POWER ALLOCATION FOR JOINT ESTIMATION WITH ENERGY HARVESTING CONSTRAINTS

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ABSTRACT

This paper considers joint estimation with multiple sensors powered by energy harvesters in wireless sensor networks. In particular, we focus on a network with K sensor nodes, which communicate with a fusion center via K orthogonal channels and power themselves by harvesting energy from the environment. Assuming a deterministic energy-harvesting model under which the harvested energy profile is known prior to transmission, the worst-case mean-square error (MSE) minimization problem over a finite horizon of T estimation periods is investigated. We consider the cases that the sensors have either infinite or finite battery capacity, and develop efficient iterative algorithms to compute the optimal power allocation strategy, with numerical results presented to validate our analysis.

***Index Terms*—** Energy harvesting, joint estimation, power allocation, mean-square error (MSE).

1. INTRODUCTION

Energy harvesters, which carry the capability to harvest energy from the environment [1], e.g., from wind or solar sources, have been recognized as a low-cost and convenient solution to provide almost unlimited lifetime for resource-limited wireless communication systems such as wireless sensor networks (WSNs). However, compared to the case with conventional time-invariant energy sources, energy replenished by harvesters is intermittent and varying over time. As a result, wireless devices powered by such renewable energy are subject to the newly introduced energy-harvesting (EH) constraints over time: The total energy consumed up to any time must be less than the energy harvested by that time [2–6].

Single-snap shot estimation over a constant unknown parameter has been thoroughly investigated for the past years (see [7] and the references therein). In this paper, we consider the joint estimation across a finite number of T estimation periods (over a time-varying unknown parameter). Moreover, we assume a deterministic EH model [2–6], corresponding to practical scenarios where the harvested energy levels can be predicted with negligible errors over time, and leave the more general random cases for the future study. We examine the optimal power allocation to minimize the worst-case mean-square error (MSE) over the T estimation periods and show that this problem is convex. Furthermore, we develop efficient iterative algorithms to calculate the optimal solution.

The remainder of the paper is organized as follows. Section 2 presents the system model and summarizes the main assumptions in this paper. Section 3 formulates the worst-case MSE minimization problem, and develops iterative algorithms to solve both of the cases with infinite and finite battery capacity. Numerical results are presented in Section 4. Finally, Section 5 concludes the paper.

Notation: $\min\{x,y\}$ and $\max\{x,y\}$ denote the minimum and maximum between two real numbers x and y, respectively; $(x)^+ = \min(x,0)$.

2. SYSTEM MODEL

We consider a wireless sensor network with K sensors, connecting to one fusion center via K orthogonal channels, respectively. At the fusion center, we intend to estimate a sequence of parameters $\{\theta(t)\}$, $t = 1, \cdots, T$, over T estimation periods. Each estimation period consists of two time slots: 1) EH slot, where each sensor harvests energy with duration τ_E; 2) transmission slot, where each sensor transmits the observed signal to the fusion center with duration τ_0. During the transmission slot, no energy is harvested.

The input-output relationship during each transmission slot is given as follows. The observation $x_k(t)$ for the t-th parameter $\theta(t)$ at the k-th sensor is given by

$$x_k(t) = \theta(t) + n_k(t),\ t = 1, 2, \cdots, T, \tag{1}$$

where $n_k(t)$ is the independent and identically distributed (i.i.d.) noise. In this paper, we assume that $\{\theta(t)\}$ are i.i.d. across t, which also makes the results in this paper valid for the case where they are correlated but with unknown prior statistic information. Moreover, it is assumed that $\theta(t)$ and $n_k(t)$ are independent of each other with zero mean and variances σ_θ^2 and σ_k^2, respectively. As such, the local observation signal-to-noise ratio (SNR) at the k-th sensor is defined as $\gamma_k = \frac{\sigma_\theta^2}{\sigma_k^2}$.

