OPTIMAL CHUNK-BASED RESOURCE ALLOCATION FOR OFDMA SYSTEMS WITH MULTIPLE BER REQUIREMENTS

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ABSTRACT

We investigate the chunk-based resource allocation for OFDMA downlink, where data streams contain packets with diverse bit-error-rate (BER) requirements. Supposing adaptive transmissions based on a number of discrete modulation and coding modes, we derive the optimal scheme that maximizes the weighted sum of average user rates under the multiple BER and total power constraints. With the relevant optimization problem cast as an integer linear program, we show that the optimal strategy can be obtained through Lagrange dual-based gradient iterations with fast convergence and low computational complexity per iteration. Furthermore, a novel on-line algorithm is developed to approach the optimal strategy without knowledge of intended wireless channels a priori.

Keywords: Chunk-based resource allocation, OFDMA, Lagrange dual approach, stochastic optimization.

1. INTRODUCTION

Efficient resource allocation for diverse data streams in high-speed environment has attracted interest in the next-generation wireless network design. Since orthogonal frequency division multiple-access (OFDMA) can convert the frequency selective channel to multiple flat fading channels and eliminate inter-symbol interference in broadband channels, it has been widely adopted for wireless applications in high-speed (railway) systems [1].

Resource allocation for OFDMA networks has attracted a growing research interest [2, 3]. Most of the prior works assumed that individual subcarriers can be assigned to a user. In practical OFDMA systems, however, single-subcarrier based allocation schemes incur significant signaling overhead and entail complicated implementation [4]. To mitigate these defects, the correlation between adjacent subcarriers is utilized in wireless standards by grouping a set of subcarriers into one chunk, and making a chunk as the minimum allocation unit. Resource allocation for the chunk-based OFDMA systems was addressed in [5, 6], where heuristic algorithms were proposed to maximize the total throughput. Optimal chunk allocation strategies were developed to maximize a utility function of average user rates for a wireless OFDMA system under different power control policies in [7], where it is assumed that continuous rate adaptation up to Shannon's limit can be supported. With practical modulation schemes employed for rate adaptation, a recent work [1] proposed a heuristic algorithm to minimize the total transmit power for real-time data streams with multiple bit-error-rate (BER) requirements.

In this paper, we investigate the chunk-based resource allocation for OFDMA downlink, where data streams contain packets with diverse BER requirements. Supposing that adaptive transmissions are based on a number of discrete modulation and coding modes, we derive the optimal resource allocation scheme that maximizes the weighted sum of average user rates under the multiple BER and total power constraints. Specifically, we formulate the intended optimization problem as an integer linear program (ILP). Provided that the ergodic wireless fading process has a continuous cumulative distribution function (cdf), we show that the optimal strategy for this problem adopts a greedy approach, and it can be obtained through Lagrange dual-based gradient iterations with fast convergence and low computational complexity per iteration. Relying on the stochastic optimization tools, we further develop an on-line algorithm capable of dynamically learning the underlying channel distribution and asymptotically approaching the optimal strategy without knowledge of intended wireless channels a priori.

2. SYSTEM MODELS

We consider a downlink OFDMA system consisting of an access point (AP) and K wireless users $k = 1, \cdots, K$. The overall bandwidth B is divided into $M \times J$ orthogonal narrowband subcarriers, each with sub-bandwidth $\triangle f = B/(MJ)$ small enough for each subcarrier to experience only flat fading. As pre-determined by the practical systems or wireless standards, the subcarriers are grouped into M chunks. Each chunk m consists of J adjacent subcarriers $j = (m-1)J + 1, \ldots, mJ$. The data streams from the AP to users contain packets with different quality of service requirements, e.g., video streams consisting of the base and enhancement layers of packets [8], where the base layer is to provide coarse visual quality whereas enhancement layers are to enhance visual quality when the base layer is available. The reliable transmission of the base layer is clearly important and it should have a high quality (HQ), i.e., smaller BER, requirement than the enhancement layers with relative low quality (LQ) requirements. For this reason, we assume that each data stream has two types of packets: HQ and LQ packets, which are queued into two separate buffers (i.e., queues 0 and 1) with dif-

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ferent BER requirements $\check{\epsilon}^{(0)}$ and $\check{\epsilon}^{(1)}$ ($\check{\epsilon}^{(0)} \leq \check{\epsilon}^{(1)}$), respectively. In addition, depending on the given encoding and decoding schemes, a constant ratio η needs to be maintained between the numbers of HQ and LQ packets of the same data stream delivered to each user.

