A BALANCED PRECODING SCHEME FOR THE TWO-USER SISO X CHANNEL WITH OPTIMAL DOF AND A SMALL POWER OFFSET

Ahmed Medra and Timothy N. Davidson

Department of Electrical and Computer Engineering
McMaster University, Hamilton, Ontario, Canada.

ABSTRACT

In this paper we propose a structured low-latency interference alignment scheme for the two-user single-input single-output (SISO) time-invariant X channel. The scheme is based on asymmetric (non-circular) complex signals generated by linear precoders and linear zero-forcing receivers. In addition to achieving the optimal (sum) degrees of freedom (DoF) of 4/3, the precoders are balanced in the sense that for pairs of symbols the signal-to-noise ratio at the input to the decoder is the same. Furthermore, the sum rate is shown to have a small power offset—in one simple scenario that offset is at least 6 dB smaller than the offset of an existing scheme. The principles that underlie this scheme are extended to a time-varying channel with delayed feedback in which either asymmetric symmetric signalling is used.

Index Terms— interference alignment, degrees of freedom, power offset

1. INTRODUCTION

In the quest to improve the spectral efficiency of wireless networks, the simplicity of signalling schemes that avoid interference is being challenged by the potential of schemes that manage interference to achieve high utilization efficiencies. An intriguing approach to managing interference is that of interference alignment (IA) [1, 2, 3]. The basic principle that underlies (linear) IA is to arrange the signalling in such a way that the interference components arriving at a particular receiver are "aligned" in the sense that they lie in a proper subspace of the space spanned by the received signal. If the portion of the received signal that is dependent of the desired signal has a component that lies outside that "interference subspace", then that component can be extracted without interference using a simple projection (linear receiver). In its natural form, the design of an interference alignment scheme is dependent on the availability of accurate channel state information (CSI), but some of the principles are being extended to other cases; e.g., [4, 5]

In this paper, we develop structured interference alignment schemes for the two-user single-input single-output (SISO) X channel, which is illustrated in Fig. 1. Under the X channel model, each transmitter has independent messages to be sent to each of the receivers. In the assessment of communication schemes for this channel model, and indeed schemes for many other channel models, one of the first steps is often an assessment of the degrees of freedom (DoF), which corresponds to the slope, at high signal-to-noise ratios (SNRs), of the achievable rate in bits-per-channel-use against the SNR in "3-dB units"; i.e., $\log_2(\text{SNR})$. For the two-user SISO time-invariant channel with perfect CSI, the optimal DoF for the sum of the rates of the four messages in Fig. 1 is $4/3$. This optimal value for the DoF can be achieved using asymmetric (non-circular) complex signalling over blocks of three channel uses [6]. In the absence of CSI for the design of the transmitters, the optimal DoF for this system collapses to one.

Although assessing a communication scheme in terms of its DoF is an important step, it is widely acknowledged as a somewhat coarse metric in that it only characterizes the high SNR slope of the achievable rate [7]. As a result, the design guidance provided by an analysis of the DoF is useful, but incomplete. Indeed, in the case of the two-user SISO X channel a large class of systems that achieve the optimal DoF can be obtained by choosing some of the precoders arbitrarily, and then applying the (linear) interference alignment conditions to obtain the other precoders. A refined assessment of the performance, and additional design guidance, can be obtained by seeking an affine approximation of the achievable rate as a function of $\log_2(\text{SNR})$ at high SNR [8]. This approximation takes the form of $S_\infty\big(\log_2(\text{SNR}) - \mathcal{L}_\infty\big)$, where the slope S_∞ is the DoF and $\mathcal{L}_\infty$ is called the power offset. This approximation suggests that if we seek schemes that perform well at high SNRs, then we should search among the schemes that achieve the optimal DoF for schemes that have small power offsets.

Like some existing schemes, the precoding schemes proposed in this paper for the time-invariant two-user SISO X channel achieve the optimal DoF using asymmetric (non-circular) signalling over blocks of three channel uses. However, the proposed schemes have the additional desirable property that pairs of real symbols have the same decision point SNR. This property can be used to simplify the encoding requirements at the transmitter. For this reason, the proposed schemes will be said to be "balanced". In addition to being balanced, we will show by simulation that under a simple i.i.d. Rayliegh fading model, the proposed scheme achieves a reduction in the power offset of more than 6 dB over that of the existing scheme in [6], while maintaining the optimal DoF.

The principles that underlie the proposed design for the time-invariant case with perfect CSI are then applied to the delayed CSI scenario in [5]. We propose both circularly symmetric and asymmetric signalling schemes for this scenario that achieve the optimal DoF and have small power offsets.

