JOINT POWER AND ADMISSION CONTROL VIA P NORM MINIMIZATION DEFLATION

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ABSTRACT

In an interference network, joint power and admission control aims to support a maximum number of links at their specified signal to interference plus noise ratio (SINR) targets while using a minimum total transmission power. In our previous work, we formulated the joint control problem as a sparse ℓ_0-minimization problem and relaxed it to a ℓ_1-minimization problem. In this work, we propose to approximate the ℓ_0-optimization problem by a p norm minimization problem where $0 < p < 1$, since intuitively p norm will approximate 0 norm better than 1 norm. We first show that the ℓ_p-minimization problem is strongly NP-hard and then derive a reformulation of it such that the well developed interior-point algorithms can be applied to solve it. The solution to the ℓ_p-minimization problem can efficiently guide the link's removals (deflation). Numerical simulations show the proposed heuristic outperforms the existing algorithms.

1. INTRODUCTION

Power control is an effective tool for interference management in cellular, ad-hoc, and cognitive underlay networks [1, 2, 3, 4, 5, 6, 7, 8]. The prevailing formulation of power control aims to use a minimum total transmission power to support all links in an interference network at their desired SINR targets. A longstanding issue associated with power control is that the problem often becomes *infeasible*, i.e., it is not possible to simultaneously support all links in the network at their SINR targets. In this case, we must adopt a joint power and admission control approach to *selectively* remove some links from the network so that the remaining ones can be simultaneously supported at their desired SINR levels. Our goal is to maximize the number of simultaneously supportable links at their required SINR targets while using a minimum total transmission power.

Theoretically, the joint power and admission control problem is known to be NP-hard to solve to global optimality [1, 3] and to approximate to constant ratio global optimality [6], so various heuristic algorithms [1, 2, 3, 6, 7, 8] have been proposed for this problem. Among them, the reference [1] proposed a convex approximation-based algorithm for the joint power and admission control problem. Instead of directly solving the original NP-hard problem, the basic idea of the proposed linear programming deflation (LPD) algorithm in [1] is to approximate the problem by an appropriate convex problem. The solution to the approximation problem can be used to check the feasibility of the original problem and guide link's removals. The removal procedure is terminated until all the remaining links in the network are simultaneously supportable. The recent work [6] developed another LP approximation-based new linear programming deflation (NLPD) algorithm for the joint power and admission control problem. In [6], the joint power and admission control problem is first equivalently reformulated as a sparse ℓ_0-minimization problem and then its ℓ_1-convex approximation is used to derive a LP, which is different from the one in [1]. Again, the solution to the derived LP can guide an iterative link removal procedure, and the removal procedure is terminated if all the remaining links in the network are simultaneously supportable.

Based on the sparse ℓ_0-minimization reformulation in [6], this paper proposes a new deflation algorithm based on p $(0 < p < 1)$ norm minimization for the joint power and admission control problem. Compared to the ℓ_1-minimization problem, the p norm minimization problem is closer to the original ℓ_0-optimization problem. The ℓ_p-approximation problem is solved by applying the efficient interior-point algorithm in [9] to solve its equivalent reformulation. Numerical results show that the proposed algorithm compares favorably with the existing approaches [1, 2, 6] in terms of the number of supported links, the total transmission power, and the CPU time.

Notations: We adopt the following notations in this paper. We denote the index set $\{1, 2, \cdots, K\}$ by $\mathcal{K}$. Lowercase boldface and uppercase boldface are used for vectors and matrices, respectively. For a given vector $\mathbf{x}$, the notations $\max\{\mathbf{x}\}$, $[\mathbf{x}]_k$ and $\|\mathbf{x}\|_p := \sum_k |[\mathbf{x}]_k|^p$ $(0 \leq p < 1)$[1] stand for its maximum entry, its k-th entry, and its p norm, respectively. In particular, when $p = 0$, $\|\mathbf{x}\|_0$ stands for the number of nonzero entries in $\mathbf{x}$. Finally, we use $\mathbf{e}$ to represent the vector of an appropriate size with all components being one and $\mathbf{I}$ to represent the identity matrix of an appropriate size, respectively.

