ALTERNATING RATE PROFILE OPTIMIZATION IN SINGLE STREAM MIMO INTERFERENCE CHANNELS

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ABSTRACT

We consider a set of transmitter-receiver pairs operating concurrently in the same spectral band. The transmitters and receivers are equipped with multiple antennas and are restricted to apply single stream beamforming. This setting corresponds to the single stream multiple-input multiple-output (MIMO) interference channel. We assume perfect channel state information at the transmitters and the single-user decoding receivers. Efficient operating points in this setting correspond to points on the Pareto boundary of the achievable rate region. Characterizing all Pareto optimal points in the MIMO interference channel is still an unsolved problem. An approach to attain different Pareto optimal points in the MIMO interference channel is rate profile optimization. Given the nonconvexity of the problem, we propose an alternating approach based on successive optimization of the transmit and receive beamforming vectors. For fixed receive beamforming vectors, a solution for the rate profile optimization exists and is solved by a set of convex feasibility problems. For fixed transmit beamforming vectors, we show that the rate profile optimization can be solved by a set of feasibility problems each corresponding to an inverse field of values problem. The convergence of the alternating algorithm is guaranteed to a stationary point of the original problem.

***Index Terms*—** MIMO interference channel; single stream beamforming; rate profile optimization; alternating optimization; Pareto optimality

1. INTRODUCTION

We consider multiple transmitter-receiver pairs (links) which simultaneously utilize the same spectral band. Each transmitter sends a single data stream of useful information to its intended receiver, and each receiver treats the interference signals from unintended transmitters as additive noise. We assume the transmitters and receivers use multiple antennas. This setting corresponds to the single stream multiple-input multiple-output (MIMO) interference channel.

Using models from noncooperative game theory, strategic games between the links are studied in [1, 2]. The conditions for the global stability of the Nash equilibrium in MIMO interference channels are characterized in [1]. In order to improve the performance of the noncooperative links, interference pricing schemes have been studied in [2]. Cooperative mechanisms in the MIMO interference channel are proposed in [3, 4]. In [3], the transmit and receive strategies are updated to balance the egoistic and altruistic behaviour of the links. In [4], different generalisations of interference alignment algorithms are proposed.

Efficient operating points in the MIMO interference channel correspond to the Pareto optimal points. At these points, it is impossible to increase the rate of one link without degrading the performance of at least another link. In the MIMO interference channel, the problem of finding specific Pareto optimal points such as the maximum sum-rate and the proportional fair operating points are NP-hard problems even for single antenna receivers [5]. Also, it has been proven that finding the max-min fair operating point in the MIMO interference channel is a strongly NP-hard problem [6].

Existing approaches to characterize the Pareto optimal points in the MIMO interference channel are reported in [7, 8, 9]. In [7], MIMO interfering broadcast channels are studied and an alternating optimization is proposed to maximize the weighted sum-rate of the system. In [8], a two-user single stream MIMO interference channel is considered and an alternating optimization is proposed where in each iteration the rate of one link is optimized while fixing the rate of the other link. The approach in [8] has the advantage over the weighted sum-rate approach in achieving the points on the nonconvex part of the rate region [10]. The algorithm in [8] is however not extensible to more than two links. Recently in [9], the necessary transmit covariance matrices to achieve all Pareto optimal points in MIMO interference channels are characterized and parameterized.

We consider rate profile optimization for characterizing all Pareto optimal points in the K-user single stream MIMO interference channel rate region. Since this problem is nonconvex, we propose an alternating approach based on iteratively optimizing the transmit and receive beamforming vectors. For fixed receivers, the setting corresponds to the multiple-input single-output (MISO) interference channel and rate profile optimization is solved by a set of convex feasibility problems in [11]. For fixed transmit beamforming vectors, the setting corresponds to a single-input multiple-output (SIMO) interference channel. We show that the rate profile optimization in SIMO channels can be solved by a set of feasibility problems each corresponding to an inverse field of value problem. The alternating rate profile optimization is guaranteed to converge to a stationary point of the original problem.