2. SYSTEM MODEL

We consider the two-user SISO memoryless X channel illustrated in Fig. 1. In this simple network, each of the transmitters has independent messages to send to each of the receivers. The channel from transmitter i to receiver j is modeled as being linear and memoryless, with the gain at channel use t being $h_{ji}[t]$. The noise at each

This work was supported in part by the Natural Sciences and Engineering Research Council of Canada.

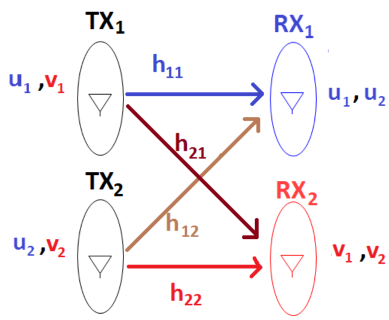

Fig. 1. A model for the two user SISO X channel

receiver is modeled as being additive, white, and independent between receivers. The (complex) output at receiver j at channel use t can be modelled as:

$$y_j[t] = h_{j1}[t]x_1[t] + h_{j2}[t]x_2[t] + z_j[t], \tag{1}$$

where $x_i[t]$ denotes the (complex) symbol transmitted by transmitter i at channel use t, and $z_j[t]$ denotes the additive noise at the jth receiver. In Section 3 we will focus on the case of the time-invariant channel, with $h_{ji}[t] = h_{ji}$, and in Section 4 we will consider the case of the time-varying channel.

The schemes that we will consider in this paper are based on 3 uses of the channel model in (1), which we will nominally index as $t = 1, 2$ and 3. If we define $\mathbf{y}_j = \big[y_j[1], y_j[2], y_j[3]\big]^T$, and define $\mathbf{x}_i$ and $\mathbf{z}_j$ analogously, then we can write the signal at receiver j as

$$\mathbf{y}_j = \mathcal{H}_{j1}\mathbf{x}_1 + \mathcal{H}_{j2}\mathbf{x}_2 + \mathbf{z}_j, \tag{2}$$

where $\mathcal{H}_{ji} = \mathrm{Diag}\,\big(h_{ji}[1], h_{ji}[2], h_{ji}[3]\big)$. As we will consider the possibility of asymmetric signalling [6], it is more convenient to rewrite the model in terms of the real and imaginary parts of the signal. In particular, if we define $\tilde{\mathbf{y}}_j = \big[\mathrm{Re}(y_j[1]), \mathrm{Im}(y_j[1]), \mathrm{Re}(y_j[2]), \mathrm{Im}(y_j[2]), \mathrm{Re}(y_j[3]), \mathrm{Im}(y_j[3])\big]^T$, and define $\tilde{\mathbf{x}}_i$ and $\tilde{\mathbf{z}}_j$ analogously, then the model in (1) can be rewritten as

$$\tilde{\mathbf{y}}_j = \tilde{\mathcal{H}}_{j1}\tilde{\mathbf{x}}_1 + \tilde{\mathcal{H}}_{j2}\tilde{\mathbf{x}}_2 + \tilde{\mathbf{z}}_j, \tag{3}$$

where $\tilde{\mathcal{H}}_{ji} \in \mathbb{R}^{6\times 6}$ is a block diagonal matrix whose kth diagonal block is $\tilde{\mathbf{H}}_{ji}[k] = \begin{bmatrix} \mathrm{Re}(h_{ji}[k]) & -\mathrm{Im}(h_{ji}[k]) \\ \mathrm{Im}(h_{ji}[k]) & \mathrm{Re}(h_{ji}[k]) \end{bmatrix}$.

Since the optimal DoF for the case of a constant channel is $4/3$ [6], and since we would like each message to have equal access to the channel, we will first consider systems in which each transmitter sends a pair of real symbols to each receiver in the three complex channel uses. (See Section 4 for the complex case.) We will denote the symbols destined for receiver 1 by the vectors $\tilde{\mathbf{u}}_1$, $\tilde{\mathbf{u}}_2 \in \mathbb{R}^{2\times 1}$, where the index denotes the transmitter that sends the symbols and the symbols destined for receiver 2 by $\tilde{\mathbf{v}}_1$, $\tilde{\mathbf{v}}_2 \in \mathbb{R}^{2\times 1}$; see Fig. 1.