Let $h_{k,n}$ denote the frequency-domain channel coefficient for the kth user over the nth subcarrier, $k = 1, \ldots, K, n = 1, \ldots, MJ$. Assume a block fading model, where the fading coefficients $\boldsymbol{h} = \{h_{k,n}^{(j)},\ k = 1, \ldots, K,\ n = 1, \ldots, MJ\}$ are fixed per (coherent time) slot n, but are allowed to change randomly from slot to slot according to a stationary and ergodic random process with a continuous cdf. The wireless channel is assumed to be frequency selective, and the correlation coefficient between any two, say the n_1th and n_2th subcarriers per user k, is given by [9]:

$$\rho_{n_1,n_2} = \mathbb{E}\{h_{k,n_1}^* h_{k,n_2}\} = \frac{1}{\sqrt{1 + (\frac{(n_1-n_2)\triangle f}{f_c})^2}} \quad (1)$$

where f_c is the channel coherence bandwidth, and $\triangle f$ is the frequency separation between two adjacent subcarriers.

Relying on a finite set of modulation and coding pairs (modes), adaptive modulation and coding (AMC) is adopted by the AP for downlink transmission with rate ρ_l, $l = 1, \ldots, L$. For a given channel power gain $\gamma_{k,n} = |h_{k,n}|^2$ and the transmit-power $\pi_{k,n}$, let $\epsilon_{k,n,l}(\gamma_{k,n}\pi_{k,n})$ denote the instantaneous BER for the transmission to the kth user over the nth subcarrier when the lth mode is employed. Assuming without loss of generality that the additive white Gaussian noise (AWGN) at the receiver has unit variance, it is known that $\epsilon_{k,n,l}$ can be approximated in a closed form [10]:

$$\epsilon_{k,n,l}(\gamma_{k,n}\pi_{k,n}) \simeq \kappa_1 \exp(-\frac{\kappa_2\gamma_{k,n}\pi_{k,n}}{2^{\rho_l} - 1}) \quad (2)$$

where κ_1 and κ_2 are mode-dependent constants.

Consider the case that $\gamma_{k,n}$ are highly correlated to each other within a chunk. Then it was proven that the average channel gain $\gamma_{k,m} \simeq \frac{1}{J}\sum_{n=(m-1)J+1}^{mJ} \gamma_{k,n}$, can be used to calculate the average BER across the subcarriers for the chunk m; i.e., with a power $\pi_{k,m}$ per subcarrier, the average BER for transmission to the kth user with the lth AMC mode over the mth chunk is given by $\bar{\epsilon}(k,m,l) \simeq \kappa_1 \exp(-\frac{\kappa_2\gamma_{k,m}\pi_{k,m}}{2^{\rho_l}-1})$ [1]. To meet the prescribed BER requirement $\check{\epsilon}^{(q)}$, $q = 1, 2$, for the HQ or LQ data packets, we can then solve the equation $\bar{\epsilon}(k,m,l) = \check{\epsilon}^{(q)}$ to obtain the minimum transmit-power required for each (k,m,l,q) quadruplet as:

$$\pi_{k,m,l}^{(q)}(\boldsymbol{\gamma}) := \pi_{k,m,l}^{(q)}(\gamma_{k,m}) = (\frac{\Gamma^{(q)}}{\gamma_{k,m}})(2^{\rho_l} - 1) \quad (3)$$

where $\Gamma^{(q)} := \kappa_2^{-1}\ln(\frac{\kappa_1}{\check{\epsilon}^{(q)}})$.

For notational convenience, in addition to AMC modes $l = 1, \ldots, L$ with non-zeros rates $\rho_l > 0$, we let $l = 0$ mode to denote no transmission, thus $\rho_0 = \pi_{k,m,l}^{(q)}(\boldsymbol{\gamma}) = 0, \forall k, m, \boldsymbol{\gamma}$.