Outline: In Section 2, we describe the system model and state the problem formulation. In Section 3, rate profile optimization in MISO and SIMO interference channels are studied. The alternating rate profile optimization algorithm is presented with numerical results in Section 4. In Section 5, we draw the conclusions.

2. PRELIMINARIES

Notations: Column vectors and matrices are given in lowercase and uppercase boldface letters, respectively. $\|\boldsymbol{a}\|$ is the Euclidean norm of $\boldsymbol{a} \in \mathbb{C}^N$. $|b|$ is the absolute value of $b \in \mathbb{C}$. $(\cdot)^H$ denotes Hermitian transpose. $\boldsymbol{I}$ is an identity matrix. Define the collection $\{\boldsymbol{a}\}_{\mathcal{K}} := (a_1, \ldots, a_{|\mathcal{K}|})$. $\mathcal{CN}(0, \boldsymbol{A})$ denotes a circularly-symmetric Gaussian complex random vector with covariance matrix $\boldsymbol{A}$.

2.1. System and Channel Model

Consider a set $\mathcal{K} = \{1, \ldots, K\}$ of interfering links. Each transmitter j uses n_j antennas and each receiver k uses m_k receive antennas. The flat fading channel matrix from transmitter j to receiver k is $\boldsymbol{H}_{jk} \in \mathbb{C}^{m_k \times n_j}$. We assume that each transmitter transmits a single data stream to its intended receiver. The beamforming vector used at a transmitter j is $\boldsymbol{w}_j$ from the set

$$\boldsymbol{w}_j \in \mathcal{W}_j := \left\{\boldsymbol{w} \in \mathbb{C}^{n_j} : \|\boldsymbol{w}\|^2 \leq 1\right\}, \tag{1}$$

where we assumed a total transmit power constraint of one without loss of generality. The received signal at receiver k is written as

$$\boldsymbol{y}_k = \sum\nolimits_{j=1}^{K} \boldsymbol{H}_{jk} \boldsymbol{w}_j x_j + \boldsymbol{z}_k, \tag{2}$$

where $x_j \sim \mathcal{CN}(0,1)$ is the signal from transmitter j and $\boldsymbol{z}_k \sim \mathcal{CN}(0, \boldsymbol{I}\sigma^2)$ is additive white Gaussian noise. The signal-to-interference-plus-noise ratio (SINR) at receiver k after equalization with the receive beamforming vector $\boldsymbol{v}_k$ is

$$\phi_k(\boldsymbol{v}_k, \{\boldsymbol{w}\}_{\mathcal{K}}) = \frac{|\boldsymbol{v}_k^H \boldsymbol{H}_{kk} \boldsymbol{w}_k|^2}{\sigma^2 \|\boldsymbol{v}_k\|^2 + \sum_{j \neq k} |\boldsymbol{v}_k^H \boldsymbol{H}_{jk} \boldsymbol{w}_j|^2}. \tag{3}$$

Since the SINR in (3) is not affected by the amplitude of the receive beamforming vector, $\boldsymbol{v}_k$ is chosen from the set

$$\boldsymbol{v}_k \in \mathcal{V}_k := \left\{\boldsymbol{v} \in \mathbb{C}^{m_k} : \|\boldsymbol{v}\|^2 = 1\right\}, \tag{4}$$

where the receive power is normalized to one without loss of generality. The achievable rate of link k is

$$R_k(\boldsymbol{v}_k, \{\boldsymbol{w}\}_{\mathcal{K}}) = \log_2\left(1 + \phi_k(\boldsymbol{v}_k, \{\boldsymbol{w}\}_{\mathcal{K}})\right), \tag{5}$$

where single-user decoding (SUD) is assumed at the receiver. The rate region

$$\mathcal{R} := \{(R_1(\boldsymbol{v}_1, \{\boldsymbol{w}\}_{\mathcal{K}}), \ldots, R_K(\boldsymbol{v}_K, \{\boldsymbol{w}\}_{\mathcal{K}})) \subset \mathbb{R}_+^K : \boldsymbol{w}_k \in \mathcal{W}_k, \boldsymbol{v}_k \in \mathcal{V}_k, k \in \mathcal{K}\}, \tag{6}$$