For simplicity we will focus on linear precoding schemes and hence the transmitted signal vectors $\mathbf{x}_i \in \mathbb{C}^3$ can be written as:

$$\mathbf{x}_1 = \mathbf{F}_{11}\tilde{\mathbf{u}}_1 + \mathbf{F}_{21}\tilde{\mathbf{v}}_1 \tag{4a}$$
$$\mathbf{x}_2 = \mathbf{F}_{12}\tilde{\mathbf{u}}_2 + \mathbf{F}_{22}\tilde{\mathbf{v}}_2 \tag{4b}$$

Since $\tilde{\mathbf{u}}_i$ and $\tilde{\mathbf{v}}_i$ are real vectors, the distributions of the transmitted signals are not necessarily circularly symmetric [6]. If the symbols are normalized so that $E\{\tilde{\mathbf{u}}_i\tilde{\mathbf{u}}_i^T\} = \mathbf{I}$ and $E\{\tilde{\mathbf{v}}_i\tilde{\mathbf{v}}_i^T\} = \mathbf{I}$, then the average transmission power assigned to $\tilde{\mathbf{u}}_i$ is $\mathrm{Tr}\,\big(\mathbf{F}_{1i}\mathbf{F}_{1i}^H\big)/3$ and the average transmission power assigned to $\tilde{\mathbf{v}}_i$ is $\mathrm{Tr}\,\big(\mathbf{F}_{2i}\mathbf{F}_{2i}^H\big)/3$. Returning to the real-valued model, we can also rewrite (4) as:

$$\tilde{\mathbf{x}}_1 = \tilde{\mathbf{F}}_{11}\tilde{\mathbf{u}}_1 + \tilde{\mathbf{F}}_{21}\tilde{\mathbf{v}}_1 \tag{5a}$$
$$\tilde{\mathbf{x}}_2 = \tilde{\mathbf{F}}_{12}\tilde{\mathbf{u}}_2 + \tilde{\mathbf{F}}_{22}\tilde{\mathbf{v}}_2 \tag{5b}$$

where $\tilde{\mathbf{F}}_{ji}$ is defined conformally with $\tilde{\mathbf{x}}_i$.

In terms of achieving the optimal DoF for this scenario, it is sufficient to restrict attention to receivers that separate the estimates of the individual transmitted symbols by using a linear zero-forcing equalizer and then perform separate scalar decoding of the desired symbols. Since

$$\tilde{\mathbf{y}}_1 = \tilde{\mathcal{H}}_{11}\big(\tilde{\mathbf{F}}_{11}\tilde{\mathbf{u}}_1 + \tilde{\mathbf{F}}_{21}\tilde{\mathbf{v}}_1\big) + \tilde{\mathcal{H}}_{12}\big(\tilde{\mathbf{F}}_{12}\tilde{\mathbf{u}}_2 + \tilde{\mathbf{F}}_{22}\tilde{\mathbf{v}}_2\big), \tag{6a}$$
$$\tilde{\mathbf{y}}_2 = \tilde{\mathcal{H}}_{21}\big(\tilde{\mathbf{F}}_{11}\tilde{\mathbf{u}}_1 + \tilde{\mathbf{F}}_{21}\tilde{\mathbf{v}}_1\big) + \tilde{\mathcal{H}}_{22}\big(\tilde{\mathbf{F}}_{12}\tilde{\mathbf{u}}_2 + \tilde{\mathbf{F}}_{22}\tilde{\mathbf{v}}_2\big), \tag{6b}$$

this zero forcing architecture can achieve the optimal DoF using separate ideal Gaussian codebooks for each of the transmitted symbols, if there exist precoders $\tilde{\mathbf{F}}_{ji} \in \mathbb{R}^{6\times 2}$ and equalizers $\tilde{\mathbf{G}}_j \in \mathbb{R}^{4\times 6}$ that satisfy the linear IA conditions, e.g., [3],

$$\mathrm{rank}\big(\tilde{\mathbf{G}}_j\,[\tilde{\mathcal{H}}_{j1}\tilde{\mathbf{F}}_{j1} \quad \tilde{\mathcal{H}}_{j2}\tilde{\mathbf{F}}_{j2}]\big) = 4, \quad j = 1, 2, \tag{7a}$$
$$\tilde{\mathbf{G}}_1\,[\tilde{\mathcal{H}}_{11}\tilde{\mathbf{F}}_{21} \quad \tilde{\mathcal{H}}_{12}\tilde{\mathbf{F}}_{22}] = 0, \tag{7b}$$
$$\tilde{\mathbf{G}}_2\,[\tilde{\mathcal{H}}_{21}\tilde{\mathbf{F}}_{11} \quad \tilde{\mathcal{H}}_{22}\tilde{\mathbf{F}}_{12}.] = 0. \tag{7c}$$