At each sensor, the amplify-and-forward (AF) strategy is adopted, and the transmit power of the k-th sensor at time t is given by $P_{k,t} = \alpha'_{k,t}\left(\sigma_\theta^2 + \sigma_k^2\right) = \alpha_{k,t}\left(1+\gamma_k^{-1}\right)$, where $\alpha'_{k,t}$ is the power amplifying factor and $\alpha_{k,t} = \alpha'_{k,t}\sigma_\theta^2$. By denoting the fixed (over T periods) channel gains from the k-th sensor to the fusion center as g_k, $k = 1, \cdots, K$, the received signal $y_k(t)$ at the fusion center from the k-th sensor in the t-th estimation period is given by

$$y_k(t) = \sqrt{g_k \alpha_{k,t}} x_k(t) + w_k(t), \tag{2}$$

where $w_k(t)$ is the i.i.d. noise in the transmission slot of the t-th estimation period and over the k-th channel, with zero mean and variance ξ_k^2. For convenience, $s_k = g_k/\xi_k^2$ is defined as the link SNR between the k-th sensor and the fusion center.

Since we intend to make the estimator universal (in the sense of being independent of particular observation noise distributions) and simple, the BLUE [7, 8] is adopted at the fusion center. Then, the MSE of the k-th estimation period is given as [7]

$$\mathrm{Var}\left[\widehat{\theta}(t)\right] = \sigma_\theta^2 \left(\sum_{k=1}^{K} \frac{\alpha_{k,t}s_k}{\gamma_k^{-1}\alpha_{k,t}s_k + 1}\right)^{-1}. \tag{3}$$

In addition to the sequential estimation model, we assume that the harvested energy at the k-th sensor, $k = 1, \cdots, K$, within the EH slot of the t-th estimation period is known with the amount $E_{k,t}$, $t = 1, \cdots, T$. In this paper, we assume that the consumed energy at each sensor other than the transmission energy is relatively small and thus negligible (or it can be modeled as a constant that could be easily handled by our optimization framework). Thus, the amount of energy available for each transmission is constrained by the following causal EH constraints:

$$\left(1+\gamma_k^{-1}\right)\tau_0 \sum_{t=1}^{j} \alpha_{k,t} \le \sum_{t=1}^{j} E_{k,t},\ \forall k, t. \tag{4}$$

Furthermore, we assume that the battery capacity of each sensor is B_0. Without of loss of generality, we assume that $E_{k,t} \le B_0$ for $\forall k, t$. Similar to [5], we obtain the following energy-storage constraints for $j = 2, \cdots, T$,

$$B_0 \ge \sum_{t=1}^{j} E_{k,t} - \left(1+\gamma_k^{-1}\tau_0\right)\sum_{t=1}^{j-1}\alpha_{k,t},\ \forall k. \tag{5}$$

In particular, for the case of $B_0 = \infty$, the constraints in (5) become redundant.

3. PROBLEM FORMULATION AND OPTIMAL SOLUTION

3.1. Problem Formulation

Considering T estimation periods, we could obtain T MSEs $\mathrm{Var}[\widehat{\theta}(t)]$, $t = 1, \cdots, T$. For the purpose of exposition, we adopt the worst-case MSE as the minimization objective, and the worst-case MSE minimization problem over the sequence of T estimations with the EH constraints can be written as:

$$\text{(P1)}\ \min_{\{\alpha_{k,t}\}} \max_{1\le t\le T} \mathrm{Var}\left[\widehat{\theta}(t)\right]$$
$$\text{s.t. } (4),\ (5),\ \alpha_{k,t} \ge 0,\ \forall\, k \text{ and } t.$$

By defining $D = \min_{1\le t\le T}\sum_{k=1}^{K}\frac{\alpha_{k,t}s_k}{\gamma_k^{-1}\alpha_{k,t}s_k+1}$, Problem (P1) could be shown to have the same optimal solution as the following problem

$$\text{(P2)}\ \max_{\{\alpha_{k,t}\},D}\ D$$
$$\text{s.t. } \sum_{k=1}^{K}\frac{\alpha_{k,t}s_k}{\gamma_k^{-1}\alpha_{k,t}s_k+1} \ge D, \forall\, t,$$
$$(4),\ (5),\ \alpha_{k,t} \ge 0,\ \forall\, k \text{ and } t.$$

