BACKHAUL-CONSTRAINED OPTIMIZATION FOR HYBRID ACCESS SMALL CELLS

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ABSTRACT

In this paper, we investigate the limited-capacity backhaul's impact on small cell holders' (SHs') utilities and the mobile network operator's (MNO's) net revenue under a refunding framework. SHs are reluctant to share accesses with guest users due to selfish nature. To advocate better resource utilization, the MNO refunds SHs to motivate hybrid access as incentives. We model the interactions between the MNO and SHs as a Stackelberg game: in Stage I, the MNO refunds SHs and we propose a lookup table approach to decide individualized refunding and interference temperature constraints to different SHs; in Stage II, SHs admit guest users and we propose a near-optimal two-phase guest user admission algorithm where guest users are gradually admitted in terms of the minimum increment of sum-log power. Simulation results show that under the limited-capacity backhaul, a higher refunding can increase SHs' utilities while decrease the MNO's net revenue. Hence, the MNO implicitly controls the number of admitted guest users through individualized refunding to maximize its net revenue.

***Index Terms*—** Small Cell, Stackelberg Game, Limited-Capacity Backhaul, Hybrid Access, Refunding

1. INTRODUCTION

Small cells which encompass femtocells, picocells and microcells are low-powered access points operated in licensed spectrum. Recently, small cells are increasingly installed in offices, subways and residential sites to cope with the unprecedented data traffic growth [1, 2]. They use the broadband networks, i.e., asymmetric digital subscriber line or cable modem, to backhaul data to the MNO's core network. Compared to open and closed access modes, the hybrid access mode looks more promising [3, 4], which not only guarantees privileged access to home users while serve guest users with restrictions.

Multitudes of scholarly works have appeared in the literature on the economics of femtocells in [5]-[8]. In particular, the authors propose a utility-aware refunding framework for the hybrid access in femtocell networks [7]. The MNO refunds femtocell holders to motivate hybrid access and then femtocell holders compete for refunding. In [8], it investigates the economic incentive for a cellular operator to add femtocell service on top of its existing macrocell service. However, those works ignore the effect of limited-capacity backhaul.

In fact, limited-capacity backhaul is one of the key constraints for small cell networks, if one small cell's data traffic exceeds its backhaul capacity, it can cause huge delayed data delivery. In literature, its effect usually appears as constraints to the sum-rate in [9, 10] or delay. In [11], it investigates the cooperation framework between the MNO and the backhaul provider in order to provide a desired backhual capacity. The Nash bargaining model is used to solve the revenue division problem. These motivate us to study backhaul-constrained optimization for hybrid access small cell networks under a refunding framework.

The main contributions of this paper are as follows:

- *Novel Stackelberg Game Formulation*: We provide a novel Stackelberg game formulation combining refunding with technical specifications, including SINR, interference temperature, and backhaul constraints;
- *Joint Admission and Power Allocation*: For each SH, we maximize its utility which is a tradeoff between the refunding from the MNO and degradation of home user's Quality of Service (QoS). We propose a low complexity two-phase guest user admission and power allocation algorithm.
- *Individualized Refunding*: We find the near-optimal individualized refunding to different SHs, through which the MNO maximizes its net revenue, which is a tradeoff between the system performance gain and refunding payment. We propose a lookup table approach.

2. PROBLEM FORMULATION

We consider an uplink heterogeneous cellular network, which consists of one macrocell managed by the MNO overlaided with M small cells indexed as $\mathcal{M} = \{1, 2, \cdots, M\}$. Small cells are connected to the macrocell through wired backhaul links. The macrocell and small cells operate in two separate

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frequency bands. Small cells are densely deployed and are affected by intra-cell interference. We consider one time slot and assume that small cell $m \in \mathcal{M}$ has only one home user and N_m guest users indexed as $\mathcal{N}_m = \{1, 2, \cdots, N_m\}$. Note that one guest user belongs to one small cell at a time slot. But N_m can vary at different time slots.

The MNO refunds SHs to motivate hybrid access. Once observed refunding, the SH $m \in \mathcal{M}$ decides guest user admission set Ω_m and corresponding transmission powers $\boldsymbol{p}^m_{\Omega_m}$. The MNO and SHs are selfish and aim to maximize their own utilities. Hence, we formulate a Stackelberg game to analyze the interactions between the MNO and SHs:

- *Stage I*: the MNO decides individualized refunding and interference temperature constraints to different SHs. It maximizes net revenue.
- *Stage II*: Each SH decides its guest user admission set and corresponding transmission powers, respectively. SHs compete in a non-cooperative manner and maximize their utilities, respectively.