2. PROBLEM FORMULATION

Consider a K-link (a link corresponds to a transmitter-receiver pair) interference channel with channel gains $g_{kj} \geq 0$ (from transmitter j to receiver k), noise power $\eta_k > 0$, SINR target $\gamma_k > 0$, and power budget $\bar{p}_k > 0$ for $k, j \in \mathcal{K} := \{1, 2, \cdots, K\}$. Denote the power

[1] Strictly speaking, $\|\mathbf{x}\|_p$ with $0 \leq p < 1$ is not a norm, since it does not satisfy the triangle inequality. However, we still call it p norm for convenience in this paper.

allocation vector by $\mathbf{p} = (p_1, p_2, \cdots, p_K)^T$ and the power budget vector by $\bar{\mathbf{p}} = (\bar{p}_1, \bar{p}_2, \cdots, \bar{p}_K)^T$. Treating interference as noise, we can write the SINR at the k-th receiver as

$$\text{SINR}_k = \frac{g_{kk} p_k}{\eta_k + \sum_{j \neq k} g_{kj} p_j}, \quad \forall\, k \in \mathcal{K}.$$

The joint power and admission control problem can be mathematically formulated as a two-stage optimization problem [1]. Specifically, the first stage maximizes the number of admitted links:

$$\begin{array}{ll} \max\limits_{\mathbf{p},\, \mathcal{S}} & |\mathcal{S}| \\ \text{s.t.} & \text{SINR}_k \geq \gamma_k,\ k \in \mathcal{S} \subseteq \mathcal{K}, \\ & \mathbf{0} \leq \mathbf{p} \leq \bar{\mathbf{p}}. \end{array} \tag{1}$$

We use $\mathcal{S}_0$ to denote the optimal solution for problem (1) and call it *maximum admissible set*. Notice that the solution for (1) might not be unique. The second stage minimizes the total transmission power required to support the admitted links in $\mathcal{S}_0$:

$$\begin{array}{ll} \min\limits_{\{p_k\}_{k \in \mathcal{S}_0}} & \sum_{k \in \mathcal{S}_0} p_k \\ \text{s.t.} & \text{SINR}_k \geq \gamma_k,\ k \in \mathcal{S}_0, \\ & 0 \leq p_k \leq \bar{p}_k,\ k \in \mathcal{S}_0. \end{array} \tag{2}$$

Due to the special choice of $\mathcal{S}_0$, power control problem (2) is feasible and can be efficiently and distributively solved by the Foschini-Miljanic algorithm [4].

3. REVIEW OF THE NLPD ALGORITHM

Since the developed algorithm in this paper follows the similar idea as the NLPD algorithm in [6], we first briefly review the NLPD algorithm in this section. The basic idea of the NLPD algorithm is to update the power and check whether all links can be supported or not. If the answer is yes, then terminate the algorithm; else drop one link from the network and update the power again. The above process is repeated until all the remaining links are supported.

We begin with the introduction of an equivalent normalized channel on which the NLPD algorithm is based. In particular, we use $\mathbf{q} = (q_1, q_2, \cdots, q_K)^T$ with $q_k = p_k / \bar{p}_k$ to denote the normalized power allocation vector, and use $\mathbf{c} = (c_1, c_2, \cdots, c_K)^T$ with $c_k = (\gamma_k \eta_k)/(g_{kk} \bar{p}_k) > 0$ to denote the normalized noise vector. We denote the normalized channel matrix by $\mathbf{A} \in \mathbb{R}^{K \times K}$ with the (k, j)-th entry

$$a_{kj} = \begin{cases} 1, & \text{if } k = j; \\ -\dfrac{\gamma_k g_{kj} \bar{p}_j}{g_{kk} \bar{p}_k}, & \text{if } k \neq j. \end{cases}$$

In fact, $|a_{kj}|$ is the normalized channel gain. It is simple to check that $\text{SINR}_k \geq \gamma_k$ if and only if $[\mathbf{A}\mathbf{q} - \mathbf{c}]_k \geq 0$.