3. OPTIMAL CHUNK-BASED RESOURCE ALLOCATION

Upon channel realization $\boldsymbol{\gamma}$, let $\alpha_{k,m,l}^{(q)}(\boldsymbol{\gamma})$ be the chunk allocation decision for transmission of packets from the kth user's qth (i.e., HQ or LQ) queue with the lth mode over chunk m, and let $\boldsymbol{\alpha}(\boldsymbol{\gamma}) := \{\alpha_{k,m,l}^{(q)}(\boldsymbol{\gamma}),\ k = 1, \cdots, K,\ m = 1, \cdots, M,\ , l = 0, \cdots, L,\ q = 0, 1\}$. We consider that only packets from one queue can be chosen to be transmitted with a single mode when user k is scheduled for transmission over chunk m. In addition, at most one user can be allocated to a single chunk m ($m = 1, \cdots, M$) in practical systems. This then implies an exclusive chunk allocation; i.e.,

$$\alpha_{k,m,l}^{(q)}(\boldsymbol{\gamma}) \in \{0, 1\}, \quad \sum_{k=1}^{K}\sum_{l=0}^{L}\sum_{q=0}^{1} \alpha_{k,m,l}^{(q)}(\boldsymbol{\gamma}) = 1,\ \forall m. \quad (4)$$

Let $\mathcal{A}$ denote the set of all chunk schedules satisfying the latter constraints. For a given chunk schedule $\boldsymbol{\alpha}$, the average HQ, LQ and total data rates for user $k = 1, \ldots, K$ are:

$$\bar{r}_k^{\mathrm{HQ}}(\boldsymbol{\alpha}) = \mathbb{E}_{\boldsymbol{\gamma}}\Big[\sum_{m=1}^{M}\sum_{l=0}^{L} \alpha_{k,m,l}^{(0)}(\boldsymbol{\gamma})J\rho_l\Big], \quad (5)$$

$$\bar{r}_k^{\mathrm{LQ}}(\boldsymbol{\alpha}) = \mathbb{E}_{\boldsymbol{\gamma}}\Big[\sum_{m=1}^{M}\sum_{l=0}^{L} \alpha_{k,m,l}^{(1)}(\boldsymbol{\gamma})J\rho_l\Big], \quad (6)$$

$$\bar{r}_k(\boldsymbol{\alpha}) = \bar{r}_k^{\mathrm{HQ}} + \bar{r}_k^{\mathrm{LQ}} = \mathbb{E}_{\boldsymbol{\gamma}}\Big[\sum_{m=1}^{M}\sum_{l=1}^{L}\sum_{q=0}^{1} \alpha_{k,m,l}^{(q)}J\rho_l\Big], \quad (7)$$

and the total power spent for downlink transmissions is:

$$\bar{P}(\boldsymbol{\alpha}) = \mathbb{E}_{\boldsymbol{\gamma}}\Big[\sum_{k=1}^{K}\sum_{m=1}^{M}\sum_{l=0}^{L}\sum_{q=0}^{1} \alpha_{k,m,l}^{(q)}(\boldsymbol{\gamma})J\pi_{k,m,l}^{(q)}(\boldsymbol{\gamma})\Big], \quad (8)$$

where $\mathbb{E}_{\boldsymbol{\gamma}}[\cdot]$ denotes expectation over fading realization $\boldsymbol{\gamma}$.

The AP is to decide the optimal schedule $\boldsymbol{\alpha} \in \mathcal{A}$ that maximizes the weighted sum of average user rates; i.e., we wish to solve:

$$\begin{aligned} &\max_{\boldsymbol{\alpha}\in\mathcal{A}} \quad \sum_{k=1}^{K} w_k \bar{r}_k(\boldsymbol{\alpha}) \\ &\text{s. t.} \quad \bar{P}(\boldsymbol{\alpha}) \leq \check{P}, \quad \bar{r}_k^{\mathrm{HQ}}(\boldsymbol{\alpha}) = \eta\bar{r}_k^{\mathrm{LQ}}(\boldsymbol{\alpha}),\ \ \forall k \end{aligned} \quad (9)$$

where w_k is the given priority weight for user k, $\check{P}$ is the total power budget at the AP, and η is the prescribed HQ-LQ packet ratio that needs to be maintained for all users.

