ROBUST NONCOOPERATIVE RATE-MAXIMIZATION GAME FOR MIMO GAUSSIAN INTERFERENCE CHANNELS UNDER BOUNDED CHANNEL UNCERTAINTY

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ABSTRACT

We propose a robust formulation for the noncooperative rate-maximization game in MIMO Gaussian interference channels under bounded channel uncertainty. The proposed robust game needs little additional computation and requires no additional information exchange among users when compared to the nominal game and thus maintains the low-complexity and distributed nature of the MIMO waterfilling algorithm. The robust rate-maximization game is shown to be equivalent to the nominal game with modified direct-channel matrices. The equilibrium solution of the robust rate-maximization game and the required iterative algorithm to obtain the solution are presented. Sufficient conditions for the uniqueness of the equilibrium and the convergence of the algorithm are also presented. Simulation results indicate that the robust solution in the presence of channel uncertainty performs better than the nominal solution with zero uncertainty, due to the users being more conservative in their power allocation when there is channel uncertainty.

Index Terms— Game theory, resource allocation, MIMO systems, Nash equilibrium, waterfilling, robustness.

1. INTRODUCTION

The competitive rate-maximization problem for the Gaussian interference channel with single-antenna users has been well-studied and characterized over the past decade using non-cooperative game theory (See [1] and references therein). This has been extended to the multi-antenna scenario [2, 3] where the multiple-input multiple-output (MIMO) waterfilling algorithm has been developed and characterized. However, these schemes assume the availability of perfect channel knowledge, which is not possible in a practical scenario due to various reasons such as estimation errors, feedback quantization and latency between channel estimation and signal transmission.

Uncertainty in rate-maximization games for the Gaussian interference channel has been investigated [4]. Our previous work presented and analyzed a robust rate-maximization game in the frequency-selective Gaussian interference channel under bounded channel uncertainty [5, 6]. However, these solutions were limited to SISO (single-input single-output) systems and cannot be directly applied to MIMO systems.

A dynamic robust game for rate-maximization in MIMO systems has been proposed [7] where a learning framework has been used to develop suitable power allocations in repeated games in the presence of channel uncertainty and imperfect payoffs (information rate) with time delays. Such an approach has no closed-form solution, both for the static and dynamic case, and needs to be computed numerically, which makes further characterization of the equilibrium difficult. In addition, the simulation results presented therein are for the dynamic case where robustness under a learning framework is investigated. However, the effect of the degree of uncertainty on the properties of the equilibrium have not been investigated.

Here, we propose a distribution-free robust rate-maximization game for the MIMO Gaussian interference channel under bounded channel uncertainty and show that this is equivalent to a nominal MIMO rate-maximization game [2] with modified channel matrices. The proposed robust solution also needs no additional information (other than the uncertainty bound) such as the strategies and channel matrices of other users and does not add much to the complexity of the nominal solution. The closed-form equilibrium solution for this game is presented along with sufficient conditions for the uniqueness of the equilibrium and the convergence of the iterative waterfilling algorithm (IWFA) proposed for computing it. We also investigate the effect of uncertainty on the guaranteed convergence of the algorithm and the sum-rate of the system. We show that the robust solution leads to higher sum-rate with increasing uncertainty, but at the cost of guaranteed convergence of the algorithm.

This paper is organized as follows: The system model and its underlying assumptions are described in Section 2. In Section 3, the robust rate-maximization game for the MIMO Gaussian interference channel is formulated. In Section 4, the equilibrium solution for the robust MIMO rate-maximization game is presented and characterized. In Section 5, the behaviour of the proposed solution is analyzed under various conditions through simulations. Finally, conclusions are drawn in Section 6.