It is easy to check that Problem (P2) is convex [9]. To have an efficient solution, we turn to an iterative algorithm, whose main idea is summarized as follows. Fix the power profiles at the other $K-1$ sensors, and optimize over the power profile belonging to the k_0-th sensor, $1 \le k_0 \le K$; repeat the above procedure until the pre-defined error tolerance is met. The convex structure of the problem guarantees the convergence of the above algorithm. At the i-th iteration, the power allocation problem for the k_0-th sensor could be cast from Problem (P2) as:

$$\text{(P3)}\ \max_{\{\alpha_{k_0,t}^i\},D^i}\ D^i \tag{6}$$

$$\text{s.t. } \frac{\alpha_{k_0,t}^i s_{k_0}}{\gamma_{k_0}^{-1}\alpha_{k_0,t}^i s_{k_0}+1} \ge D^i - \sum_{k\ne k_0}\frac{\alpha_{k,t}^i s_k}{\gamma_k^{-1}\alpha_{k,t}^i s_k+1}, \tag{7}$$

$$\sum_{t=1}^{j}\alpha_{k_0,t}^i \le \frac{1}{(1+\gamma_{k_0}^{-1})\tau_0}\sum_{t=1}^{j}E_{k_0,t}, \tag{8}$$

$$\sum_{t=1}^{j-1}\alpha_{k_0,t}^i \ge \frac{1}{\left(1+\gamma_{k_0}^{-1}\right)\tau_0}\left[\sum_{t=1}^{j}E_{k_0,t} - B_0\right], \tag{9}$$

$$\alpha_{k_0,t}^i \ge 0, \forall\, t, j, \tag{10}$$

which will be shown efficiently solvable in the next two subsections for both cases with infinite and finite battery capacity. Note that in the iterative algorithm, $\alpha_{k_0,t}^i$'s could be initialized by greedy power allocation: For each sensor, it transmits with its maximum power available at each time slot, i.e., $\alpha_{k_0,t} = \frac{E_{k_0,t}}{\left(1+\gamma_{k_0}^{-1}\right)\tau_0}$.

Remark 3.1 Since Problem (P2) is convex, we conclude that the solution provided by the above iterative algorithm will converge to the optimal solution of Problem (P2) if the optimal solution of Problem (P3) can be obtained in each iteration. In the following two subsections, we will show that this requirement could be satisfied.

3.2. Infinite Battery Capacity Case

In this subsection, we consider the case with infinite battery capacity, i.e., constraints (9) are inactive all the time. Thus, the power allocation problem for the k_0-th sensor is given as follows.

$$\text{(P4)} \quad \max_{\{\alpha^i_{k_0,t}\}, D^i} \; D^i \tag{11}$$

$$\text{s.t.} \;\; (7),\; (8),\; \alpha^i_{k_0,t} \geq 0, \forall\, t, \tag{12}$$

The optimal power allocation of Problem (P4) can be obtained by bi-section searching [9] over D^i, $0 \leq D^i \leq D_{\max}$, where $D_{\max}$ can be obtained by using the greedy power allocation. The main idea of the bisection search algorithm is described as follows: For a given D^i, we calculate the candidate power profile by letting (7) achieve with equality, i.e.

$$\alpha^i_{k_0,t} = \gamma_{k_0} s^{-1}_{k_0} \cdot \left[\gamma_{k_0} \left(\left(D^i - \sum_{k \neq k_0} \frac{\alpha^i_{k,t} s_k}{\gamma_k^{-1} \alpha^i_{k,t} s_k + 1} \right)^+ \right)^{-1} - 1 \right]^{-1}, \tag{13}$$

which provides the minimum power requirement in each estimation period to achieve the given target D^i. As such, the obtained $\{\alpha^i_{k_0,t}\}$ in (13) is feasible for Problem (P4) if and only if the EH constraints in (8) are satisfied. Then, if for a given D^i, (8) is true with the obtained $\{\alpha^i_{k_0,t}\}$ in (13), we increase D^i in the next iteration; otherwise, we decrease D^i in the next iteration.