In [6], we reformulate the two-stage joint power and admission control problem (1) and (2) as a single-stage optimization problem

$$\begin{array}{ll} \min\limits_{\mathbf{q}_e,\, \mathbf{q}} & \|\mathbf{q}_e\|_0 + \alpha \bar{\mathbf{p}}^T \mathbf{q} \\ \text{s.t.} & \mathbf{q}_e = \mathbf{c} - \mathbf{A}\mathbf{q}, \\ & \mathbf{0} \leq \mathbf{q} \leq \mathbf{e}, \end{array} \tag{3}$$

where $0 < \alpha < \alpha_1 := 1/\mathbf{e}^T \bar{\mathbf{p}}$, and $[\mathbf{q}_e]_k$ measures the excess transmission power [1] that the transmitter of link k needs in the normalized channel in order to be served with its desired SINR target (assuming all other links keep their transmission powers unchanged). Notice that the formulation (3) is capable of finding the maximum admissible set with minimum total transmission power. Since problem (3) is still NP-hard, we further consider its ℓ_1-convex approximation (equivalent to a LP)

$$\begin{array}{ll} \min\limits_{\mathbf{q}_e,\, \mathbf{q}} & \|\mathbf{q}_e\|_1 + \alpha \bar{\mathbf{p}}^T \mathbf{q} \\ \text{s.t.} & \mathbf{q}_e = \mathbf{c} - \mathbf{A}\mathbf{q}, \\ & \mathbf{0} \leq \mathbf{q} \leq \mathbf{e}. \end{array} \tag{4}$$

By solving (4), we know whether all links in the network can be simultaneously supported or not. If not, we drop the link

$$k_0 = \arg\max_{k \in \mathcal{K}} \left\{ \sum_{j \neq k} \left(|a_{kj}| \, [\mathbf{q}_e]_j + |a_{jk}| \, [\mathbf{q}_e]_k \right) \right\}. \tag{5}$$

An easy-to-check necessary condition

$$\left(\boldsymbol{\mu}^+\right)^T \mathbf{e} - \left(\boldsymbol{\mu}^- + \mathbf{e}\right)^T \mathbf{c} \geq 0 \tag{6}$$

for all links in the network to be simultaneously supported is also derived in [6], where $\boldsymbol{\mu}^+ = \max\{\boldsymbol{\mu}, \mathbf{0}\}$, $\boldsymbol{\mu}^- = \max\{-\boldsymbol{\mu}, \mathbf{0}\}$, and $\boldsymbol{\mu} = \mathbf{A}^T \mathbf{e}$. The necessary condition allows us to iteratively remove strong interfering links from the network. In particular, we remove the link k_0 according to the scheme

$$k_0 = \arg\max_{k \in \mathcal{K}} \left\{ \sum_{j \neq k} |a_{kj}| + \sum_{j \neq k} |a_{jk}| + c_k \right\} \tag{7}$$

until (6) becomes true.

The NLPD algorithm can be described as follows.

The NLPD Algorithm

Step 1. Initialization: Input data $(\mathbf{A}, \mathbf{c}, \bar{\mathbf{p}})$.

Step 2. Preprocessing: Remove link k_0 iteratively according to (7) until condition (6) holds true.

Step 3. Power control: Solve problem (4); check whether all links are supported: if yes, go to **Step 5**; else go to **Step 4**.

Step 4. Admission control: Remove link k_0 according to (5), set $\mathcal{K} = \mathcal{K} / \{k_0\}$, and go to **Step 3**.

Step 5. Postprocessing: Check the removed links for possible admission.

4. A P NORM MINIMIZATION DEFLATION ALGORITHM

In this section, we develop a new deflation algorithm based on ℓ_p-minimization for the joint control problem (1) and (2). As seen in Section 3, the original ℓ_0-minimization problem (3) is *successively* approximated by the ℓ_1-minimization problem (4) in the NLPD algorithm. Intuitively, the p $(0 < p < 1)$ norm minimization problem

$$\begin{array}{ll} \min\limits_{\mathbf{q}_e,\, \mathbf{q}} & \|\mathbf{q}_e\|_p + \alpha \bar{\mathbf{p}}^T \mathbf{q} \\ \text{s.t.} & \mathbf{q}_e = \mathbf{c} - \mathbf{A}\mathbf{q}, \\ & \mathbf{0} \leq \mathbf{q} \leq \mathbf{e} \end{array} \tag{8}$$

should approximate (3) better than (4). This is the motivation for the development of the new deflation algorithm based on the p norm minimization for the joint power and admission control problem.

Comparing the 1 norm minimization problem (4) and the p norm minimization problem (8), we see that problem (4) is convex while problem (8) is nonconvex (for its objective function is nonconvex with respect to $\mathbf{q}_e$). Generally speaking, convex problems are relatively easy to solve, while nonconvex optimization problems are difficult to solve. However, not all nonconvex problems are hard since the lack of convexity may be due to an inappropriate formulation. In fact, many nonconvex optimization problems admit a convex reformulation. Therefore, convexity is useful but unreliable to test the computational intractability of an optimization problem. A more robust tool is the computational complexity theory [10].