is the K-dimensional set composed of all achievable rate tuples. The set of Pareto optimal points in $\mathcal{R}$ is defined as [12, p. 14]

$$\mathcal{W}(\mathcal{R}) := \{\boldsymbol{x} \in \mathcal{R} : \text{ there is no } \boldsymbol{y} \in \mathcal{R} \text{ with } \boldsymbol{y} > \boldsymbol{x}\}, \tag{7}$$

with componentwise inequality in (7). At a Pareto optimal point it is impossible to strictly improve the performance of at least one user without degrading the performance of another user. The set of *strong* Pareto optimal points is a subset of the Pareto optimal points in (7) and defined as:

$$\mathcal{P}(\mathcal{R}) := \{\boldsymbol{x} \in \mathcal{R} : \text{ there is no } \boldsymbol{y} \in \mathcal{R} \text{ with } \boldsymbol{y} \geq \boldsymbol{x}, \boldsymbol{y} \neq \boldsymbol{x}\}, \tag{8}$$

where the inequality in (8) is componentwise. An illustration of the Pareto boundary in a two-user rate region is given in Figure 1.

2.2. Problem Formulation

Any point on the Pareto boundary of the rate region $\mathcal{R}$ can be attained by the solution of the rate profile optimization problem[1]:

$$\underset{\{\boldsymbol{v}\}_{\mathcal{K}}, \{\boldsymbol{w}\}_{\mathcal{K}}, \bar{R}}{\text{maximize}} \quad \bar{R} \tag{9a}$$

$$s.t. \quad R_k(\boldsymbol{v}_k, \{\boldsymbol{w}\}_{\mathcal{K}}) = \alpha_k \bar{R}, \quad k \in \mathcal{K}, \tag{9b}$$

$$\boldsymbol{w}_k \in \mathcal{W}_k, \quad \boldsymbol{v}_k \in \mathcal{V}_k \quad k \in \mathcal{K}. \tag{9c}$$

[1]The rate profile approach has been first proposed for broadcast and multiple-access channels in [13] and for MISO interference channels in [14].

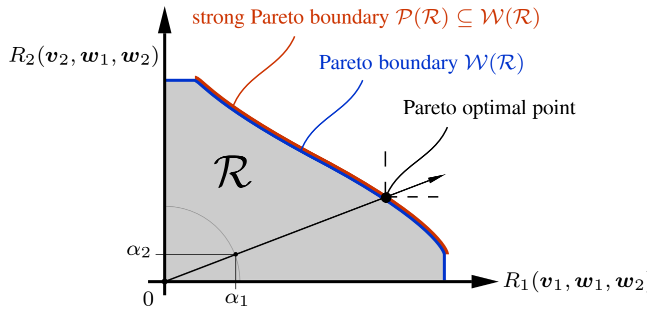


Fig. 1. Illustration of Pareto optimality in a two-user rate region.

In (9), the rate profile $(\alpha_1, \ldots, \alpha_K)$ satisfies $\alpha_k \geq 0, k \in \mathcal{K}$ and $\sum_{k=1}^{K} \alpha_k = 1$. The objective $\bar{R}$ corresponds to the links' sum-rate. The rate profile specifies the direction for a ray starting in the origin of the rate region, and the point of intersection of the ray and the Pareto boundary corresponds to the solution of the optimization problem. An illustration for the solution of a rate profile optimization is given in Figure 1. Solving the optimization problem in (9) for all possible rate profiles achieves all points on the Pareto boundary of the rate region $\mathcal{R}$. The problem in (9) is however not convex, and hence no method is known that can attain its solution efficiently.

We propose to decompose the problem in (9) into two problems which are solved alternatingly. In the first problem, the transmit beamforming vectors which solve the rate profile optimizaiton problem are found for fixed receive beamforming vectors. In the second problem, the receive beamforming vecotrs are optimized for fixed transmit beamforming vectors. Next, we discuss the two problems independently. Later in Section 4, the solutions of the two problems are used to construct the alternating algorithm.