After proper formulation, it follows from (4)–(9) that the problem becomes an ILP. We next show that this ILP can be efficiently solved through dual-based Lagrange approach.

3.1. Lagrange Dual Approach

Let λ denote the Lagrange multiplier associated with the power constraint, and let $\boldsymbol{\mu} := \{\mu_k,\ \forall k\}$ collect the Lagrange multipliers associated with HQ-LQ ratio constraints. With the convenient notation $\boldsymbol{\Lambda} := \{\lambda,\ \boldsymbol{\mu}\}$, the Lagrangian function of (9) becomes:

$$L(\boldsymbol{\alpha}, \boldsymbol{\Lambda}) = \sum_{k=1}^{K} w_k\bar{r}_k(\boldsymbol{\alpha}) - \lambda[\bar{P}(\boldsymbol{\alpha}) - \check{P}] - \sum_{k=1}^{K} \mu_k[\bar{r}_k^{\mathrm{HQ}}(\boldsymbol{\alpha}) - \eta\bar{r}_k^{\mathrm{LQ}}(\boldsymbol{\alpha})] \quad (10)$$

The Lagrange dual function is then given by:

$$D(\boldsymbol{\Lambda}) = \max_{\boldsymbol{\alpha}\in\mathcal{A}} L(\boldsymbol{\alpha}, \boldsymbol{\Lambda}), \quad (11)$$

and the dual problem of (9) is :

$$\min_{\boldsymbol{\Lambda}} D(\boldsymbol{\Lambda}). \quad (12)$$

Upon defining:

$$\varphi_{k,m,l}^{(q)}(\mathbf{\Lambda};\boldsymbol{\gamma}) := \begin{cases} w_k J\rho_l - \lambda J\pi_{k,m,l}^{(0)}(\boldsymbol{\gamma}) - \mu_k J\rho_l, & q=0, \\ w_k J\rho_l - \lambda J\pi_{k,m,l}^{(1)}(\boldsymbol{\gamma}) + \mu_k \eta J\rho_l, & q=1, \end{cases} \tag{13}$$

the Lagrangian (10) can be rewritten as:

$$L(\boldsymbol{\alpha},\mathbf{\Lambda}) = \lambda\check{P} + \mathbb{E}_{\boldsymbol{\gamma}}\Big[\sum_{m=1}^{M}\Big\{\sum_{k,l,q}\alpha_{k,m,l}^{(q)}(\boldsymbol{\gamma})\varphi_{k,m,l}^{(q)}(\mathbf{\Lambda};\boldsymbol{\gamma})\Big\}\Big].$$

For a given $\mathbf{\Lambda}$, the dual-optimal $\boldsymbol{\alpha}^*(\mathbf{\Lambda})$ then solves per $\boldsymbol{\gamma}$:

$$\max_{\boldsymbol{\alpha}\in\mathcal{A}} \sum_{m=1}^{M}\Big\{\sum_{k,l,q}\alpha_{k,m,l}^{(q)}(\boldsymbol{\gamma})\varphi_{k,m,l}^{(q)}(\mathbf{\Lambda};\boldsymbol{\gamma})\Big\}$$

Under the constraints $\alpha_{k,m,l}^{(q)}(\boldsymbol{\gamma}) \in \{0,1\}$ and $\sum_{k,l,q}\alpha_{k,m,l}^{(q)}(\boldsymbol{\gamma})$ $=1$ per m, clearly the optimal chunk allocation should adopt a "winner-takes-all" strategy; i.e., chunk m is assigned to a triplet

$$\{k_m^*, l_m^*, q_m^*\}(\mathbf{\Lambda};\boldsymbol{\gamma}) = \arg\max_{(k,l,q)} \varphi_{k,m,l}^{(q)}(\mathbf{\Lambda};\boldsymbol{\gamma}), \quad \forall m. \tag{14}$$