We show that problem (8) is strongly NP-hard. The proof is based on a polynomial time reduction from the MAX-2UNANIMITY problem, which is shown to be strongly NP-hard in [11].

Theorem 4.1 *The ℓ_p-minimization problem* (8) *is strongly NP-hard if* $0 \leq p < 1$.

The complexity result in Theorem 4.1 motivates us to approximately solve problem (8). Next, we first give a reformulation of problem (8), and then propose to use the interior-point algorithm developed in [9] to solve it.

Theorem 4.2 *The ℓ_p-minimization problem* (8) *can be equivalently reformulated as*

$$\begin{array}{ll} \min\limits_{\mathbf{q}_e,\,\mathbf{q}} & \sum_k [\mathbf{q}_e]_k^p + \alpha \bar{\mathbf{p}}^T \mathbf{q} \\ \textit{s.t.} & \mathbf{q}_e = \mathbf{c} - \mathbf{A}\mathbf{q}, \\ & \mathbf{0} \leq \mathbf{q} \leq \mathbf{e},\ \mathbf{q}_e \geq \mathbf{0}. \end{array} \tag{9}$$

A rigorous proof of Theorem 4.2 shall be given in the journal version. Here we just shed some light on why it does not harm optimality to restrict $\mathbf{q}_e \geq \mathbf{0}$. Notice that problem (9) is equivalent to

$$\begin{array}{ll} \min\limits_{\mathbf{q}_e,\,\mathbf{q}} & \|\mathbf{q}_e\|_p + \alpha\, \bar{\mathbf{p}}^T \mathbf{q} \\ \text{s.t.} & \mathbf{q}_e = \mathbf{c} - \mathbf{A}\mathbf{q}, \\ & \mathbf{0} \leq \mathbf{q} \leq \mathbf{e},\ \mathbf{q}_e \geq \mathbf{0}. \end{array}$$

Thus, to show the equivalence of (8) and (9), it suffices to show that any optimal solution $(\tilde{\mathbf{q}}_e, \tilde{\mathbf{q}})$ of (8) always satisfies $\tilde{\mathbf{q}}_e = \mathbf{c} - \mathbf{A}\tilde{\mathbf{q}} \geq \mathbf{0}$. In fact, assume the contrary that $|\mathcal{K}^+| \geq 1$, where $\mathcal{K}^+ = \{\, k \mid [\tilde{\mathbf{q}}_e]_k > 0 \,\}$, $\mathcal{K}^= = \{\, k \mid [\tilde{\mathbf{q}}_e]_k = 0 \,\}$. Then by the Balancing Lemma (see [6, Lemma 1]), we can appropriately reduce the power of links in $\mathcal{K}^+ \cup \mathcal{K}^=$ so that both the first term and the second term in the objective of (8) are strictly decreased.

Based on Theorem 4.2, by introducing a slack variable $\mathbf{s} \geq \mathbf{0}$ to problem (9), we see that problem (8) is actually equivalent to

$$\begin{array}{ll} \min\limits_{\mathbf{q}_e,\,\mathbf{q},\,\mathbf{s}} & \sum_k [\mathbf{q}_e]_k^p + \alpha \bar{\mathbf{p}}^T \mathbf{q} \\ \text{s.t.} & \mathbf{q}_e = \mathbf{c} - \mathbf{A}\mathbf{q},\, \mathbf{s} + \mathbf{q} = \mathbf{e} \\ & \mathbf{q} \geq \mathbf{0},\, \mathbf{q}_e \geq \mathbf{0},\, \mathbf{s} \geq \mathbf{0}. \end{array} \tag{10}$$

Now, we can apply the interior-point algorithm in [9] to solve problem (10). Similar to [9], we can prove that the potential reduction interior-point algorithm returns an ϵ-KKT [12] or ϵ-global solution of problem (10) in no more than $O\left((\frac{3K}{\epsilon}) \log(\frac{1}{\epsilon}) \right)$ iterations.

One may ask why we wish to use interior-point algorithms to solve problem (10)? The reasons are the following. First, the objective function of problem (10) is *differentiable* in the interior feasible region. Moreover, we are actually interested in finding a sparse solution $\mathbf{q}_e$ of problem (10); if we start from a solution, some of whose entries are already zero, then it is very hard to make it nonzero. In contrast, if we start from an interior point, the interior-point algorithm may generate a sequence of interior points that bypasses solutions with the wrong zero supporting set and converges to the true one. This is exactly the idea of the interior-point algorithm developed in [12] for the nonconvex quadratic programming.