If such matrices exist, then we can permute the entries of $\tilde{\mathbf{G}}_j$ so that the estimates of the transmitted symbols at the inputs to the decoders can be written as:

$$\begin{bmatrix} \hat{\mathbf{u}}_1 \\ \hat{\mathbf{u}}_2 \end{bmatrix} = \tilde{\mathbf{G}}_1\tilde{\mathbf{y}}_1 = \mathrm{Diag}(\alpha_1, \alpha_2, \alpha_3, \alpha_4)\begin{bmatrix} \tilde{\mathbf{u}}_1 \\ \tilde{\mathbf{u}}_2 \end{bmatrix} + \tilde{\mathbf{G}}_1\tilde{\mathbf{z}}_1, \tag{8a}$$
$$\begin{bmatrix} \hat{\mathbf{v}}_1 \\ \hat{\mathbf{v}}_2 \end{bmatrix} = \tilde{\mathbf{G}}_2\tilde{\mathbf{y}}_2 = \mathrm{Diag}(\beta_1, \beta_2, \beta_3, \beta_4)\begin{bmatrix} \tilde{\mathbf{v}}_1 \\ \tilde{\mathbf{v}}_2 \end{bmatrix} + \tilde{\mathbf{G}}_2\tilde{\mathbf{z}}_2, \tag{8b}$$

where, for example, $\alpha_1 = \tilde{\mathbf{g}}_1\tilde{\mathbf{H}}_{11}\tilde{\mathbf{f}}_{11}^1$, where $\tilde{\mathbf{g}}_1$ is the first row of $\tilde{\mathbf{G}}_1$ and $\tilde{\mathbf{f}}_{11}^1$ is the first column of $\tilde{\mathbf{F}}_{11}$. Any set of real matrices $\{\tilde{\mathbf{F}}_{ji}, \tilde{\mathbf{G}}_{ji}\}_{i,j\in\{1,2\}}$ that satisfies the conditions in (7) enables the optimal DoF to be achieved, and a scheme for choosing such matrices was described in [6]. That scheme is based on randomly selecting some of the precoding matrices, followed by an application of the (linear) IA conditions to obtain the others. However, as discussed in the introduction, the DoF can be a rather coarse measure of the system performance [7]. Since we are looking at the scalar decoding schemes, the actual achievable rate depends on the SNRs at the decoder inputs. For receivers 1 and 2, these are:

$$\rho_{1,i} = \frac{\alpha_i^2}{\sigma_1^2\,\big[\tilde{\mathbf{G}}_1\tilde{\mathbf{G}}_1^T\big]_{ii}} \quad \text{and} \quad \rho_{2,i} = \frac{\beta_i^2}{\sigma_2^2\,\big[\tilde{\mathbf{G}}_2\tilde{\mathbf{G}}_2^T\big]_{ii}}, \tag{9}$$

respectively, where σ_j^2 is the variance of the additive white noise at receiver j.

3. THE PROPOSED BALANCED SNR SCHEME

In this section, we propose a precoding scheme for the time-invariant SISO X channel that is structured so that pairs of real symbols have the same transmitted power and the same SNR at the intended receiver, and hence can support same rate. In particular,

$$\rho_{j,1} = \rho_{j,2}, \quad \rho_{j,3} = \rho_{j,4} \quad j \in \{1, 2\}. \tag{10}$$

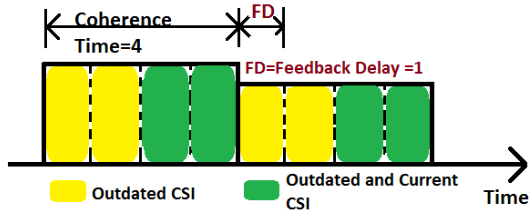

Fig. 2. Delayed CSIT model. (Adapted from [9].)

In order to achieve the optimal $4/3$ DoF, each transmitter sends 4 real symbols in 3 channel uses. Using the notation of the model in (4), we propose to structure the precoders for $\tilde{\mathbf{u}}_1$ and $\tilde{\mathbf{v}}_2$ as:

$$\mathbf{F}_{11} = \begin{bmatrix} f_1 c_1 & 0 \\ f_1 & \mathrm{j} f_1 \\ 0 & \mathrm{j} f_1 c_1 \end{bmatrix} \quad \text{and} \quad \mathbf{F}_{22} = \begin{bmatrix} f_2 & 0 \\ f_2 c_2 & \mathrm{j} f_2 c_2 \\ 0 & \mathrm{j} f_2 \end{bmatrix}, \qquad (11)$$