3. OPTIMALITY IN MISO AND SIMO CHANNELS

3.1. Rate Profile Optimization in MISO Interference Channels

In this section, we assume the receive beamforming vectors are fixed. The considered MIMO setting reduces to a MISO interference channel, and the rate region is a subset of $\mathcal{R}$ defined as:

$$\mathcal{R}^{\text{MISO}} := \{(R_1(\boldsymbol{v}_1, \{\boldsymbol{w}\}_{\mathcal{K}}), \ldots, R_K(\boldsymbol{v}_K, \{\boldsymbol{w}\}_{\mathcal{K}})) \in \mathcal{R} : \boldsymbol{w}_k \in \mathcal{W}_k, k \in \mathcal{K}\}. \tag{10}$$

Rate profile optimization in MISO interference channels has been studied in [11]. For a rate profile $(\alpha_1, \ldots, \alpha_K)$ the problem is

$$\underset{\{\boldsymbol{w}\}_{\mathcal{K}}, \bar{R}_{\text{MISO}}}{\text{maximize}} \quad \bar{R}_{\text{MISO}} \tag{11a}$$

$$s.t. \quad R_k(\boldsymbol{v}_k, \{\boldsymbol{w}\}_{\mathcal{K}}) \geq \alpha_k \bar{R}_{\text{MISO}}, \quad k \in \mathcal{K}, \tag{11b}$$

$$\boldsymbol{w}_k \in \mathcal{W}_k, \quad k \in \mathcal{K}. \tag{11c}$$

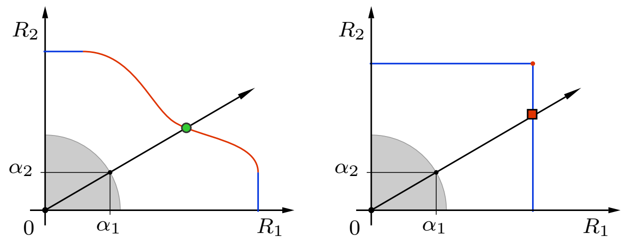
It is shown in [11] that the problem in (11) can be solved by a set of feasibility problems:

$$\text{find} \quad \boldsymbol{w}_1, \ldots, \boldsymbol{w}_K \tag{12a}$$

$$s.t. \quad R_k(\boldsymbol{v}_k, \{\boldsymbol{w}\}_{\mathcal{K}}) \geq \alpha_k t, \quad k \in \mathcal{K}, \tag{12b}$$

$$\boldsymbol{w}_k \in \mathcal{W}_k, \quad k \in \mathcal{K}, \tag{12c}$$

where the parameter $t > 0$ is updated based on a bisection method. The problem in (12) is transformed in [11, Section II.D] to a second order cone programm (SOCP) and solved efficiently. The convex feasibility problem is used in [15] to characterize the necessary



(a) MISO channels (b) SIMO channels

Fig. 2. Illustration of the solutions of rate profile optimization in two-user rate regions of MISO and SIMO interference channels.

beamforming vectors to achieve all Pareto optimal points in MISO multicell settings with different transmit power constraints.

In Figure 2(a), the rate profile optimization is illustrated. Note that only points on the strong Pareto boundary according to (8) are achieved with (11). This is because the constraint in (11b) is an inequality constraint as opposed to the equality constraint in (9b) in the original problem. However, since at least one constraint in (11b) is satisfied with equality with the solution of (11), the value of the objective $\bar{R}_{\text{MISO}}$ is the same as for the optimization problem with the constraints in (9b) replaced with equality constraints.

3.2. Rate Profile Optimization in SIMO Interference Channels

In this section, we assume the transmit beamforming vectors $\{\boldsymbol{w}\}_{\mathcal{K}}$ are fixed. The setting corresponds to a SIMO interference channel. The rate region in the SIMO setting is a subset of the rate region $\mathcal{R}$ in (6) and has the following property.