Remark 3.2 It is worth noting that with the above obtained optimal power allocation $\{\alpha^i_{k_0,t}\}$ for Problem (P4), the last energy harvesting constraint in (8) may not be satisfied with equality.

3.3. Finite Battery Capacity Case

For the finite battery capacity case, Problem (P3) has extra energy-storage constraints (9) compared to the case in the previous subsection. We propose a forward search algorithm (summarized as Algorithm 1 in Table 1 to find the optimal power allocation of Problem (P3), which is based on solving a sequence of problems each with a similar form as Problem (P4).

The main idea of Algorithm 1 is shown as follows. Starting from the first period $t_0 = 1$, search the estimation period with the index t^*, after which the MSE related parameter D^i may change, i.e., from the t_0-th to the t^*-th estimation period, their corresponding MSEs keep identical while that of the $(t^* + 1)$-th estimation period is different. It is noted that due to constraints (8) and (9), the consumed energy at the k_0-th sensor up to the m-th estimation period must be within the interval $\left[\sum_{t=1}^{m+1} E_{k_0,t} - B_0, \sum_{t=1}^{m} E_{k_0,t}\right]$. As such, consider the following power allocation problem among the t_0-th to the m-th time slots, $m = t_0, \cdots, T$, with a modified harvested energy profile by adding an upper bound $\widetilde{E}_{k_0,m}$ on the total consumed energy at each estimation period and removing the consumed energy before the t_0-th estimation period, i.e.,

$$\text{(P5.}m\text{)} \;\; \mathcal{D}^i_m \left(\widetilde{E}_{k_0,m}\right) = \max_{\{\alpha^i_{k_0,t}\}, D^i} \; D^i,\; m = t_0, \cdots, T$$

$$\text{s.t.} \;\; \sum_{t=t_0}^{j} \alpha^i_{k_0,t} \leq \frac{\min\left\{E_j, \widetilde{E}_{k_0,m}\right\} - E_{t_0}}{(1+\gamma_k^{-1})\tau_0},$$
$$(7),\; \alpha_{k_0,t} \geq 0,\; t_0 \leq t \leq m,\; t_0 \leq j \leq m,$$

where $E_j = \sum_{t=1}^{j} E_{k_0,t}$ is the harvested energy up to the j-th estimation period, and E_{t_0} is energy consumed up to the $(t_0 - 1)$-th estimation period with $E_{t_0} = (1 + \gamma_{k_0}^{-1})\tau_0 \sum_{t=1}^{t_0-1} \alpha_{k_0,t}$.

It is worth noting that the optimal values of Problems (P5.m) to (P5.T) are nondecreasing functions of $\widetilde{E}_{k_0,m}$ over $\left[\sum_{t=1}^{m+1} E_{k_0,t} - B_0, \sum_{t=1}^{m} E_{k_0,t}\right]$, since a larger $\widetilde{E}_{k_0,m}$ always leads to a larger feasible set. Then, we define two sets $\boldsymbol{D}^{i,\text{up}} = \left[D^{i,\text{up}}_{t_0}, \cdots, D^{i,\text{up}}_{T}\right]$ and $\boldsymbol{D}^{i,\text{low}} = \left[D^{i,\text{low}}_{t_0}, \cdots, D^{i,\text{low}}_{T}\right]$, where $D^{i,\text{up}}_m = \mathcal{D}^i_m\left(\sum_{t=1}^{m} E_{k_0,t}\right)$, $t_0 \leq m \leq T$, $D^{i,\text{low}}_m = \mathcal{D}^i_m\left(\sum_{t=1}^{m+1} E_{k_0,t} - B_0\right)$, $t_0 \leq m \leq T-1$, and $D^{i,\text{low}}_T = D^{i,\text{up}}_T$. Thus, the MSE related parameter $\widetilde{D}^i$ over the t_0-th to the t^*-th estimation periods should satisfy $\widetilde{D}^i \in \left[D^{i,\text{low}}_t, D^{i,\text{up}}_t\right]$ for $t_0 \leq t \leq t^*$. In order to maximize $\widetilde{D}^i$, t^* should be chosen as