where $\mathrm{j} = \sqrt{-1}$ and f_i and c_i are complex scalars that can be chosen to optimize other aspects of the system performance subject to a chosen power constraint on the average transmission powers; see Section 2. With this choice of $\mathbf{F}_{11}$ and $\mathbf{F}_{22}$, we design the precoders for $\tilde{\mathbf{u}}_2$ and $\tilde{\mathbf{v}}_1$ so that the interfering terms appear at the receiver in the same subspace. That is, we construct $\mathbf{F}_{12}$ and $\mathbf{F}_{21}$ according to the following interference alignment conditions:

$$\tilde{\mathcal{H}}_{11} \tilde{\mathbf{F}}_{21} \equiv \tilde{\mathcal{H}}_{12} \tilde{\mathbf{F}}_{22} \quad \text{and} \quad \tilde{\mathcal{H}}_{21} \tilde{\mathbf{F}}_{11} \equiv \tilde{\mathcal{H}}_{22} \tilde{\mathbf{F}}_{12}, \qquad (12)$$

where the symbol $\equiv$ is used to denote the condition that the subspaces spanned by the columns of the matrices are the same.

Once the precoding matrices have been determined in this way, the zero-forcing equalizers can be found using (7). In particular, the interference space for receiver 1 is the span of the columns of the 6×4 real matrix $\left[\tilde{\mathcal{H}}_{11} \tilde{\mathbf{F}}_{21},\, \tilde{\mathcal{H}}_{12} \tilde{\mathbf{F}}_{22}\right]$. The interference alignment conditions ensure that the (column) rank of this matrix is 2, and hence one can construct a zero-forcing equalizer $\tilde{\mathbf{G}}_1 \in \mathbb{R}^{4 \times 6}$ from the null space of that matrix.

Due to the specific structure of the precoding matrices on the transmitter side, the received powers for each of the two real elements of $\tilde{\mathbf{u}}_1 = \left[u_1^1,\, u_1^2\right]^T$ are the same. Indeed, using (8), (11) and (12) we have

$$\tilde{\mathbf{g}}_1 \tilde{\mathcal{H}}_{11} \tilde{\mathbf{f}}_{11}^1 = \tilde{\mathbf{g}}_2 \tilde{\mathcal{H}}_{11} \tilde{\mathbf{f}}_{11}^2. \qquad (13)$$

That is, $\alpha_1 = \alpha_2$. Hence, the SNRs and the achievable rates for u_1^1 and u_1^2 are equal. The same conclusion holds for the three other pairs of symbols. Having the ability to decode 8 real symbols in 3 channel uses, we can see that our model still able to achieve the optimal $4/3$ DoF. In the simulation section, we will show that the proposed scheme has a significant performance improvement over the basic scheme in [6].

4. BLOCK FADING CHANNEL WITH DELAYED FEEDBACK

Asymmetric complex signaling is a key ingredient for achieving the DoF of the X channel in case of a time-invariant channel. When the channel is time-varying, the fluctuations in the channel coefficients are sufficient to provide the receivers with linearly independent equations to decode their desired signals and the question that arises is whether or not asymmetric signaling may be useful. We now examine that question in the case of block fading channel model with delayed CSI at the transmitter (CSIT); e.g., [5, 9]; see Fig. 2.

According to this block fading model, the i.i.d. channel coefficients remain constant over the coherence time of the channel, then change independently to new values. The receivers have instantaneous CSI, which is fed back to the transmitters with a delay. At the start of fading block n, the transmitters have the knowledge of the CSI up to the block $(n-1)$, and after the second channel use, they obtain the CSI for block n.

Now, we have to distinguish two different channel scenarios. If the channel is complex, then symmetric signaling can be used to achieve the optimal DoF. However, this DoF cannot be achieved using asymmetric complex signaling because IA is not feasible in that case. On the other hand, if the channel is real, both symmetric and asymmetric signalling can be used to achieve the optimal DoF.