For the formulated Stackelberg game, the Stackelberg equilibrium (SE) is defined as follows:

Definition 1 *Denote $(\boldsymbol{B}^\star, \boldsymbol{I}^\star)$ be a feasible solution of $\mathcal{P}_1$ and $(\boldsymbol{p}_{\boldsymbol{\Omega}^\star}, \boldsymbol{\Omega}^\star)$ be a feasible solution of $\mathcal{P}_2$ of the M small cells. Then, the point $(\boldsymbol{B}^\star, \boldsymbol{I}^\star, \boldsymbol{p}_{\boldsymbol{\Omega}^\star}, \boldsymbol{\Omega}^\star)$ is a SE for the formulated Stackelberg game if for any other feasible solution $\left(\boldsymbol{B}', \boldsymbol{I}', \boldsymbol{p}_{\boldsymbol{\Omega}'}, \boldsymbol{\Omega}'\right)$, the following conditions are satisfied:*

$$\begin{aligned} &U_{\text{MNO}}\left(\boldsymbol{B}^\star, \boldsymbol{I}^\star, \boldsymbol{p}_{\boldsymbol{\Omega}^\star}, \boldsymbol{\Omega}^\star\right) \geq U_{\text{MNO}}\left(\boldsymbol{B}'_m, \boldsymbol{I}'_m, \boldsymbol{p}_{\boldsymbol{\Omega}^\star}, \boldsymbol{\Omega}^\star\right) \\ &U_m\left(\boldsymbol{B}^\star, \boldsymbol{I}^\star, \boldsymbol{p}^m_{\boldsymbol{\Omega}^\star_m}, \boldsymbol{\Omega}^\star_m\right) \geq U_m\left(\boldsymbol{B}^\star, \boldsymbol{I}^\star, \boldsymbol{p}^m_{\boldsymbol{\Omega}'_m}, \boldsymbol{\Omega}'_m\right) \forall m \end{aligned} \tag{1}$$

where $\boldsymbol{B}^\star = [B^\star_1, B^\star_2, \cdots, B^\star_M]$, $\boldsymbol{I}^\star = [I^\star_1, I^\star_2, \cdots, I^\star_M]$, $\boldsymbol{p}_{\boldsymbol{\Omega}^\star} = [\boldsymbol{p}^1_{\Omega^\star_1}, \boldsymbol{p}^2_{\Omega^\star_2}, \cdots, \boldsymbol{p}^M_{\Omega^\star_M}]$, and $\boldsymbol{\Omega}^\star = [\Omega^\star_1, \Omega^\star_2, \cdots, \Omega^\star_M]$.

To obtain SE of the formulated Stackelberg game, it is analyzed by backward induction and the goal is to find the subgame perfect equilibrium for the two-stage game. Hence, $\mathcal{P}_2$ is solved for a given $(\boldsymbol{B}, \boldsymbol{I})$ first. Then based on each SH's best response function, we solve $\mathcal{P}_1$ for the optimal $(\boldsymbol{B}^\star, \boldsymbol{I}^\star)$. It is not difficult to see that the MNO assigns (B_m, I_m) to different SH.

2.1. Stage I Problem

The Stage I optimization problem at the MNO is formulated as follows:

$$\mathcal{P}_1 := \begin{cases} \underset{\{B_m\},\{I_m\}}{\text{maximize}} & U_{\text{MNO}} = \sum_{m=1}^{M} (\pi - B_m) \left|\Omega_m\right| \\ \text{subject to} & I_m \geq \nu_m p^m_0 \\ & \sum_{m=1}^{M} I_m \leq Q \\ & 0 \leq B_m \leq \pi \end{cases} \tag{2}$$

where π is the usage-based fee per time slot, B_m is the individualized refunding price, $|\Omega_m|$ is the cardinality of admission set Ω_m, I_m is the interference temperature constraint for small cell m, $v_m p^m_0$ is the lower bound of I_m which accounts for the interference generated by the home user, and Q is the upper bound of the aggregate interference.

The objective function (2) consists of two terms: the first term $\sum_{m=1}^{M} \pi|\Omega_m|$ is the total service revenue from the guest users and the second term $\sum_{m=1}^{M} B_m\left|\Omega_m\right|$ is the total refunding to SHs where small cell m receives $B_m|\Omega_m|$ refunding. It implies that the MNO treats every guest user in the same small cell equally. Note that $|\Omega_m|$ is a implicit integer function of I_m and B_m. Therefore, $\mathcal{P}_1$ is a mixed integer optimization problem and it is generally difficult to solve. Obviously, the MNO has other revenues, such as macrocell service revenues and equipment costs, however, it is not within the scope of this paper and the refunding between the MNO and SHs will not be affected by these operations.