$$t^* = \arg \max_{t_0 \leq j \leq T} \left\{\cap_{m=t_0}^{j} \left[D^{i,\text{low}}_m, D^{i,\text{up}}_m\right] \neq \Phi\right\}. \tag{14}$$

Denote

$$D^i_{\text{max}} = \max \cap_{m=t_0}^{t^*} \left[D^{i,\text{low}}_m, D^{i,\text{up}}_m\right], \tag{15}$$

$$D^i_{\text{min}} = \min \cap_{m=t_0}^{t^*} \left[D^{i,\text{low}}_m, D^{i,\text{up}}_m\right]. \tag{16}$$

Thus, if $t^* = T$, we have $\widetilde{D}^i = D^i_{\text{max}}$, and the optimal power factor over the t_0-th to the T-th time slots for Problem (P3) is given by Problem (P5.T) when $\widetilde{E}_{k_0,m} = \sum_{t=1}^{T} E_{k_0,t}$; otherwise, we have the following two cases:
1) If $D^{i,\text{low}}_{t^*+1} > D^i_{\text{max}}$, we have $\widetilde{D}^i = D^i_{\text{max}}$, $t = t_0, \cdots, t^*$, and the optimal power factor over the t_0-th to the t^*-th time slots for Problem (P3) is given by Problem (P5.t^*) when $\widetilde{E}_{k_0,t^*} = \sum_{t=1}^{t^*} E_{k_0,t}$;
2) If $D^{i,\text{up}}_{t^*+1} < D^i_{\text{min}}$, we have $\widetilde{D}^i = D^i_{\text{min}}$, $t = t_0, \cdots, t^*$, and the optimal power factor over the t_0-th to the t^*-th time slots for Problem (P3) is given by Problem (P5.t^*) when $\widetilde{E}_{k_0,t^*} = \sum_{t=1}^{t^*+1} E_{k_0,t} - B_0$.

After checking the two cases, let $t_0 = t^* + 1$, and repeat the above procedure until the end of the T estimation period.

Table 1. Algorithm 1: Compute the optimal power allocation for Problem (P3).

1. Let $t_0 = 1$; repeat steps 2-4, until $t_0 = T$.
2. Compute the vectors $\boldsymbol{D}^{i,\text{up}}$ and $\boldsymbol{D}^{i,\text{low}}$ by solving Problem (P5.m), $m = t_0, \cdots, T$, by applying the same algorithm for Problem (P4) and letting $\widetilde{E}_{k_0,m} = \sum_{t=1}^{m} E_{k_0,t}$ and $\widetilde{E}_{k_0,m} = \sum_{t=1}^{m} E_{k_0,t} - B_0$, respectively.
3. Compute t^*, D^i_{max}, and D^i_{min} by using (14), (15), and (16), respectively, and check that
   3.1 If $t^* = T$ or $D^{i,\text{low}}_{t^*+1} > D^i_{\text{max}}$, the optimal power factor $\alpha^i_{k_0,t}$, $t = t_0, \cdots, t^*$ is given by Problem (P5.t^*) when $\widetilde{E}_{k_0,t^*} = \sum_{t=1}^{t^*} E_{k_0,t}$;
   3.2 If $t^* < T$ and $D^{i,\text{up}}_{t^*+1} < D^i_{\text{min}}$, the optimal power factor $\alpha^i_{k_0,t}$, $t = t_0, \cdots, t^*$ is given by Problem (P5.t^*) when $\widetilde{E}_{k_0,t^*} = \sum_{t=1}^{t^*} E_{k_0,t} - B_0$.
4. Let $t_0 = t^* + 1$;
5. Algorithm ends.

