\ \vdots \\ \mathbf{H}^{(i,i)}[n+K]\mathbf{V}^{(i)}[n_K] - \mathbf{H}^{(i,i)}[n+1] \end{bmatrix}}_{\mathbf{H}^{(i)}_{eff}} \mathbf{s}^{(i)}$$

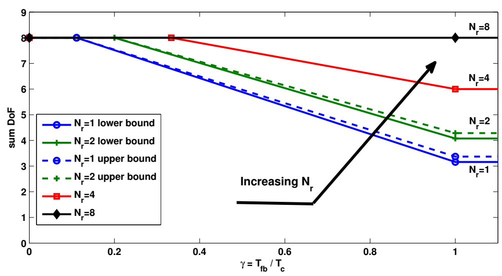

Fig. 2. Time sharing between STIA and MAT in the BC for $N_t = 8$.

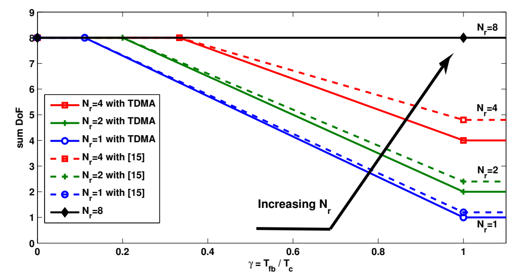

Fig. 3. Time sharing between STIA and TDMA or [15] in the IC for $N_t = 8$.

and user i can decode $\mathbf{s}^{(i)}$ since the rank of the $N_t \times N_t$ matrix $\mathbf{H}_{eff}^{(i)}$ is almost surely N_t because all channel vectors are independent with a continuous distribution. This scheme allows to transmit a total of $N_t K$ independent data symbols in K channels uses thus yielding N_t DoF. This is optimal for the configurations considered, as shown in [13], for $\frac{N_t}{N_r} \in \mathbb{N}$ the maximum sum DoF for the IC with $K = \frac{N_t}{N_r}$ or $K = \frac{N_t}{N_r} + 1$ users is N_t.

4.3. Performances

As in the BC, in the IC we are able to make use of the multiple receive antennas to widen the range of feedback delay for which we can preserve the N_t sum DoF up to

$$\gamma_{max} = \frac{1}{\frac{N_t}{N_r} + 1} = \frac{N_r}{N_t + N_r} \tag{6}$$

of the coherence time T_c which grows with the number of receive antennas N_r. From (6) we see that for a large N_t and small values of N_r, γ grow almost linearly with N_r.

5. LONGER FEEDBACK DELAYS

Feedback delays longer than $\frac{N_r}{N_t+N_r}$ can simply be dealt with by doing time sharing between STIA and a scheme designed for completely outdated CSIT, MAT, as suggested in [11] for the MISO BC case. If [7] is focused on the MISO BC, most of the schemes presented can be extended to the case of receivers with multiple antennas. Theorem 4 in [7] give a lower bound for the sum DoF. Here DoF(.,.,.) refers to MIMO BC whereas $\text{DoF}_1(.,.)$ refers to MISO BC. For multiple antenna receivers the lower bound becomes $DoF^L(N_t, K, N_r) = N_r DoF_1^L(\frac{N_t}{N_r}, K)$ where

$$DoF_1^L(M,K) = \frac{M}{\sum_{i=1}^{K-M} \frac{1}{i} \left(\frac{M-1}{M}\right)^{i-1} + \left(\frac{M-1}{M}\right)^{K-M} \left(\sum_{i=K-M+1}^{K} \frac{1}{i}\right)}$$

in the MIMO BC with N_t transmit antennas and K receivers equipped with N_r antennas. The following upper bound can be derived from [14], $DoF^U(N_t, K, N_r) = N_r DoF_1^U(\frac{N_t}{N_r}, K)$ where

$$DoF_1^U(M,K) = \frac{K}{\frac{1}{\min\{1,M\}} + \frac{1}{\min\{2,M\}} + \cdots + \frac{1}{\min\{K,M\}}}$$

In Fig. 2 lower and upper bounds on the DoF region for $N_t = 8$ and different N_r as a function of γ are given. We observe that increasing the number of receive antennas allows to win on both sides, preserving the full sum DoF on a wider range of γ and also increasing the DoF reached by MAT. For $N_r = 4$ there is only one curve because the upper bound is $DoF^U(8,3,4) = 4\, DoF_1^U(2,3)$ and $DoF_1^U(2,3)$ is achievable according to Theorem 5 in [7]. For $N_r = 8$ there is only one curve because $DoF = \min\{N_t = 8, N_r = 8\} = 8$ is achievable without any CSIT.

A similar strategy can be used in the MIMO IC. With completely delayed CSIT, N_r DoF can be assured by TDMA transmission and for the cases with 3 or more users $\frac{6}{5} N_r$ DoF can be reached relying on delayed output feedback according to [15]. In Fig. 3 lower bounds on the DoF region for $N_t = 8$ and different N_r as a function of γ are given. Note that, for $\gamma \geq 1$, the scheme in [16] could be used as it yields a slightly larger DoF for large K, for example with $K = 9$, $\frac{573}{470} \approx 1.2191$ could be reached instead of the 1.2 of [15] that we used for $K \geq 3$. Again, increasing the number of receive antennas allows to preserve the full sum DoF on a wider range of γ.

6. CONCLUSION

The STIA scheme by Lee and Heath is very interesting as it proved that up to a certain delay in the feedback the full DoF of the MISO BC is still attainable. By extending this result to multiple antenna receivers (MIMO BC), we managed to widen the range of feedback delays for which the full sum DoF can be preserved. We also described an extension to a combination with MAT to cover all possible feedback delays. Finally we provided a minor variation of the scheme to adapt it to the IC, allowing to maintain N_t DoF for the same range of feedback delays as in the BC.

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