2.2. Stage II Problem

The SINR of the guest user i in small cell m is approximated as

$$\text{SINR}^m_i(\boldsymbol{p}^m_{\Omega_m}) = \frac{p^m_i h^m_i g^m_i}{\sum_{j\neq i,\, j\in\Omega_m} p^m_j h^m_j g^m_j + p^m_0 h^m_0 g^m_0 + n_m}$$

where $\boldsymbol{p}^m_{\Omega_m}$ is the subvector of $\boldsymbol{p}^m = \left[p^m_1, p^m_2, \cdots, p^m_{N_m}\right]$, h^m_i and g^m_i are fast and slow fading gain from user i to small cell m. The aggregate noise power is calculated as $n_m = \sigma^2_m + \sum_{n\neq m} I_n$, where σ^2_m is the additive white Gaussian noise power. We approximate SINR by considering the maximum amount of interference from neighboring small cells.

With Rayleigh fading assumption, we assume h_i as independent exponentially random variable with unit mean, hence, the outage probability of home user is given by [12]

$$\text{P}^m_{\text{out}}(\Omega_m) = 1 - e^{\left(-\gamma^m_0 n_m / p^m_0 g^m_0\right)} \prod_{i\in\Omega_m} \left(1 + \frac{\gamma^m_0 p^m_i g^m_i}{p^m_o g^m_0}\right)^{-1}$$

where p^m_0 and γ^m_0 are the fixed transmission power and target SINR of the home user.

The Stage II optimization problem at small cell m is formulated as

$$\mathcal{P}^m_2 := \begin{cases} \underset{\boldsymbol{p}^m_{\Omega_m},\,\Omega_m}{\text{maximize}} & U_{\text{m}}(\boldsymbol{p}^m_{\Omega_m}, \Omega_m) \\ \text{subject to} & \Omega_m \subset \mathcal{N}_m \\ & 0 \leq p^m_i \leq p^{\text{MAX}},\ \forall i \in \Omega_m \\ & \text{SINR}^m_i(\boldsymbol{p}^m_{\Omega_m}) \geq \gamma^m_i,\ \forall i \in \Omega_m \\ & v_m \left(\sum_{i\in\Omega_m} p^m_i + p^m_0\right) \leq I_m \\ & \sum_{i\in\Omega_m} \log\left(1 + \gamma^m_i\right) \leq \tilde{C}_m \end{cases} \tag{3}$$

where p^{MAX} is the maximum amount of transmission power, γ^m_i is the guest user i's target SINR, v_m is the parameter to convert total transmission power into interference, and $\tilde{C}_m =$

$C_m - \log(1+\gamma_0)$ is the remaining backhaul capacity after serving the home user.

The objective function $U_m(\boldsymbol{p}_{\Omega_m}^m, \Omega_m)$ is the summation of two terms: the first term $B_m|\Omega_m|$ is the refunding from the MNO and the second term is given by

$$\begin{aligned}&\log\{[1-\mathrm{P}_{\text{out}}^m(\Omega_m)]\log(1+\gamma_0^m)\}\\&= -\sum_{i\in\Omega_m}\log\left(1+\frac{\gamma_0^m p_i^m g_i^m}{p_0^m g_0^m}\right)+A_m\end{aligned}$$

which is the logarithmic utility of home user in [13], where $A_m = -\frac{\gamma_0^m n_m}{p_0^m g_0^m} + \log[\log(1+\gamma_0^m)]$ is a constant. There is a tradeoff between refunding from the MNO and home user's expected long-term throughput. Admitting more guest users can receive more refunding from the MNO with the degradation of home user's QoS. However, rejecting guest users can secure a better QoS for home user with the sacrifice of lossing refunding. We model the network of small cell m as a single whole entity to the environment, i.e., to the neighboring small cells. Therefore, the aggregate power generated by small cell m is $\sum_{i\in\Omega_m} p_i + p_0$ and the interference to the environment is approximated as $v_m\left(\sum_{i\in\Omega_m} p_i + p_0\right)$, where v_m can be regarded as the virtual fading gain from small cell m to the environment and $I_m \geq \nu_m p_0^m$. The last constraint in (3) is backhaul constraint. $\mathcal{P}_2^m$ is non-concave and mixed integer optimization problem. Different from the admission control and power allocation problem in [14, 15], $\mathcal{P}_2^m$ is a tradeoff between the number of admitted users and refunding.