2. SYSTEM MODEL

Notations used: The operators $(\cdot)^H, \mathrm{E}\{\cdot\}, \mathrm{Tr}(\cdot)$ and $\|\cdot\|_F$ denote the Hermitian, statistical expectation, trace and Frobenius norm operations respectively. $\mathbb{R}_+^{m\times n}$ is the set of $m \times n$ matrices with real non-negative elements. The largest, i-th largest and smallest eigenvalues of matrix $\mathbf{A}$ are denoted by $\lambda_{\max}(\mathbf{A}), \lambda_i(\mathbf{A})$ and $\lambda_{\min}(\mathbf{A})$ respectively. The largest and smallest singular values of matrix $\mathbf{A}$ are denoted by $\sigma_{\max(\mathbf{A})}$ and $\sigma_{\min}(\mathbf{A})$ respectively. The spectral radius of matrix $\mathbf{A}$ is denoted by $\rho(\mathbf{A})$. The operation $(x)^+ \triangleq \max(0, x)$. A random variable x drawn from a complex normal distribution with mean μ and variance σ^2 is denoted by $x \sim N_C(\mu, \sigma^2)$.

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Consider a MIMO Gaussian interference channel composed of Q MIMO transmit-receive pairs operating in the same environment. The signal vector $\mathbf{y}_q \in \mathbb{C}^{n_{R_q} \times 1}$ measured at the receiver of user q is

$$\mathbf{y}_q = \widetilde{\mathbf{H}}_{qq}\mathbf{x}_q + \sum_{r \neq q} \mathbf{H}_{rq}\mathbf{x}_r + \mathbf{n}_q \tag{1}$$

where $\widetilde{\mathbf{H}}_{qq} \in \mathbb{C}^{n_{R_q} \times n_{T_q}}$ is the direct-channel matrix between source q and destination q, $\mathbf{H}_{rq} \in \mathbb{C}^{n_{R_q} \times n_{T_r}}$ is the cross-channel matrix between source r and destination q, $\mathbf{x}_q \in \mathbb{C}^{n_{T_q} \times 1}$ is the signal vector transmitted by source q and $\mathbf{n}_q \in \mathbb{C}^{n_{R_q} \times 1}$ is the receiver noise vector of user q, which is assumed to be a zero-mean complex Gaussian vector with an arbitrary (nonsingular) covariance matrix $\mathbf{R}_{n_q}$. The multi-user interference (MUI) observed at the destination q, which is treated as additive spatially coloured noise at the receiver of user q, is represented by the second term in the right hand side of (1).

The system is assumed to be quasi-stationary for the duration of the transmission. Each receiver is assumed to be able to measure accurately the covariance matrix of the noise plus MUI generated by the other users. The direct-channel matrices $\{\widetilde{\mathbf{H}}_{qq}\}_{q=1}^{Q}$ are assumed to be *square* and *nonsingular* and to have a bounded uncertainty of unknown distribution. [1] The uncertainty set $\mathcal{H}_q$ of the direct-channel matrix $\widetilde{\mathbf{H}}_{qq}$ is deterministically modelled as an ellipsoid centered around the nominal value $\mathbf{H}_{qq}$,

$$\mathcal{H}_q \triangleq \left\{ \widetilde{\mathbf{H}}_{qq} \triangleq \mathbf{H}_{qq} + \mathbf{\Delta}_q : \|\mathbf{\Delta}_q\|_F \leq \epsilon_q \right\} \tag{2}$$

where ϵ_q is the uncertainty bound.

Each destination q computes the optimal covariance matrix $\mathbf{Q}_q \triangleq \mathrm{E}\{\mathbf{x}_q\mathbf{x}_q^H\}$ for its own link and transmits it back to its transmitter over a low bit-rate error-free feedback channel. From this optimal covariance matrix, the beamformer weights of the transmitter can be computed as $\mathbf{x}_q = \sum_{i=1}^{n_{T_q}} \sqrt{\lambda_{q_i}}\mathbf{v}_{q_i}$ where λ_{q_i} is the i-th eigenvalue of $\mathbf{Q}_q$ and $\mathbf{v}_{q_i}$ is its associated eigenvector.