The proposed p norm minimization deflation (PNMD) algorithm is given as follows. There are two unclear points in the PNMD algorithm. One is how to compute the parameter α in problem (10), and the other is which removal strategy will be used in the admission control step. Next, we make clear of these two points, i.e., we shall use α given in (12) and the removal strategy (13) in the new deflation algorithm.

In the NLPD algorithm [6], the parameter α is given by

$$\alpha = \begin{cases} c_1 \alpha_1, & \text{if } \rho(\mathbf{I} - \mathbf{A}) \geq 1, \\ c_2 \min\{\alpha_1,\, \alpha_2\}, & \text{if } \rho(\mathbf{I} - \mathbf{A}) < 1, \end{cases} \tag{11}$$

where $0 < c_1 \leq c_2 < 1$ are two constants, and α_1 is determined by the equivalence between problem (3) and the joint problem (1) and (2), and α_2 is determined by the so-called "Never-Over-Removal" property. Since the ℓ_p-minimization problem (8) is closer to the ℓ_0-minimization problem (3), we relax the parameter α in (11) to

$$\alpha = \begin{cases} c_1 \alpha_1, & \text{if } \rho(\mathbf{I} - \mathbf{A}) \geq 1, \\ \min\{c_2 \alpha_1,\, c_3 \alpha_2\}, & \text{if } \rho(\mathbf{I} - \mathbf{A}) < 1, \end{cases} \tag{12}$$

where $c_3 > c_2$, $0 < c_1, c_2 < 1$ are three constants.

Having obtained the solution $(\mathbf{q}_e,\ \mathbf{q},\ \mathbf{s})$ of problem (10), we use the removal strategy called SMART rule in [3] to drop the link k_0 according to

$$k_0 = \arg\max_{k \in \mathcal{K}} \left\{ \sum_{j \neq k} |a_{kj}| q_j + \sum_{j \neq k} |a_{jk}| q_k + c_k \right\}. \tag{13}$$

The above operation can be interpreted as removing the link with the largest interference plus noise footprint in the *normalized* network.

The PNMD Algorithm

Step 1. Initialization: Input data $(\mathbf{A}, \mathbf{c}, \bar{\mathbf{p}})$.

Step 2. Preprocessing: Remove link k_0 iteratively according to (7) until condition (6) holds true.

Step 3. Power control: Compute the parameter α and solve problem (10); check whether all links are supported: if yes, go to **Step 5**; else go to **Step 4**.

Step 4. Admission control: Remove link k_0 according to some removal strategy, set $\mathcal{K} = \mathcal{K} / \{k_0\}$, and go to **Step 3**.

Step 5. Postprocessing: Check the removed links for possible admission.

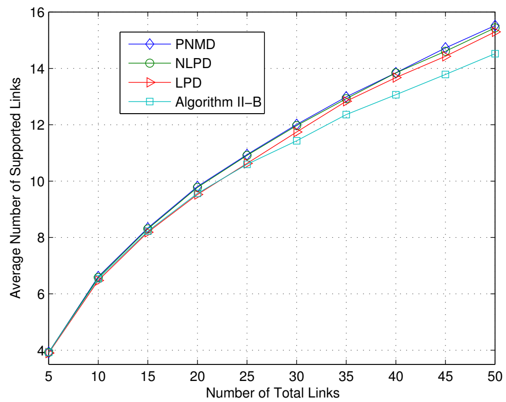

Fig. 1. Average number of supported links versus the number of total links.

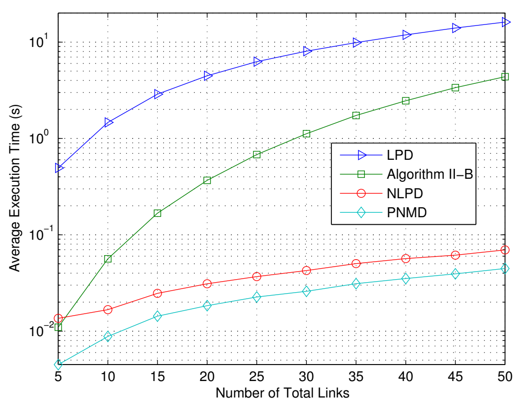

Fig. 2. Average CPU time versus the number of total links.