For the case of a complex channel and symmetric signaling, we consider the input signals u_1, u_2, v_1 and v_2 that are complex, Gaussian and circularly symmetric. In this setting, the same 4/3 DoF that was achieved in the case of the time-invariant channel with instantaneous CSI by using asymmetric signalling can be achieved in three channel uses that cross a boundary between fading blocks (e.g., slots 1, 2 and 7 in Figure 2). The transmitters have full knowledge of the outdated and current CSI at the end of the second channel use of each fading block. In the first two channel uses of the scheme, transmitter 1 sends the signal:

$$\begin{bmatrix} f_{11}(1) \\ f_{11}(2) \end{bmatrix} u_1 + \begin{bmatrix} f_{21}(1) \\ f_{21}(2) \end{bmatrix} v_1, \qquad (14)$$

while transmitter 2 sends the signal:

$$\begin{bmatrix} f_{12}(1) \\ f_{12}(2) \end{bmatrix} u_2 + \begin{bmatrix} f_{22}(1) \\ f_{22}(2) \end{bmatrix} v_2, \qquad (15)$$

where $f_{ii}(t)$ may be chosen arbitrarily and $f_{ij}(t)$ are chosen according to:

$$\begin{bmatrix} f_{21}(1) \\ f_{21}(2) \end{bmatrix} = \begin{bmatrix} f_{22}(1) \\ f_{22}(2) \end{bmatrix}, \quad \begin{bmatrix} f_{12}(1) \\ f_{12}(2) \end{bmatrix} = \begin{bmatrix} f_{11}(1) \\ f_{11}(2) \end{bmatrix}. \qquad (16)$$

In the third channel use, transmitter 1 sends $f_{11}(3)u_1 + f_{21}(3)v_1$ and transmitter 2 sends $f_{12}u_2 + f_{22}(3)v_2$ where $f_{12}(3)$ and $f_{21}(3)$ depend on the previous and the current CSI and designed to allow IA at the receivers. The signal that arrives at receiver 1, $\left[y_1(1),\, y_1(2),\, y_1(3)\right]^T$ is:

$$\begin{bmatrix} h_{11}\left(\left[\begin{smallmatrix} f_{11}(1) \\ f_{11}(2) \end{smallmatrix}\right] u_1 + \left[\begin{smallmatrix} f_{21}(1) \\ f_{21}(2) \end{smallmatrix}\right] v_1\right) + h_{12}\left(\left[\begin{smallmatrix} f_{22}(1) \\ f_{22}(2) \end{smallmatrix}\right] v_2 + \left[\begin{smallmatrix} f_{12}(1) \\ f_{12}(2) \end{smallmatrix}\right] u_2\right) \\ g_{11}\big(f_{11}(3)u_1 + f_{21}(3)v_1\big) + g_{12}\big(f_{22}(3)v_2 + f_{12}(3)u_2\big) \end{bmatrix} \qquad (17)$$

where g_{ji} is the channel coefficient from transmitter i to receiver j during the third channel use. Since the channel gains h_{ji} and g_{ji} are complex scalars, the following choices for $f_{ij}(3)$ are sufficient for the decodability of u_1 and u_2 at receiver 1 and v_1 and v_2 at receiver 2 and hence to achieve the optimal DoF:

$$f_{21}(3) = \frac{h_{11} f_{21}(2) g_{12} f_{22}(3)}{h_{12} f_{22}(2) g_{11}}, \quad f_{12}(3) = \frac{h_{22} f_{12}(2) g_{21} f_{11}(3)}{h_{21} f_{11}(2) g_{22}}. \qquad (18)$$

When dealing with the real channel case, asymmetric signaling can be used to achieve the optimal DoF in a delayed CSIT model. The precoding process is quite similar to the symmetric signaling case, but each precoder is now a matrix with two columns. For consistency with the previous case, we denote the (t, k)th entry of $\mathbf{F}_{ij}$ as $f_{ij}^k(t)$. Following the above approach, the transmitters choose the entries in the first and second rows of $\mathbf{F}_{11}$ and $\mathbf{F}_{22}$ arbitrarily. These entries correspond to the first two channel uses in which the CSI is unknown. The entries of the first two rows of the other precoding matrices are chosen so that

$$\begin{bmatrix} f_{21}^k(1) \\ f_{21}^k(2) \end{bmatrix} = \begin{bmatrix} f_{22}^k(1) \\ f_{22}^k(2) \end{bmatrix}, \quad \begin{bmatrix} f_{12}^k(1) \\ f_{12}^k(2) \end{bmatrix} = \begin{bmatrix} f_{11}^k(1) \\ f_{11}^k(2) \end{bmatrix}, \quad k \in \{1,2\}. \tag{19}$$

At the third channel use, the following conditions must hold in order for IA to be feasible:

$$f_{21}^k(3) = \frac{h_{11} f_{21}^k(2) g_{12} f_{22}^k(3)}{h_{12} f_{22}^k(2) g_{11}}, \; f_{12}^k(3) = \frac{h_{22} f_{12}^k(2) g_{21} f_{11}^k(3)}{h_{21} f_{11}^k(2) g_{22}}. \tag{20}$$