Lemma 1 *To maximize $L(\boldsymbol{\alpha},\mathbf{\Lambda})$ for a given $\mathbf{\Lambda}$ in (11), the optimal chunk allocation at AP amounts to a greedy strategy:*

$$\begin{cases} \alpha_{k,m,l}^{(q)*}(\mathbf{\Lambda};\boldsymbol{\gamma}) = 1, & \{k,l,q\} = \{k_m^*, l_m^*, q_m^*\}(\mathbf{\Lambda};\boldsymbol{\gamma}), \\ \alpha_{k,m,l}^{(q)*}(\mathbf{\Lambda};\boldsymbol{\gamma}) = 0, & \textit{otherwise}; \end{cases} \tag{15}$$

where the "winner" $\{k,l,q\} = \{k_m^*, l_m^*, q_m^*\}(\mathbf{\Lambda};\boldsymbol{\gamma})$ per chunk m per fading realization $\boldsymbol{\gamma}$ is chosen from (14).

Lemma 1 states that the optimal chunk schedule $\alpha_{k,m,l}^{(q)*}(\mathbf{\Lambda};\boldsymbol{\gamma})$ admits a greedy policy, where $\varphi_{k,m,l}^{(q)*}(\mathbf{\Lambda};\boldsymbol{\gamma})$ can be seen as net-reward (reward minus cost) that packet transmission from the kth user's qth queue with the lth mode can obtain over the chunk m per $\boldsymbol{\gamma}$. Comparing with net-rewards across users, modes, and queues, chunk m is then assigned to the triplet $\{k_m^*, l_m^*, q_m^*\}(\mathbf{\Lambda};\boldsymbol{\gamma})$ with the largest net-reward.

With $\boldsymbol{\alpha}^*(\mathbf{\Lambda})$ from Lemma 1 for a given $\mathbf{\Lambda}$, the dual problem in (12) can be solved through the subgradient iterations. Define

$$\boldsymbol{g}(\boldsymbol{\alpha}) := [\check{P}-\bar{P}(\boldsymbol{\alpha}), \eta\bar{r}_1^{\mathrm{LQ}}(\boldsymbol{\alpha})-\bar{r}_1^{\mathrm{HQ}}(\boldsymbol{\alpha}), \ldots, \eta\bar{r}_K^{\mathrm{LQ}}(\boldsymbol{\alpha})-\bar{r}_K^{\mathrm{HQ}}(\boldsymbol{\alpha})]^T.$$

Using $\boldsymbol{\alpha}^*(\mathbf{\Lambda})$, it can be shown that $\boldsymbol{g}(\boldsymbol{\alpha}^*(\mathbf{\Lambda}))$ is a (sub-)gradient of dual function $D(\mathbf{\Lambda})$. Therefore, the dual problem (12) can be solved through the following (sub-)gradient descent iteration[11]:

$$\mathbf{\Lambda}[n+1] = [\mathbf{\Lambda}[n] - \beta\boldsymbol{g}(\boldsymbol{\alpha}^*(\mathbf{\Lambda}[n]))]^+; \tag{16}$$

specifically, we have (with short-hand notation $\boldsymbol{\alpha}^*[n] := \boldsymbol{\alpha}^*(\mathbf{\Lambda}[n])$):

$$\begin{aligned} \lambda[n+1] &= \left[\lambda[n] + \beta\left(\bar{P}(\boldsymbol{\alpha}^*[n]) - \check{P}\right)\right]^+ \\ \mu_k[n+1] &= \mu_k[n] + \beta(\bar{r}_k^{\mathrm{HQ}}(\boldsymbol{\alpha}^*[n]) - \eta\bar{r}_k^{\mathrm{LQ}}(\boldsymbol{\alpha}^*[n])) \end{aligned} \tag{17}$$

where β is a small stepsize, n is iteration index, and $[x]^+ = \max(0,x)$. Convergence of gradient iteration (17) to the optimal $\mathbf{\Lambda}^* := \{\lambda^*, \boldsymbol{\mu}^*\}$ for (12) is guaranteed from any initial $\mathbf{\Lambda}[0] \geq 0$ [11].