3. TWO-PHASE GUEST USER ADMISSION ALGORITHM

In this section, we focus on Stage II problem. To solve $\mathcal{P}_2^m$, we firstly derive a power update rule and then propose a near-optimal two-phase guest user admission algorithm. We state two theorems below and leave the proofs in [16].

Theorem 1 *(Feasibility and Optimality) Denote the feasible solution set of $\mathcal{P}_2^m$ as $\Psi_m = \left\{\left(\Omega_m, \boldsymbol{p}_{\Omega_m}^m\right)\right\}|(3)\big\}$, there exists at least one feasible solution $(\Omega_m^\star, \boldsymbol{p}_{\Omega_m^\star}^m)$ to be the global maximizer of $\mathcal{P}_2^m$ with $\boldsymbol{p}_{\Omega_m^\star}^m = n_m'(\boldsymbol{T}_{\Omega_m^\star}^m)^{-1}\mathbf{1}_{|\Omega_m^\star|}$. Here, $[\boldsymbol{T}^m]_{ij} = \begin{cases} t_i^m/\gamma_i^m & i=j \\ -t_j^m & i\neq j\end{cases}$ is the coefficient matrix, $t_i^m = h_i^m g_i^m$, and $n_m' = p_0^m h_0^m g_0^m + n_m$.*

The theorem is based on the fact that the minimizers of concave objective function lie in the extreme points. From Theorem 1, for fixed admission set Ω_m, minimizing the sum-log power $\sum_{i\in\Omega_m}\log\left(1+\frac{\gamma_0^m p_i^m g_i^m}{p_0^m g_0^m}\right)$ is equivalent to the classical power allocation problem, which finds the extreme point.

Theorem 2 *(Power Update Rule) Denote $\bar{\boldsymbol{p}}_{\mathcal{K}}^m$ as transmission powers of already admitted $K = |\mathcal{K}|$ guest users and corresponding coefficient matrix as $\boldsymbol{T}_{\mathcal{K}}^m$. If the SH admits one new guest user, the transmission power for each guest user is updated as follows:*

$$\begin{aligned}\boldsymbol{p}_{\mathcal{K}+1}^m(k) &= \bar{\boldsymbol{p}}_{\mathcal{K}}(k)\left(1+\frac{t_{K+1}^m}{n_m'}\boldsymbol{p}_{\mathcal{K}+1}^m(K+1)\right)\ \forall k\in\mathcal{K}\\ \boldsymbol{p}_{\mathcal{K}+1}^m(K+1) &= D\Big((\boldsymbol{t}_{\mathcal{K}}^m)^T\bar{\boldsymbol{p}}_{\mathcal{K}}^m + n_m'\Big)\end{aligned} \tag{4}$$

where $D = 1/\left(\frac{t_{K+1}^m}{\gamma_{K+1}^m} - (\boldsymbol{t}_{\mathcal{K}}^m)^T(\boldsymbol{T}_{\mathcal{K}}^m)^{-1}t_{K+1}^m\mathbf{1}_K\right)$.

It is derived from the block matrix inverse of $\boldsymbol{T}_{\mathcal{K}+1}^m$. From Theorem 2, by admitting one new guest user, the transmission powers of previous K guest users are amplified by the same factor $1+\frac{t_{K+1}^m}{n_m'}\boldsymbol{p}_{\mathcal{K}+1}^m(K+1)$ and the transmission power of the newly admitted guest user is jointly determined by $\frac{t_{K+1}^m}{\gamma_{K+1}^m}$, $\boldsymbol{T}_{\mathcal{K}}^m$ and $\bar{\boldsymbol{p}}_{\mathcal{K}}^m$. Therefore, it is impossible to derive a simple admission criterion. This theorem tells us how to update transmission powers if the SH admits guest users gradually. To cope with this difficulty, we propose a near-optimal two-phase guest user admission algorithm as follows:

Algorithm 1 Two-phase guest user admission and power allocation algorithm

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Phase I: Sort guest users in terms of the minimum increment of the sum-log powers