The basic scheme described above is DoF optimal, but its performance can be greatly improved by imposing structure on the precoders that are arbitrarily chosen. To achieve the attractive properties of the balanced SNR scheme, we suggest the following structure:

$$\mathbf{F}_{11} = \begin{bmatrix} c_1 & 0 \\ 0 & \mathrm{j}c_1 \\ c_1 & \mathrm{j}c_1 \end{bmatrix} \quad \text{and} \quad \mathbf{F}_{22} = \begin{bmatrix} c_2 & 0 \\ 0 & \mathrm{j}c_2 \\ c_2 & \mathrm{j}c_2, \end{bmatrix}, \tag{21}$$

where c_1 and c_2 are complex scalars that can be chosen arbitrarily. The design of the remaining precoders follows the standard requirements in (7). In order to achieve equal pairwise decision point SNRs at the intended receivers, each receiver should exchange the order of the second and the third received signals before processing. For example, the signal at receiver 1 can be written as:

$$\tilde{\mathbf{y}}_1 = \begin{bmatrix} \tilde{\mathbf{y}}_1(1) \\ \tilde{\mathbf{y}}_1(2) \\ \tilde{\mathbf{y}}_1(3) \end{bmatrix} = \tilde{\mathcal{H}}_{11}\big(\tilde{\mathbf{F}}_{11}\tilde{\mathbf{u}}_1 + \tilde{\mathbf{F}}_{21}\tilde{\mathbf{v}}_1\big) + \tilde{\mathcal{H}}_{12}\big(\tilde{\mathbf{F}}_{12}\tilde{\mathbf{u}}_2 + \tilde{\mathbf{F}}_{22}\tilde{\mathbf{v}}_2\big) \tag{22}$$

where $\tilde{\mathbf{y}}_1 \in \mathbb{R}^{6\times1}$ and $\tilde{\mathcal{H}}_{ji} \in \mathbb{R}^{6\times6}$ is a block diagonal matrix whose diagonal blocks are $\{\tilde{\mathbf{H}}_{ji}, \tilde{\mathbf{H}}_{ji}, \tilde{\mathbf{G}}_{ji}\}$. When linear zero forcing equalization is applied, the signal at the decoder inputs is

$$\mathbf{K}_{11}\big(\overline{\mathbf{F}}_{11}\tilde{\mathbf{u}}_1 + \overline{\mathbf{F}}_{21}\tilde{\mathbf{v}}_1\big) + \mathbf{K}_{12}\big(\overline{\mathbf{F}}_{12}\tilde{\mathbf{u}}_2 + \overline{\mathbf{F}}_{22}\tilde{\mathbf{v}}_2\big), \tag{23}$$

where $\mathbf{K}_{ji} \in \mathbb{R}^{6\times6}$ is a block diagonal matrix whose diagonal blocks are $\{\tilde{\mathbf{H}}_{ji}, \tilde{\mathbf{G}}_{ji}, \tilde{\mathbf{H}}_{ji}\}$. Similarly , $\overline{\mathbf{F}}_{11}$,$\overline{\mathbf{F}}_{21}$,$\overline{\mathbf{F}}_{12}$ and $\overline{\mathbf{F}}_{22}$ are $\tilde{\mathbf{F}}_{11}, \tilde{\mathbf{F}}_{21}, \tilde{\mathbf{F}}_{12}$ and $\tilde{\mathbf{F}}_{22}$ with the real and imaginary components of the second and the third channel uses being swapped. Now, following the same analysis presented in Section 3, the SNRs of every pair of real symbols will be exactly the same and the optimal DoF can be achieved using idealized scalar Gaussian codes. Furthermore, in the simulation section, we will show that the proposed scheme has a significantly smaller power offset than the basic scheme, in both the symmetric and asymmetric cases.

5. SIMULATION RESULTS

We now compare the sum rate performance of the proposed schemes with that of the corresponding existing "basic" schemes that involve more arbitrary choices of some of the parameters [6]. In all the comparisons, the channels are i.i.d. Rayleigh fading and we evaluate the average of the sum rates over 1,000 realizations. We begin with the time-invariant channel model presented in Section 3. In Fig. 3 we provide the performance of the proposed scheme, with f_1 and f_2 either being randomly chosen or being matched to the channel coefficients, i.e., $f_i = h_{ii}^*$. For comparison, we also provide the performance of the basic asymmetric complex signaling scheme. The figure shows the that proposed balanced scheme provides a large reduction in the power offset—more than 6 dB—relative to that of the basic scheme, even in the matched channel case. This suggests that the specific structure of the precoding scheme plays a more important role than matching to the channel condition.