Since (9) is a (non-convex) ILP, there may exist a non-zero duality gap; therefore, solving the dual problem (12) via dual-based sub-gradient iterations (17) may not yield the optimal solution for (9). However, under the condition that random fading $\boldsymbol{\gamma}$ has a continuous cdf, we can mimic the proof of [7, Proposition 1] to show:

Proposition 1 *For ergodic fading process with a continuous cdf, problem (9) has a zero-duality gap with its dual (12), and the almost surely optimal solution for (9) is given by $\{\alpha^*(\mathbf{\Lambda}^*;\boldsymbol{\gamma}),\ \forall\boldsymbol{\gamma}\}$, where $\mathbf{\Lambda}^*$ is obtained from (17) with any initial $\mathbf{\Lambda}[0] \geq 0$.*

3.2. Stochastic Chunk Allocation

From Proposition 1, the dual-gradient iteration (17) can yield the globally optimal resource allocation strategy for (9) under condition. To implement (17), a-priori knowledge of channel cdf should be available, only with which we can evaluate the average rates $\bar{r}_k^{\mathrm{HQ}}(\boldsymbol{\alpha}^*(\mathbf{\Lambda}))$, $\bar{r}_k^{\mathrm{LQ}}(\boldsymbol{\alpha}^*(\mathbf{\Lambda}))$ and average power $\bar{P}(\boldsymbol{\alpha}^*(\mathbf{\Lambda}))$ in (17). However, practical applications motivate resource allocation schemes that can operate without the knowledge of channel cdf, but approach the optimal strategy by "learning" channel statistics on-the-fly.

To this end, we rely on a stochastic optimization paradigm [3, 12] to develop a *stochastic gradient* iteration from (17) as follows. Given a Lagrange multiplier vector $\mathbf{\Lambda}$ and the fading realization $\boldsymbol{\gamma}[n]$ per slot n, the AP transmits in accordance with the chunk allocation strategy (15). After transmissions, it collects the values of instantaneous powers per chunk m:

$$P_m(\mathbf{\Lambda};\boldsymbol{\gamma}[n]) = J\pi_{k_m^*,m,l_m^*}^{(q_m^*)}(\mathbf{\Lambda};\boldsymbol{\gamma}[n]), \tag{18}$$

and instantaneous HQ-LQ balance at chunk m per user k:

$$Q_{k,m}(\mathbf{\Lambda};\boldsymbol{\gamma}[n]) = \begin{cases} J\rho_{l_m^*} & \text{if } k = k_m^* \ \&\ q_m^* = 0, \\ -\eta J\rho_{l_m^*} & \text{if } k = k_m^* \ \&\ q_m^* = 1, \\ 0 & \text{if } k \neq k_m^*; \end{cases} \tag{19}$$

where we omit the dependence of $\{k_m^*, l_m^*, q_m^*\}$ on $(\mathbf{\Lambda};\boldsymbol{\gamma}[n])$.

With $\{P_m(\hat{\mathbf{\Lambda}}[n];\boldsymbol{\gamma}[n]), Q_{k,m}(\hat{\mathbf{\Lambda}}[n];\boldsymbol{\gamma}[n])\}$ collected at the end of slot n, we propose that the AP implements the following stochastic gradient descent iterations:

$$\begin{aligned} \hat{\lambda}[n+1] &= [\hat{\lambda}[n] + \beta(\sum_{m=1}^{M} P_m(\hat{\mathbf{\Lambda}}[n];\boldsymbol{\gamma}[n]) - \check{P})]^+ \\ \hat{\mu}_k[n+1] &= \hat{\mu}_k[n] + \beta\sum_{m=1}^{M} Q_{k,m}(\hat{\mathbf{\Lambda}}[n];\boldsymbol{\gamma}[n]), \quad \forall k \end{aligned} \tag{20}$$

where hats are to stress that these iterations are stochastic estimates of those in (17), based on *instantaneous* (instead of average) rates and powers. Notice that here, n stands for both iteration and slot indices; in other words, each iteration of (20) will be run per slot.