The nominal information rate of user q, $R_q(\mathbf{Q}_q, \mathbf{Q}_{-q})$, for this system can be written as

$$R_q(\mathbf{Q}_q, \mathbf{Q}_{-q}) = \log\det(\mathbf{I} + \mathbf{H}_{qq}^H \mathbf{R}_{-q}^{-1}(\mathbf{Q}_{-q})\mathbf{H}_{qq}\mathbf{Q}_q) \tag{3}$$

where $\mathbf{R}_{-q}(\mathbf{Q}_{-q}) \triangleq \mathbf{R}_{\mathbf{n}_q} + \sum_{r \neq q} \mathbf{H}_{rq}\mathbf{Q}_r\mathbf{H}_{rq}^H$ is the interference plus noise covariance matrix observed by destination q, and $\mathbf{Q}_{-q} \triangleq \{\mathbf{Q}_r\}_{r \neq q}$ is the set of covariance matrices of all users except the q-th user. Each player q competes rationally against other users in order to maximize its own information rate $R_q(\mathbf{Q}_q, \mathbf{Q}_{-q})$ by designing the optimal covariance matrix $\mathbf{Q}_q^\star$, given the constraint $\mathrm{E}\left\{\|\mathbf{x}_q\|_2^2\right\} = \mathrm{Tr}(\mathbf{Q}_q) \leq P_q$ where P_q is the maximum average power transmitted in units of energy per transmission for user q.

Mathematically, the nominal game without CSI uncertainty can be written as [2]

$$\mathscr{G}_{\mathrm{nom}} \qquad \begin{array}{ll} \max\limits_{\mathbf{Q}_q} & R_q(\mathbf{Q}_q, \mathbf{Q}_{-q}) \\ \mathrm{s.\,t.} & \mathbf{Q}_q \in \mathscr{Q}_q \end{array} \qquad \forall q \in \Omega \tag{4}$$

where $\Omega \triangleq \{1, \ldots, Q\}$ is the set of the Q players (i.e. MIMO links), $R_q(\mathbf{Q}_q, \mathbf{Q}_{-q})$ is the payoff function of player q as given in (3) and the set of admissible strategies of player q, $\mathscr{Q}_q$, is defined as

$$\mathscr{Q}_q \triangleq \left\{ \mathbf{Q} \in \mathbb{C}^{n_{T_q} \times n_{T_q}} : \mathbf{Q} \succeq \mathbf{0}, \quad \mathrm{Tr}(\mathbf{Q}_q) = P_q \right\}. \tag{5}$$

[1] An ellipsoid is often used to approximate unknown and potentially complicated convex uncertainty sets [8]. The ellipsoidal approximation has the advantage of parametrically modelling a complicated data set and thus is a convenient input parameter to algorithms. In addition, there are statistical reasons leading to ellipsoidal uncertainty sets in certain cases. Also, this model often results in optimization problems with convenient analytical structures [9].

3. ROBUST RATE-MAXIMIZATION GAME FORMULATION

The robust game model [10] suggests that when players have uncertainties in their payoff functions, formulating the best response to the worst-case payoff functions leads to a stable equilibrium. Motivated by this approach, a *protection function* (which is a lower bound on the payoff function) is formulated for each user, which is then maximized.

Theorem 1. *The protection function for the information rate of user q in* (3) *in given by*

$$\log\det\left(\mathbf{I} + \gamma_q \mathbf{H}_{qq}^H \mathbf{R}_{-q}^{-1}(\mathbf{Q}_{-q})\mathbf{H}_{qq}\mathbf{Q}_q\right), \tag{6}$$

where γ_q is defined as $\gamma_q \triangleq 1 - 2\epsilon_q/\sigma_{\max}(\mathbf{H}_{qq})$.