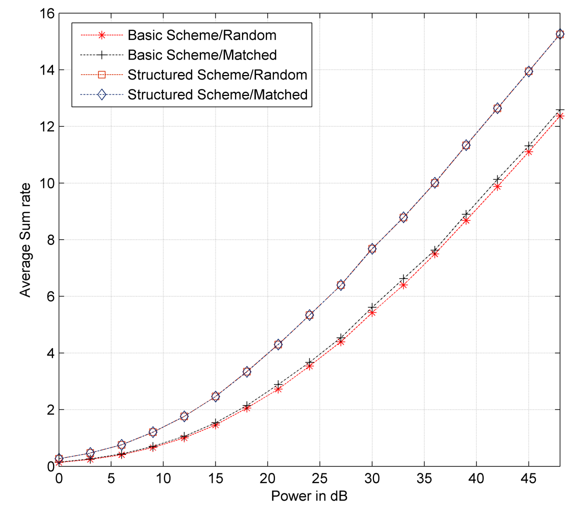

Fig. 3. Average Sum Rate in case of constant channel model

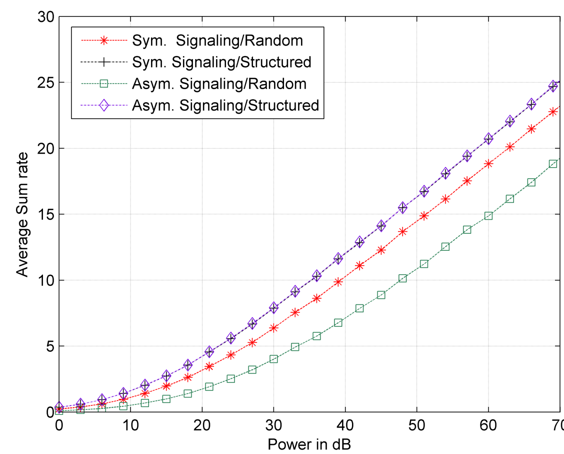

Fig. 4. Average Sum Rate for real channel with delayed CSIT

Moving to the delayed CSIT model with real channel coefficients, the advantages of the proposed balanced scheme remain. In Fig. 4, the power gap between the proposed balanced scheme and symmetric signaling with random initialization is about 4dB, while the gap between the proposed balanced scheme and the basic asymmetric signaling scheme exceeds 12dB. Now, if we applied the same structure for the precoders of the symmetric signaling, we found that the performance gap vanishes. Interestingly, the proposed balanced scheme provides a way for the initialization of the precoders of the symmetric signaling that result in improved sum rate performance.

6. REFERENCES

[1] M. A. Maddah-Ali, A. S. Motahari, and A. K. Khandani, "Communication over MIMO X channels: Interference alignment, decomposition, and performance analysis," *IEEE Trans. Inf. Theory*, vol. 54, no. 8, pp. 3457–3470, Aug. 2008.

[2] V. R. Cadambe and S. A. Jafar, "Interference alignment and degrees of freedom of the K-user interference channel," *IEEE Trans. Inf. Theory*, vol. 54, no. 8, pp. 3425–3441, Aug. 2008.

[3] S. A. Jafar, "Interference alignment: A new look at signal dimensions in a communication network," *Foundations and Trends in Communications and Information Theory*, vol. 7, no. 1, pp. 1–136, 2011.

[4] H. Maleki, S. A. Jafar, and S. Shamai, "Retrospective interference alignment over interference networks," *IEEE J. Sel. Topics Signal Process.*, vol. 6, no. 3, pp. 228–240, Jun. 2012.

[5] M. A. Maddah-Ali and D. Tse, "Completely stale transmitter channel state information is still very useful," *IEEE Trans. Inf. Theory*, vol. 58, no. 7, pp. 4418–4431, Jul. 2012.

[6] V. R. Cadambe, S. A. Jafar, and C. Wang, "Interference alignment with asymmetric complex signaling: Settling the Host-Madsen–Nosratinia conjecture," *IEEE Trans. Inf. Theory*, vol. 56, no. 9, pp. 4552–4565, Sep. 2010.

[7] D. Tse and P. Viswanath, *Fundamentals of wireless communications*. Cambridge University Press, 2005.

[8] A. Lozano, A. M. Tulino, and S. Verdù, "High-SNR power offset in multiantenna communication," *IEEE Trans. Inf. Theory*, vol. 51, no. 12, pp. 4134–4151, Dec. 2005.

[9] N. Lee and R. W. Heath Jr., "Not too delayed CSIT achieves the optimal degrees of freedom," 2012, available online at http://arxiv.org/abs/1207.2211.