As with (16), we can re-write (20) into a compact form:

$$\hat{\mathbf{\Lambda}}[n+1] = \left[\hat{\mathbf{\Lambda}}[n] - \beta\hat{\boldsymbol{g}}\left(\boldsymbol{\alpha}^*(\hat{\mathbf{\Lambda}}[n];\boldsymbol{\gamma}[n])\right)\right]^+ \tag{21}$$

where $\hat{\boldsymbol{g}}\left(\boldsymbol{\alpha}^*(\hat{\mathbf{\Lambda}}[n]);\boldsymbol{\gamma}[n]\right)$ is a stochastic gradient depending on current fading realization $\boldsymbol{\gamma}[n]$. Provided that the random fading process is ergodic, it can be shown that $\mathbb{E}_{\boldsymbol{\gamma}[n]}\left[\hat{\boldsymbol{g}}\left(\boldsymbol{\alpha}^*(\mathbf{\Lambda};\boldsymbol{\gamma}[n])\right)\right] = \boldsymbol{g}(\boldsymbol{\alpha}^*(\mathbf{\Lambda}))$; i.e., stochastic gradient $\hat{\boldsymbol{g}}$ is a random realization of the "ensemble" gradient $\boldsymbol{g}$.

The convergence of stochastic (sub-)gradient iteration (21) to the optimal $\mathbf{\Lambda}^*$ in probability can be established as the stepsize $\beta \to 0$, along the similar lines in [3, 12]. Due to the zero-duality gap result in Proposition 1, the companion chunk allocation scheme $\boldsymbol{\alpha}^*(\hat{\boldsymbol{\Lambda}};\boldsymbol{\gamma})$ also converges (in probability) to the globally optimal one for (9).

An online scheduling algorithm can be then implemented at AP as follows: s1) Starting from arbitrary $\hat{\boldsymbol{\Lambda}}[0] \geq 0$, determine the on-line chunk allocation $\boldsymbol{\alpha}^*(\hat{\boldsymbol{\Lambda}}[n];\boldsymbol{\gamma}[n])$ per slot n using greedy strategy (15); s2) Update $\hat{\boldsymbol{\Lambda}}[n+1]$ from $\hat{\boldsymbol{\Lambda}}[n]$ using (21); then implement s1) and s2) again for the next slot $n+1$. In this algorithm, the AP only needs to calculate the net-rewards $\varphi_{k,m,l}^{(q)}(\hat{\boldsymbol{\Lambda}}[n];\boldsymbol{\gamma}[n])$ for all the K users, L modes, 2 queues per chunk m, and then adopts the "winner-takes-all" strategy in Lemma 1 to determine the optimal chunk assignment policy taken per slot n. It requires a linear computational complexity of $\mathcal{O}(2KML)$ to calculate the net-rewards $\varphi_{k,m,l}^{(q)}(\hat{\boldsymbol{\Lambda}}[n];\boldsymbol{\gamma}[n])$, whereas determining the triplet $\{k_m^*, l_m^*, q_m^*\}$ per chunk m for the greedy strategy (15) needs a computational complexity of $\mathcal{O}(M \cdot \log(2KL))$. Hence, all required operations per slot have a linear computational complexity of $\mathcal{O}(2KML)$ in the number of users, chunks, modes and packet queues. Moreover, such a low complexity algorithm is capable of learning the channel distribution to approach the optimal scheduling and resource allocation without *a priori* knowledge of channel cdf.

4. NUMERICAL RESULTS

Consider a $K=4$ user OFDMA downlink with bandwidth $B=512$ KHz and the channel coherence time $T=1$ ms. The subcarriers are grouped into $M=8$ chunks, each consisting of $J=16$ subcarriers. Each data stream has two types of packets with BER requirements $\check{\epsilon}^{(0)} = 10^{-6}$ for HQ and $\check{\epsilon}^{(1)} = 10^{-2}$ for LQ packets. A ratio $\eta = 1$ needs to be maintained between the numbers of HQ and LQ packets. The frequency separation between two adjacent subcarriers $\triangle f = 4$ KHz, and channel coherence bandwidth $f_c = 80$ KHz. The L_b-ary quadrature amplitude modulation (QAM) is employed for AMC, where L_b can take values from $\{0, 2^2, \cdots, 2^{12}\}$. For mode $l=0$, $L_b = 0$, i.e., no transmission; whereas $L_b = 2^{2l}$ and AMC rate $\rho_l = 2l$, for $l = 1, \ldots, 6$.