Proof. See Appendix A. □

Note that the lower bound indicated by γ_q could be too loose if the uncertainty bound ϵ_q is too high or if the largest singular value of the direct-channel, $\sigma_{\max}(\mathbf{H}_{qq})$ is too small. In particular, this could lead to $\gamma_q \leq 0$. However, $\lambda_{\min}(\mathbf{E}_q^H\mathbf{E}_q) \geq 0$. Hence, the range of γ_q is limited to $0 < \gamma_q \leq 1$, or when $\sigma_{\max}(\mathbf{H}_{qq}) \geq 2\epsilon$.

Based on the protection function in (6), the robust MIMO rate-maximization game $\mathscr{G}_{\mathrm{rob}}$ can be formulated as, $\forall q \in \Omega$,

$$\begin{array}{ll} \max\limits_{\mathbf{Q}_q} & \log\det\left(\mathbf{I} + \gamma_q \mathbf{H}_{qq}^H \mathbf{R}_{-q}^{-1}(\mathbf{Q}_{-q})\mathbf{H}_{qq}\mathbf{Q}_q\right) \\ \mathrm{s.\,t.} & \mathbf{Q}_q \in \mathscr{Q}_q \end{array} \tag{7}$$

Note that the quantity γ_q of user q is dependent only on its own direct-channel $\mathbf{H}_{qq}$ and its uncertainty bound ϵ_q, and thus does not need any additional information (other than the uncertainty bound), such as other users' transmit covariances or channel matrices, when computing the robust solutions. Furthermore, the quantity γ_q is related to the relative uncertainty in the direct-channel matrices (determined by the ratio $\epsilon_q/\sigma_q(\mathbf{H}_{qq})$). In addition, this formulation has the advantage of not needing any additional computational hardware, as the eigendecomposition is performed anyway in every iteration of the algorithm when computing the waterfilling solutions. Moreover, the additional computational cost (when compared to nominal algorithm for a system with no uncertainty) is not going to be significant, as the quantity γ_q needs to be computed only once, at the beginning of the game.

It can be observed that the robust game $\mathscr{G}_{\mathrm{rob}}$ is equivalent to the nominal game $\mathscr{G}_{\mathrm{nom}}$ in (4), with the modified channels $\{\sqrt{\gamma_q}\,\mathbf{H}_{qq}\}_{q\in\Omega}$ instead of the original channels $\{\mathbf{H}_{qq}\}_{q\in\Omega}$.

4. EQUILIBRIUM OF GAME $\mathscr{G}_{\mathrm{rob}}$

The solution to the nominal game $\mathscr{G}_{\mathrm{nom}}$ is the Nash equilibrium [2]. Based on this, the equilibrium of the game $\mathscr{G}_{\mathrm{rob}}$ can be found. In the robust game $\mathscr{G}_{\mathrm{rob}}$, given $\mathbf{Q}_{-q} \in \mathscr{Q}_{-q} \triangleq \mathscr{Q}_1 \times \cdots \times \mathscr{Q}_{q-1} \times \mathscr{Q}_{q+1} \times \cdots \mathscr{Q}_Q$, the optimum action profile of the players $\{\mathbf{Q}_q^\star\}_{q\in\Omega}$ at the equilibrium must satisfy, $\forall q \in \Omega$

$$\mathbf{Q}_q^\star = \mathrm{RWF_q}(\mathbf{Q}_{-q}^\star), \tag{8}$$

with the robust waterfilling operator $\mathrm{RWF_q}(\cdot)$ defined as

$$\mathrm{RWF_q}(\mathbf{Q}_{-q}) \triangleq \mathbf{U}_q(\mu_q\mathbf{I} - \tfrac{1}{\gamma_q}\mathbf{D}_q^{-1})^{+}\mathbf{U}_q^{H} \tag{9}$$