Proposition 1 *The rate region of a SIMO interference channel is a K-dimensional box:*

$$\mathcal{R}^{\mathit{SIMO}} = \{(r_1, \ldots, r_K) \in \mathcal{R} : r_k \leq R_k(\boldsymbol{v}_k^{\mathit{MMSE}}, \{\boldsymbol{w}\}_{\mathcal{K}}), k \in \mathcal{K}\}, \tag{13}$$

where

$$\boldsymbol{v}_k^{\mathit{MMSE}} = \frac{\left(\sigma^2 \boldsymbol{I} + \sum_{j \neq k} \boldsymbol{H}_{jk} \boldsymbol{w}_j \boldsymbol{w}_j^H \boldsymbol{H}_{jk}^H\right)^{-1} \boldsymbol{H}_{kk} \boldsymbol{w}_k}{\left\| \left(\sigma^2 \boldsymbol{I} + \sum_{j \neq k} \boldsymbol{H}_{jk} \boldsymbol{w}_j \boldsymbol{w}_j^H \boldsymbol{H}_{jk}^H\right)^{-1} \boldsymbol{H}_{kk} \boldsymbol{w}_k \right\|}. \tag{14}$$

Proof: The proof is provided in Appendix A. □

In Figure 2 (b), an illustration of a two-user SIMO interference channel rate region is plotted. The *strong* Pareto boundary according to (8) consists of a single point corresponding to joint minimum mean square error (MMSE) receive beamforming. The rate profile optimization problem in SIMO interference channels is given as[2]:

$$\begin{aligned} \underset{\{\boldsymbol{v}\}_{\mathcal{K}}, \bar{R}_{\text{SIMO}}}{\text{maximize}} \quad & \bar{R}_{\text{SIMO}} && \text{(15a)} \\ \text{s.t.} \quad & R_k(\boldsymbol{v}_k, \{\boldsymbol{w}\}_{\mathcal{K}}) = \alpha_k \bar{R}_{\text{SIMO}}, \quad k \in \mathcal{K} && \text{(15b)} \\ & \boldsymbol{v}_k \in \mathcal{V}_k, \quad k \in \mathcal{K}. && \text{(15c)} \end{aligned}$$

As in the MISO case, the problem in (15) can be solved by a set of feasibility problems:

$$\begin{aligned} \text{find} \quad & \boldsymbol{v}_1, \ldots, \boldsymbol{v}_K && \text{(16a)} \\ \text{s.t.} \quad & R_k(\boldsymbol{v}_k, \{\boldsymbol{w}\}_{\mathcal{K}}) = \alpha_k t, \quad k \in \mathcal{K}. && \text{(16b)} \\ & \boldsymbol{v}_k \in \mathcal{V}_k, \quad k \in \mathcal{K}. && \text{(16c)} \end{aligned}$$

[2]Rate profile optimization in SIMO interference channels has been considered in [16, Section IV.B]. The problem is solved by a set of feasibility problems which are related to SINR balancing problems.

Algorithm 1: Alternating rate profile optimization.

Input: rate profile $\{\boldsymbol{\alpha}\}_{\mathcal{K}} = (\alpha_1, \ldots, \alpha_K)$ and accuracy ϵ
Initialize: $i = 0$; choose random $\{\boldsymbol{v}\}_{\mathcal{K}}^{(0)}$;
repeat
  solve (11) given $\{\boldsymbol{v}\}_{\mathcal{K}}^{(i)}$ to get $\left(\{\boldsymbol{w}\}_{\mathcal{K}}^{(i+1)}, \bar{R}_{\text{MISO}}^{(i+1)}\right)$;
  solve (15) given $\{\boldsymbol{w}\}_{\mathcal{K}}^{(i+1)}$ to get $\left(\{\boldsymbol{v}\}_{\mathcal{K}}^{(i+1)}, \bar{R}_{\text{SIMO}}^{(i+1)}\right)$;
  $i = i + 1$;
until $\bar{R}_{\mathit{SIMO}}^{(i)} - \bar{R}_{\mathit{MISO}}^{(i)} < \epsilon$;
Output: $\{\boldsymbol{w}\}_{\mathcal{K}}^{(i)}, \{\boldsymbol{v}\}_{\mathcal{K}}^{(i)}$