5. NUMERICAL SIMULATIONS

We generate the same channel parameters as in [1] in our numerical simulations, i.e., each transmitter's location obeys the uniform distribution over a 2 Km $\times$ 2 Km square and the location of its corresponding receiver is uniformly generated in a disc with radius 400 m; channel gains are given by $g_{kj} = 1/d_{kj}^4$ $(\forall\, k,\ j \in \mathcal{K})$, where d_{kj} is the Euclidean distance from the link of transmitter j to the link of receiver k. Each link's SINR target is set to be $\gamma_k = 2$ dB $(\forall\, k \in \mathcal{K})$ and the noise power is set to be $\eta_k = -90$ dBm $(\forall\, k \in \mathcal{K})$. The power budget of the link of transmitter k is $\bar{p}_k = 2p_k^{\min}$ $(\forall\, k \in \mathcal{K})$, where $p_k^{\min}$ is the minimum power needed for link k to meet its SINR requirement in the absence of any interference from other links.

The parameter p in problem (8) is set to be 0.5 and the ones in (12) are set to be $c_1 = c_2 = 0.2$ and $c_3 = 4$. The number of supported links, the total transmission power, and the CPU time are the metrics we employ to compare the performance of the proposed PNMD algorithm with that of the LPD algorithm in [1], the Algorithm II-B in [2], and the NLPD algorithm in [6]. All figures are obtained

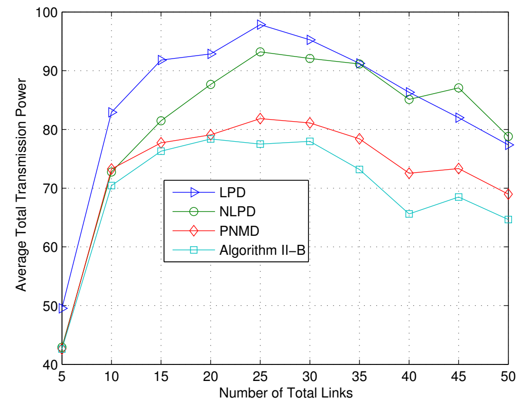

Fig. 3. Average transmission power versus the number of total links.

by averaging over 200 Monte-Carlo runs.

Figs. 1, 2, and 3 indicate that the PNMD algorithm can take less CPU time to support more links while with less total transmission power than the existing algorithms (except the Algorithm II-B). As shown in Fig. 3, the Algorithm II-B transmits the least power among the tested algorithms. This is because the Algorithm II-B supports the least number of links; see Fig. 1. In particular, compared to the NLPD algorithm[2], the proposed PNMD algorithm can support (slightly) more links with much less total transmission power, and at the same time takes less CPU time.

The performance improvement of the proposed PNMD algorithm over the NLPD algorithm is mainly attributed to the ℓ_p-approximation problem (8). The simulation results in Fig. 1 and Fig. 3 show that the admissible set $\mathcal{S}_1$ obtained by the proposed PNMD algorithm based on the ℓ_p-approximation problem (8) is "better" than the admissible set $\mathcal{S}_2$ obtained by the NLPD algorithm based on the ℓ_1-approximation problem (4), i.e., although the cardinality of the two admissible sets $\mathcal{S}_1$ and $\mathcal{S}_2$ is nearly equal to each other, it takes much less total transmission power to support the links in $\mathcal{S}_1$ than to support the links in $\mathcal{S}_2$. This is consistent with our intuition that the p $(0 < p < 1)$ norm minimization problem (8) is capable of approximating the ℓ_0-minimization problem (3) better than the ℓ_1-minimization problem (4) and the fact that the maximum admissible set for the joint power and admission control problem may not be unique.

6. CONCLUSIONS

In this paper, we have developed a p $(0 < p < 1)$ norm minimization deflation algorithm for the joint power and admission control problem. Numerical simulations show the proposed algorithm outperforms state-of-the-arts in [1, 2, 6] in terms of the number of supported links, the total transmission power, and the CPU time .

[2]To the best of our knowledge, the NLPD algorithm is so far the best removal-based algorithm for the joint power and admission control problem. It is shown in [6] that the NLPD algorithm can achieve more than 98% of global optimality in terms of the number of supported links when $K \leq 18$.

7. REFERENCES

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