where μ_q is chosen to satisfy $\mathrm{Tr}\left((\mu_q\mathbf{I} - \frac{1}{\gamma_q}\mathbf{D}_q^{-1})^{+}\right) = P_q$. The unitary matrix of eigenvectors $\mathbf{U}_q = \mathbf{U}_q(\mathbf{Q_{-q}}) \in \mathbb{C}^{n_{T_q}\times n_{T_q}}$ and the diagonal matrix $\mathbf{D}_q = \mathbf{D}_q(\mathbf{Q}_{-q}) \in \mathbb{R}_{++}^{n_{T_q}\times n_{T_q}}$ are calculated from the eigendecomposition $\mathbf{U}_q\mathbf{D}_q\mathbf{U}_q^H \triangleq \mathbf{H}_{qq}^H\mathbf{R}_{-q}^{-1}(\mathbf{Q}_{-q})\mathbf{H}_{qq}$.

The robust optimal covariance matrices $\mathbf{Q}_{q}^{*}{}_{q=1}^{Q}$ of the users can be calculated through an iterative waterfilling algorithm [2] using the robust waterfilling operator $\mathrm{RWF_q}(\cdot)$.

Given the MIMO system in (1), the non-negative matrix $\mathbf{S}_\gamma \in \mathbb{R}_{+}^{Q\times Q}$ is defined as

$$[\mathbf{S}_\gamma]_{qr} \triangleq \begin{cases} \dfrac{1}{\gamma_q}\,\rho\big(\mathbf{H}_{rq}^H\mathbf{H}_{qq}^{-H}\mathbf{H}_{qq}^{-1}\mathbf{H}_{rq}\big), & \text{if } r \neq q, \\ 0, & \text{otherwise} \end{cases} \tag{10}$$

we have the following result:

Theorem 2. *Game $\mathscr{G}^{\mathrm{rob}}$ has at least one equilibrium for any feasible set of channel matrices and transmit powers of the users. Furthermore, the equilibrium is unique and the robust MIMO IWFA converges to the unique equilibrium as $T \to \infty$ for any set of feasible initial conditions if*

$$\rho(\mathbf{S}_\gamma) < 1 \tag{11}$$

where $\mathbf{S}_\gamma$ is defined in (10).

Proof. Refer [2, Theorem 6,7]. □

It can be verified that the above condition reduces to the nominal condition [2] when there is no uncertainty ($\gamma_q = 1\ \forall q \in \Omega$). When the relative uncertainties, i.e, the ratio $\epsilon_q/\sigma_q(\mathbf{H}_{qq})$, of all users is the same, the quantities γ_q of all users are identical. In this case, the sufficient condition in (11) can be simplified as follows:

Corollary 1. *When the relative uncertainties of all the users are identical, i.e., when $\gamma_q = \gamma$, $\forall q \in \Omega$, the sufficient condition for the uniqueness of the equilibrium and the guaranteed convergence of the robust MIMO IWFA, described in* (11)*, reduces to*

$$\rho(\mathbf{S}) < \gamma \tag{12}$$

where $\mathbf{S}$ is defined as

$$[\mathbf{S}]_{qr} \triangleq \begin{cases} \rho\big(\mathbf{H}_{rq}^H\mathbf{H}_{qq}^{-H}\mathbf{H}_{qq}^{-1}\mathbf{H}_{rq}\big), & \textit{if } r \neq q, \\ 0, & \textit{otherwise} \end{cases} \tag{13}$$

This result helps analyze the effect of uncertainty on the set of channel matrices for which the equilibrium is guaranteed to be unique and the robust MIMO IWFA is guaranteed to converge. In the absence of uncertainty, this occurs when $\rho(\mathbf{S}) < 1$ [2]. When the uncertainty bound of the system increases, the value of γ reduces, and thus, the set of matrices that satisfy (12) shrinks. Thus, to achieve a robust solution, there is a trade-off between allowed uncertainty and guaranteed convergence of the algorithm.