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where the parameter $t \geq 0$ is updated according to a bisection method. The problem in (16) can be rewritten as

$$\begin{aligned} \text{find} \quad & \boldsymbol{v}_1, \ldots, \boldsymbol{v}_K && \text{(17a)} \\ \text{s.t.} \quad & \boldsymbol{v}_k^H \boldsymbol{Q}_k(t) \boldsymbol{v}_k = 0, \quad k \in \mathcal{K}. && \text{(17b)} \\ & \boldsymbol{v}_k \in \mathcal{V}_k, \quad k \in \mathcal{K}, && \text{(17c)} \end{aligned}$$

where the constraint in (16b) has been reformulated to (17b) with

$$\begin{aligned} \boldsymbol{Q}_k(t) = & \boldsymbol{H}_{kk} \boldsymbol{w}_k \boldsymbol{w}_k^H \boldsymbol{H}_{kk}^H \\ & - \left(2^{\alpha_k t} - 1\right)\left(\sigma^2 \boldsymbol{I} + \sum\nolimits_{j \neq k} \boldsymbol{H}_{jk} \boldsymbol{w}_j \boldsymbol{w}_j^H \boldsymbol{H}_{jk}^H\right). \end{aligned} \tag{18}$$

The problem in (17) is called the *inverse field of values problem* [17, 18]. In order to check the feasibility of (17) for a chosen t, it suffices to test whether $0 \in \mathcal{F}(\boldsymbol{Q}_k(t))$, where $\mathcal{F}(\boldsymbol{Q}_k(t))$ is the *field of values* of $\boldsymbol{Q}_k(t)$ defined in (24) in Appendix A. Testing whether $0 \in \mathcal{F}(\boldsymbol{Q}_k(t))$ is equivalent to checking whether zero lies between the smallest and largest eigenvalues of $\boldsymbol{Q}_k(t)$. Hence, in order to determine the optimal t through a bisection approach, the complexity of the feasibility problem in (17) corresponds to the complexity of an eigenvalue decomposition of $\boldsymbol{Q}_k(t)$. After the convergence of the bisection method which determines the optimal t, each receive beamforming vector $\boldsymbol{v}_k$ is determined by the algorithm in [17] which solves the inverse field of values problem. The algorithm requires five eigenvalue decompositions [17, Section 5].

4. ALTERNATING OPTIMIZATION

The alternating rate profile algorithm is described in Algorithm 1. Given the receive beamforming vectors, the transmit beamforming vectors are updated according to Problem (11). The optimized transmit beamforming vectors are then fixed to solve for the receive beamforming vectors according to Problem (15). Note that updating the receive beamforming vectors in each iteration according to MMSE beamforming in (14) is not appropriate since the achieved rate tuple after the receiver optimization would not be along the rate profile direction.

In Algorithm 1, the measures $\bar{R}_{\text{MISO}}^{(i)}$ and $\bar{R}_{\text{SIMO}}^{(i)}$ for MISO and SIMO rate profile optimization correspond to the distances from the origin to the Pareto boundary along the ray in the direction of the rate profile. In each iteration i, an improvement $\bar{R}_{\text{SIMO}}^{(i)} - \bar{R}_{\text{MISO}}^{(i)} \geq 0$ must be achieved. If the improvement is less than an accuracy measure ϵ, then the algorithm terminates. Since the rate region is a bounded set, the alternating algorithm is guaranteed to converge according to the monotone convergence theorem.

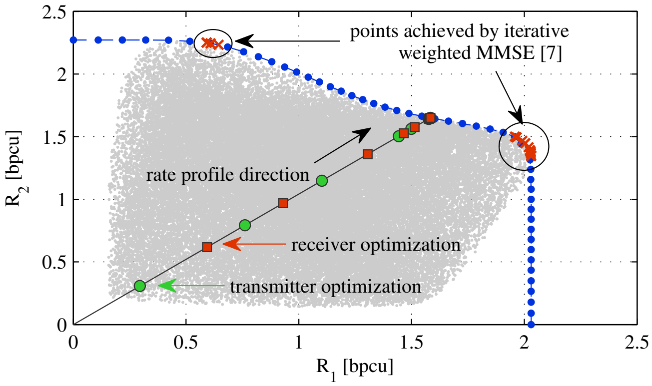

Fig. 3. Plot of two user rate region at 0 dB SNR and two antennas at each transmitter and receiver.