To gauge the performance of the proposed scheme, we compare it with three heuristic schemes: i) a fixed-user-chunk scheme (denoted by heu1), where each of the four users is exclusively assigned $M/4 = 2$ chunks and power budget $\check{P}/4$, and optimal chunk-based allocation is performed per user; ii) a fixed-queue-chunk scheme (denoted by heu2), where each of the eight user queues is exclusively assigned $M/8 = 1$ chunk and power budget $\check{P}/8$, and rate control is performed to meet the BER requirement per queue; iii) a fixed-power scheme (denoted by heu3), where each of the total 8 queues of the 4 users is exclusively assigned $M/8 = 1$ chunk, and transmit-power is fixed at $\check{P}/8$ per chunk per slot. Fig. 1 shows the resulting average sum-rates (i.e., $w_k = 1, \forall k$) for the optimal and heuristic schemes under different sum-power budget, where each result was obtained as the average of 20 independent runs, and in each run the system was simulated for a time period equivalent to 9,000 ms. It is clearly seen the proposed optimal scheme significantly outperforms the heuristic schemes for all $\check{P}$ values since it is capable of fully exploiting all the available spectral, temporal, and multi-user diversity on channel fading. The heu1 scheme ignores multi-user diversity, leading to performance loss. The heu2 scheme not only ignores multiuser diversity, but the spectral diversity; thus more performance loss is incurred. The heu3 scheme causes an extra significant performance loss due to the ignoring of all available diversity and underutilization of the power budget. Overall, Fig. 1 clearly demonstrates that the proposed optimal scheme can result in large throughput gain over heuristic schemes, and this gain becomes more pronounced as the power budget $\check{P}$ increases.

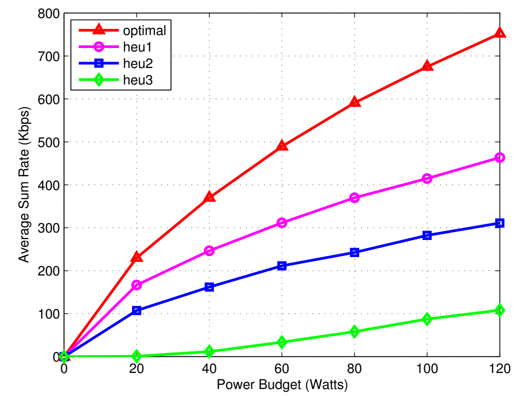

Fig. 1. Comparison of average sum-rates.

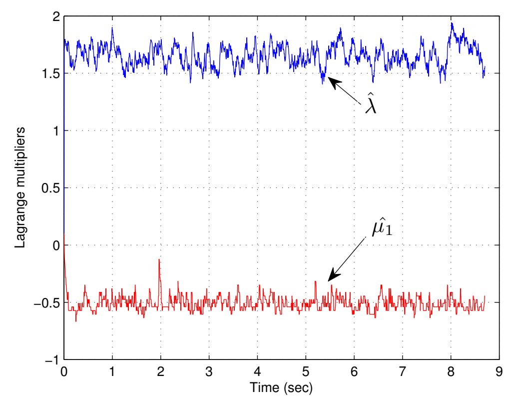

Fig. 2. Evolution of Lagrange multipliers for optimal scheme.

The channel cdf was assumed unknown a priori in simulations, and the proposed stochastic scheme is capable of learning this knowledge on-the-fly and approaching the optimal strategy. Fig. 2 depicts the evolution of Lagrange multipliers $\hat{\lambda}$ and $\hat{\mu_1}$ for the optimal scheme when the power budget $\check{P} = 40$ Watts and stepsize $\beta = 0.001$. It is clearly observed that the Lagrange multipliers quickly converge to the neighborhood of the optimal values.

5. CONCLUSIONS

We solved the optimal chunk-based allocation for OFDMA downlink, where AMC modes are employed for transmission and data streams contain packets with diverse BER requirements. It was shown that the optimal scheme can be obtained through dual-gradient iterations. Stochastic optimization method was then adopted to develop an on-line algorithm capable of dynamically learning the channel statistics and converging to the optimal benchmark.

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