5. SIMULATION RESULTS

In this section, the average behaviour of the robust MIMO algorithm under different scenarios is investigated. These results are compared with the nominal solution (i.e. using the MIMO waterfilling algorithm [2] with erroneous channel matrices). The simulation results are provided for a system with Q users averaged over 10000 trials with random channel matrices where the elements of the cross-channel matrices $[\mathbf{H}_{rq}]_{ij} \sim N_C(0,1)$ for $r \neq q$ and the direct-channel matrices $[\mathbf{H}_{qq}]_{ij} \sim N_C(0,d_r^2)$. The channel uncertainty model is $\widetilde{\mathbf{H}}_{qq} \triangleq \mathbf{H}_{qq} + \mathbf{\Delta}_q$ where $\|\mathbf{\Delta}_q\|_F \leq \epsilon$ (from (2)). The specific parameters such as number of transmit/receive antennas and number of users are provided with each figure. It is to be noted that the quantity d_r is the ratio between the standard deviation of the elements of the random direct-channel matrices and the standard deviation of the elements of the random cross-channel matrices. A higher value of d_r indicates weaker interference in the system.

The average number of iterations required to converge to the robust solution against the uncertainty bound of the system is depicted in Figure 1. It can be observed that the robust solution takes longer to converge with higher uncertainty in the system.

In Figure 2a, it can be observed that the sum-rate under the robust solution improves with rise in uncertainty while the sum-rate under the nominal solution falls with increase in uncertainty. This gap in performance can be observed to be zero under zero uncertainty since the two solutions coincide. In Figure 2b, the average sum-rate of a system with 2 users is plotted against the number of transmit/receive antennas of each user. It can be observed that the average sum-rate of the robust solution increases with the number of antennas as expected in MIMO systems. Furthermore, the robust waterfilling solution consistently performs better than the nominal solution for the observed number of transmit/receive antennas.

In Figure 2c we observe that increasing the number of users results in a lower sum-rate because a higher number of users in the system results in higher interference for all users, given a fixed value of d_r. In addition, it can be observed that the robust solution performs better than the nominal solution regardless of the number of users in the system.

In Figure 2d, the effect of the level of interference on the average sum-rate of the system can be observed. The average sum-rate at the robust solution increases with reduction in interference. Note that a higher value of d_r indicates weaker interference in the system. It can also be observed that the gap in performance between the robust solution and the nominal solution is higher when the system has higher interference and falls with reduction in interference. This is because the robust solution encourages each user to be less greedy, which results in lower interference for all users. In systems with relatively stronger cross-channel matrices, this plays a greater role in determining the observed information rates of the users, when compared to systems with weak cross-channel matrices. Thus, when d_r increases, the robust solution moves closer to the nominal solution.

6. CONCLUSIONS

This paper developed a robust formulation for the rate-maximization game in MIMO Gaussian interference channels in the presence of bounded channel uncertainty. The proposed scheme required no additional information exchange among users and does not add much complexity to the algorithm. The robust game thus developed was shown to be equivalent to the nominal MIMO rate-maximization game with modified direct-channel matrices. The equilibrium solution for this game and an iterative algorithm to compute it distributively were presented and characterized. Numerical simulations on the behaviour of this scheme indicated that the robust solution in the presence of channel uncertainty performs better than the nominal

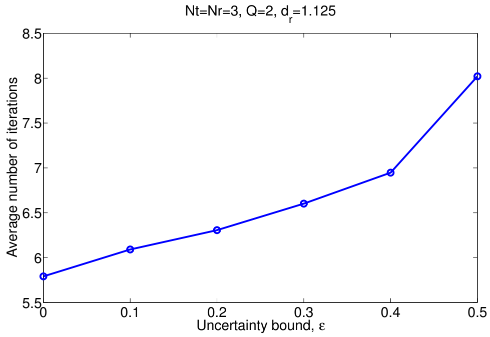


Fig. 1: Number of iterations vs. channel uncertainty bound, ϵ.

solution with perfect channel knowledge due to the users being less greedy.