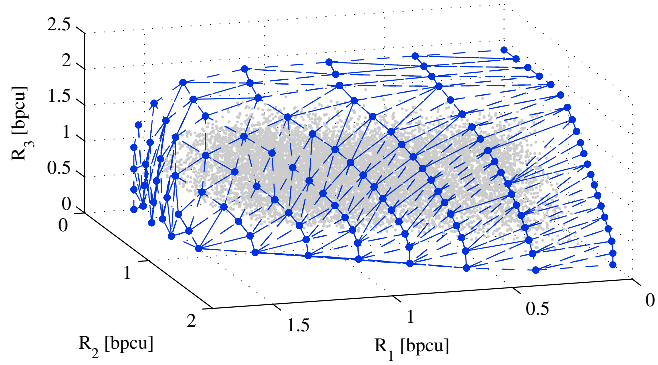

Fig. 4. A plot of a three user rate region at 0 dB SNR and two antennas at each transmitter and receiver.

Theorem 1 *The alternating rate profile optimization in Algorithm* (1) *converges to a stationary point of the original problem in* (9).

Proof: The proof follows similar steps as the proof of [19, Proposition 1], and will be provided in [20]. □

In Figure 3, a two-user rate region is plotted. All transmitters and receivers use two antennas and the signal-to-noise ratio (SNR), defined as $1/\sigma^2$, is 0 dB. Single random channel matrix realisations are selected. The cloud of points in Figure 3 correspond to random norm-one transmit beamforming vectors with MMSE receive beamforming. For a selected rate profile and accuracy measure $\epsilon = 10^{-5}$, the solutions of the transmitter and receiver optimizations are plotted during the alternating optimization. The performance improvement in each iteration can be observed and the alternating optimization terminates at a point very close to the Pareto boundary.

For 50 different samples of rate profiles, we are able to plot the points on the Pareto boundary of the rate region in Figure 3. We choose the initial receive beamforming vectors $\{\boldsymbol{v}\}_{\mathcal{K}}^{(0)}$ according to the dominant left eigenvectors of the corresponding direct channel matrices. However, since the quality of the solution of the alternating algorithm depends on the initial choice of the receive beamforming vectors, it is also possible to run the algorithm for different random initial receive beamforming vectors and then choose the solution with the best performance. In Figure 3, the points plotted with the red cross correspond to the iterative weighted MMSE algorithm proposed in [7]. The approach in [7] optimizes the weighted sum-rate and hence points on the nonconvex part of the Pareto boundary are not achieved. In Figure 4, we plot a three dimensional rate region. The transmitters and receivers use two antennas each, and the SNR is 0 dB. For 121 rate profile samples, the Pareto boundary is obtained using the alternating rate profile algorithm.

5. CONCLUSIONS

We have considered the K-user single stream MIMO interference channel. In this setting, achieving Pareto optimal points through rate profile optimization is a nonconvex problem. We have proposed an alternating optimization algorithm; For fixed receivers, we used existing results on rate profile optimization in MISO channels. For fixed transmitters, we show that rate profile optimization can be solved by a set of feasibility problems each corresponding to an inverse field of value problem. The proposed alternating rate profile optimization is guaranteed to converge to a local optimum.

A. PROOF OF PROPOSITION 1

The achievable rate of link k depends only on its receive beamforming vector $\boldsymbol{v}_k$ (given that $\{\boldsymbol{w}\}_{\mathcal{K}}$ are fixed). Thus, we have to show that the achievable rate of a link k takes values between $[0, R_k(\boldsymbol{v}_k^{\text{MMSE}}, \{\boldsymbol{w}\}_{\mathcal{K}})]$ for all $\boldsymbol{v}_k \in \mathcal{V}_k$. Since the achievable rate of link k in (5) is monotonically increasing in SINR, it is sufficient to analyze the SINR expression in (3) for the proof. We reformulate the SINR of a link k as