Proposition 3.1 The solution obtained by using Algorithm 1 is optimal to Problem (P3).

The proof will be given in the journal version [10]. With the optimality of Algorithm 1, we can obtain the optimal power allocation for Problems (P1) and (P2), which completes the claim in Remark 3.1.

4. NUMERICAL RESULTS

In this section, we compare the proposed optimal offline power allocation algorithm to a group of online algorithms, called "q-step" look-ahead schemes (first proposed in [6] for the fading channels), for which the extension to the setup considered in this paper is straight forward: At the t-th estimation period, use the current harvested energy information $E_{k,t}$, $1 \le k \le K$, and the expected harvested energy information $\widetilde{E}_{k,t}$ of the future $q-1$ periods, i.e., $\widetilde{E}_{k,t} = \mathbb{E}(E_{k,i})$, $t < i \le \min(t+q-1, T)$, and $1 \le k \le K$, as the energy profiles to compute the power allocation from the t-th to the $\min(t+q-1,T)$-th estimation periods, and adopt the solution corresponding to the t-th estimation period as the power allocation for the current period. For such online algorithms, only the average harvested energy information is assumed to be known.

For the purpose of exposition, we consider an i.i.d. Poisson energy arrival model, i.e. $\Pr\{E_{k,t} = n \cdot E_0\} = \frac{e^{-1}}{n!}$, where E_0 is the average energy harvesting rate. For other parameters, we assume that $K = 8$, $T = 8$, $g_k = 1, k = 1, \cdots, K$, $\sigma_k^2 = 0.2, i = 1, \cdots, K$, and $\xi_k^2 = 0.5, k = 1, \cdots, K$.

In Fig. 1, we plot the worst-case MSE performance versus the battery capacity B_0 for both the offline and online algorithms, with $E_0 = 6$ and $q = 1, 2$, or T (the case of $q = 1$ is equivalent to the greedy power allocation). It is observed that the worst-case MSE decreases as B_0 increases. The performances of the online algorithms with $q = 2$ and $q = T$ are close, while they perform better than the greedy power allocation algorithm, especially when B_0 is large.

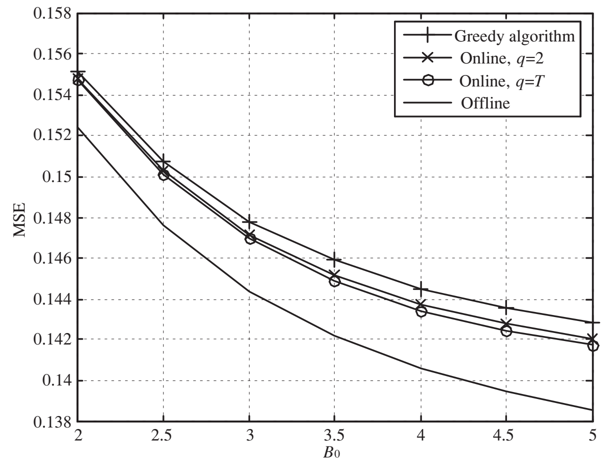

Fig. 1. The maximum MSE performance as a function of battery capacity B_0, with $E_0 = 6$.

5. CONCLUSION

In this paper, we studied the worst-case MSE minimization problem over a finite number of estimation periods in wireless sensor networks, where the power at each sensor is subject to some deterministic EH and energy-storage constraints. We proposed efficient iterative algorithms to compute the optimal power allocation, by exploiting the convex problem structure.

6. RELATION TO PRIOR WORK

This work has focused on the worst-case MSE minimization problem for joint estimation over a finite horizon of T estimation periods, and propose some efficient iterative algorithms for both the cases with infinite and finite battery capacity. Compared to the prior works for EH systems [2–6], this work considers a completely new scenario for joint estimation problem. On the other hand, we consider multiple-snap shot estimation using energy harvester, while the works in [7] and these references therein only studied single-snap estimation without using energy harvester.

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