A. APPENDIX: PROOF OF THEOREM 1

Defining the matrices $\mathbf{M}_q$ and $\mathbf{E}_q$ as

$$\mathbf{M}_q \triangleq \mathbf{H}_{qq}^H \mathbf{R}_{-q}^{-1}(\mathbf{Q}_{-q})\mathbf{H}_{qq}, \quad \text{and} \quad \mathbf{E}_q \triangleq \mathbf{I} + \mathbf{H}_{qq}^{-1}\boldsymbol{\Delta}_q, \tag{14}$$

the observed information rate of user q can be written as $\widetilde{R}_q(\mathbf{Q}_q, \mathbf{Q}_{-q})$

$$\begin{aligned}
&= \log\det(\mathbf{I} + \widetilde{\mathbf{H}}_{qq}^H \mathbf{R}_{-q}^{-1}(\mathbf{Q}_{-q})\widetilde{\mathbf{H}}_{qq}\mathbf{Q}_q), && (15)\\
&= \textstyle\sum_{i=1}^{n_q} \log \lambda_i(\mathbf{I} + \mathbf{E}_q^H \mathbf{M}_q \mathbf{E}_q \mathbf{Q}_q), && (16)\\
&= \textstyle\sum_{i=1}^{n_q} \log\left(1 + \lambda_i(\mathbf{E}_q^H \mathbf{E}_q \mathbf{M}_q \mathbf{Q}_q)\right), && (17)\\
&\geq \textstyle\sum_{i=1}^{n_q} \log\left(1 + \lambda_{\min}(\mathbf{E}_q^H \mathbf{E}_q)\lambda_i(\mathbf{M}_q \mathbf{Q}_q)\right). && (18)
\end{aligned}$$

where (16) follows from (14) and [11, Theorem 1.2.12]; (17) follows from Weyl's Theorem [11, Theorem 4.3.1]; and (18) follows from [12, Fact 8.19.17]. Now, $\lambda_{\min}(\mathbf{E}_q^H \mathbf{E}_q)$

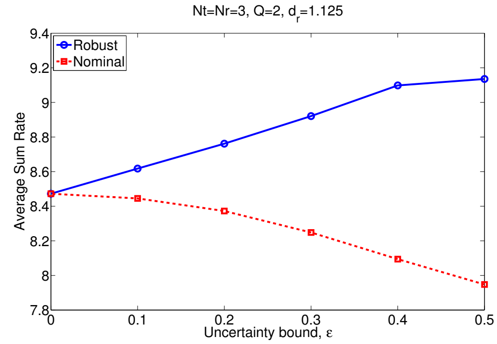
$$\begin{aligned}
&= \lambda_{\min}\big(\mathbf{I} + \boldsymbol{\Delta}_q^H \mathbf{H}_{qq}^{-H} + \mathbf{H}_{qq}^{-1}\boldsymbol{\Delta}_q + \boldsymbol{\Delta}_q^H \mathbf{H}_{qq}^{-H}\mathbf{H}_{qq}^{-1}\boldsymbol{\Delta}_q\big), && (19)\\
&\geq 1 + \lambda_{\min}\big(\boldsymbol{\Delta}_q^H \mathbf{H}_{qq}^{-H} + \mathbf{H}_{qq}^{-1}\boldsymbol{\Delta}_q\big) + \lambda_{\min}\big(\boldsymbol{\Delta}_q^H \mathbf{H}_{qq}^{-H}\mathbf{H}_{qq}^{-1}\boldsymbol{\Delta}_q\big), && (20)\\
&\geq 1 - 2\sigma_{\max}\big(\mathbf{H}_{qq}^{-1}\boldsymbol{\Delta}_q\big) + \lambda_{\min}\big(\boldsymbol{\Delta}_q^H \mathbf{H}_{qq}^{-H}\mathbf{H}_{qq}^{-1}\boldsymbol{\Delta}_q\big), && (21)\\
&\geq 1 - 2\sigma_{\max}\big(\mathbf{H}_{qq}^{-1}\boldsymbol{\Delta}_q\big) + \lambda_{\min}\big(\mathbf{H}_{qq}^{-H}\mathbf{H}_{qq}^{-1}\big)\lambda_{\min}\big(\boldsymbol{\Delta}_q^H \boldsymbol{\Delta}_q\big), && (22)\\
&\geq 1 - 2\sigma_{\min}\big(\mathbf{H}_{qq}^{-1}\big)\sigma_{\max}\big(\boldsymbol{\Delta}_q\big), && (23)\\
&\geq 1 - 2\epsilon_q/\sigma_{\max}\big(\mathbf{H}_{qq}\big), && (24)
\end{aligned}$$