$$\phi_k(\boldsymbol{v}_k, \{\boldsymbol{w}\}_{\mathcal{K}}) = \frac{\boldsymbol{v}_k^H \boldsymbol{A}_k \boldsymbol{v}_k}{\boldsymbol{v}_k^H \boldsymbol{B}_k \boldsymbol{v}_k}, \tag{19}$$

where

$$\boldsymbol{A}_k = \boldsymbol{H}_{kk} \boldsymbol{w}_k \boldsymbol{w}_k^H \boldsymbol{H}_{kk}^H \tag{20}$$

$$\boldsymbol{B}_k = \left(\sigma^2 \boldsymbol{I} + \sum\nolimits_{j \neq k} \boldsymbol{H}_{jk} \boldsymbol{w}_j \boldsymbol{w}_j^H \boldsymbol{H}_{jk}^H\right). \tag{21}$$

Since $\boldsymbol{B}_k$ is full rank, we can transform (19) to a Rayleigh-Ritz ratio [21, Chapter 4.2] by substituting $\boldsymbol{v}_k = \boldsymbol{B}_k^{-\frac{1}{2}} \boldsymbol{z}_k$ to get

$$\phi_k(\boldsymbol{v}_k, \{\boldsymbol{w}\}_{\mathcal{K}}) = \frac{\boldsymbol{z}_k^H \boldsymbol{B}_k^{-\frac{1}{2}} \boldsymbol{A}_k \boldsymbol{B}_k^{-\frac{1}{2}} \boldsymbol{z}_k}{\boldsymbol{z}_k^H \boldsymbol{z}_k}. \tag{22}$$

From the Rayleigh-Ritz Theorem for Hermitian matrices [21, Theorem 4.2.2] follows that

$$\phi_k(\boldsymbol{v}_k, \{\boldsymbol{w}\}_{\mathcal{K}}) \in \mathcal{F}\left(\boldsymbol{B}_k^{-\frac{1}{2}} \boldsymbol{A}_k \boldsymbol{B}_k^{-\frac{1}{2}}\right), \tag{23}$$

where the set $\mathcal{F}(\boldsymbol{X})$ is the *field of values* of a matrix $\boldsymbol{X}$ defined as [22, Chapter 1]:

$$\mathcal{F}(\boldsymbol{X}) = \left\{\boldsymbol{x}^H \boldsymbol{X} \boldsymbol{x} \in \mathbb{R} : \|\boldsymbol{x}\|^2 = 1\right\}. \tag{24}$$

The field of values $\mathcal{F}(\boldsymbol{X})$ is a compact convex set. If $\boldsymbol{X}$ is Hermitian, then $\mathcal{F}(\boldsymbol{X}) \subset \mathbb{R}$ with the smallest element and largest element corresponding to the smallest and largest eigenvalues of the matrix $\boldsymbol{X}$, respectively. Since $\boldsymbol{B}_k^{-\frac{1}{2}} \boldsymbol{A}_k \boldsymbol{B}_k^{-\frac{1}{2}}$ is a rank-one positive semi-definite matrix, then the SINR in (22) takes values between zero and the largest eigenvalue of $\boldsymbol{B}_k^{-\frac{1}{2}} \boldsymbol{A}_k \boldsymbol{B}_k^{-\frac{1}{2}}$. With $\boldsymbol{A}_k$ given in (20), the SINR in (22) is maximized by $\boldsymbol{z}_k = \boldsymbol{B}_k^{-\frac{1}{2}} \boldsymbol{H}_{kk} \boldsymbol{w}_k$ which is the dominant (not normalized) eigenvector of $\boldsymbol{B}_k^{-\frac{1}{2}} \boldsymbol{H}_{kk} \boldsymbol{w}_k \boldsymbol{w}_k^H \boldsymbol{H}_{kk}^H \boldsymbol{B}_k^{-\frac{1}{2}}$. Substituting $\boldsymbol{z}_k$ in $\boldsymbol{v}_k = \boldsymbol{B}_k^{-\frac{1}{2}} \boldsymbol{z}_k$ and normalizing $\boldsymbol{v}_k$ we get the expression in (14).

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