where (20) follows from Weyl's Theorem [11, 4.3.1]; (21) follows from [12, Fact 5.11.25]; (22) follows from [12, Fact 8.19.17]; (23) follows from [12, Proposition 9.6.6] and from the fact that $\lambda_{\min}\big(\boldsymbol{\Delta}_q^H \boldsymbol{\Delta}_q\big) \geq 0$; and (24) follows from the definition of Frobenius norm.

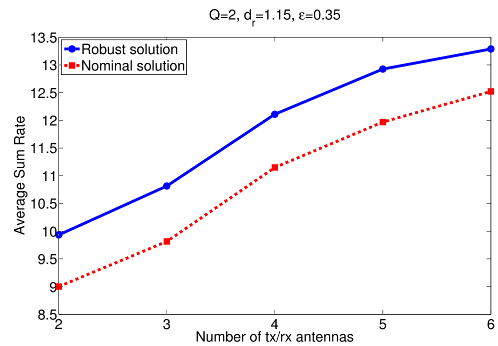
Using (24) in (18), the protection function for user q is given by $\widetilde{R}_q(\mathbf{Q}_q, \mathbf{Q}_{-q})$

$$\begin{aligned}
&\geq \textstyle\sum_{i=1}^{n_q} \log\left(1 + \gamma_q \lambda_i(\mathbf{M}_q \mathbf{Q}_q)\right), && (25)\\
&= \log\det\left(\mathbf{I} + \gamma_q \mathbf{H}_{qq}^H \mathbf{R}_{-q}^{-1}(\mathbf{Q}_{-q})\mathbf{H}_{qq}\mathbf{Q}_q\right), && (26)
\end{aligned}$$

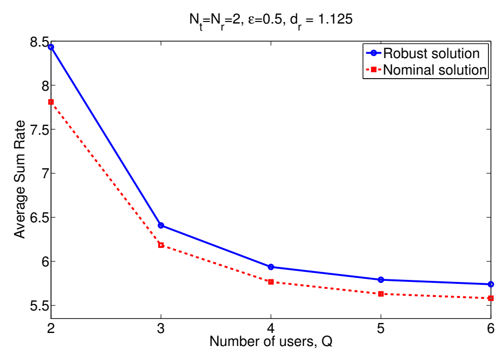
where (26) follows from [11, Theorem 1.2.12] and γ_q is defined in Theorem 1. $\square$



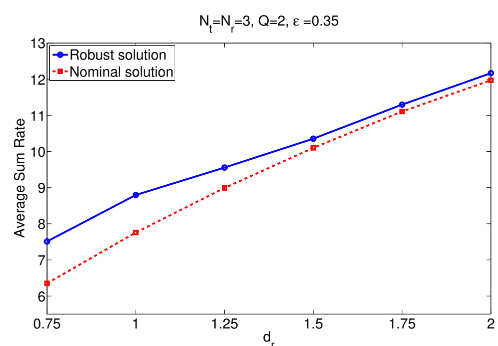
(a) Sum-rate vs. channel uncertainty bound, ϵ.



(b) Sum-rate vs. number of transmit/receive antennas.



(c) Sum-rate vs. number of users.



(d) Sum-rate vs. direct-channel matrix standard deviation, d_r.

Fig. 2: Sum-rate of the system under various scenarios.

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