1. Set $\Phi_m = \emptyset$ and $\Pi_m = \mathcal{N}_m$
2. (Initialization) $k = \arg\min_{\Pi_m}\log\left(1+\frac{\gamma_0^m\gamma_i^m n_m'}{p_0^m g_0^m h_i^m}\right)$ and $p_k^m = \frac{\gamma_k^m n_m'}{h_k^m g_k^m}$. If all the constraints in (3) hold, $\Phi_m = \Phi_m + k$, $\Pi_m = \Pi_m - k$ and go to 3; Elesif only backhaul constraint is invalid, $\Pi_m = \Pi_m - k$, go to 2; Else, terminate (it cannot support ANY guest user);
3. Choose $i\in\Pi_m$ causing the minimum increment of sum-log power by (4). If all the constraints in (3) hold, $\Phi_m = \Phi_m + i$, $\Pi_m = \Pi_m - i$ and go to 3; Elesif only backhaul constraint is invalid, $\Pi_m = \Pi_m - i$, go to 3; Else, terminate (it cannot support any MORE guest users);

Phase II: Maximize SH's utility

4. For $u = 0:1:|\Phi_m|$, calculate the SH's utility U_m in (3) by admitting first u guest users in the sorted set Φ_m.
5. Find the subset $\Omega_m \subset \Phi_m$ corresponding to the largest utility. If more than one subsets have the same largest utility, choose the one with largest cardinality. Ω_m consists first $|\Omega_m|$ sorted guest users in Φ_m.

The algorithm consists of two-phases: first sorts the guest users in terms of the minimum increment of sum-log powers and then determines the admission set which maximizes the SH's utility. The algorithm finds near-optimal solution of $\mathcal{P}_2^m$ but significantly reduces computational complexity from $\mathcal{O}\left(2^{N_m}\right)$ (enumeration search) to $\mathcal{O}\left(N_m^2\right)$.

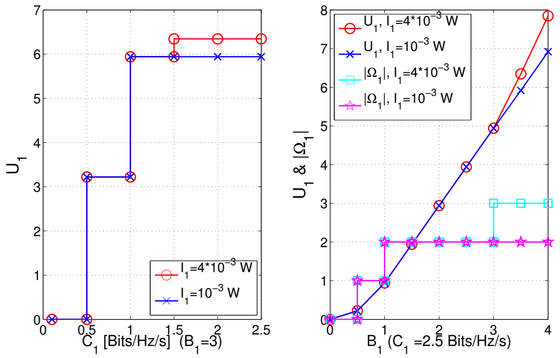

Fig. 1. U_1 with C_1 and B_1

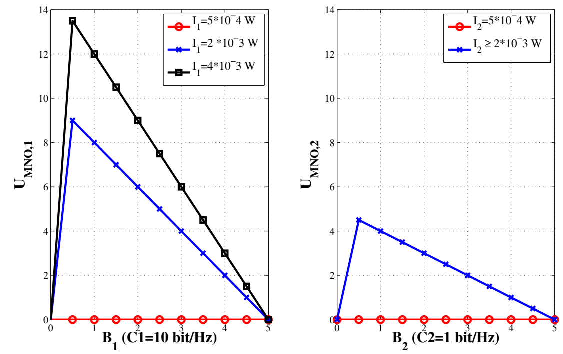

Fig. 2. U_{MNO} with refunding

4. LOOKUP TABLE APPROACH

Since $\Omega_m(B_m, I_m)$ is an implicit integer function in terms of B_m and I_m, we propose a lookup table approach to find the sub-optimal individualized refunding. The MNO divides the feasible refunding interval $[0, \pi]$ into L equal intervals with step size $\Delta\pi = \frac{\pi}{L}$ and the interference interval $[I_0, Q]$ into S equal intervals with step size $\Delta I = \frac{Q-I_0}{S}$, where $I_0 = \max(\nu_m p_0^m)$. Each SH builds up a table of $\Omega_m(l_m\Delta\pi, s_m\Delta I)$, which are best response functions to different pairs of $(l_m\Delta\pi, s_m\Delta I)$. Based on the tables from SHs, the MNO can determine refunding and interference temperature constraints to different SHs. The algorithm is summarized as follows:

Algorithm 2 Lookup Table Approach

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1. The MNO distributes π, Q, I_0, L and S to SHs;
2. Each SH calculates and builds up best response function table $\mathcal{X}_m$ for different pairs of $(l_m\Delta\pi, I_0 + s_m\Delta I)$ in Algorithm 1, where $l_m = 0, 1, \cdots, L$ and $s_m = 0, \cdots, S$, and then feedbacks to the MNO;
3. At the MNO, it searches for the sub-optimal strategies by using any fast search method.

The performance of algorithm 2 is dependent on the step size and search method. Hence, it is a tradeoff between optimality and computational complexity. As $\Delta\pi \to 0$ and $\Delta I \to 0$, it converges to SE in definition 1.

5. SIMULATION RESULTS

In this section, the performance of the proposed algorithms are investigated. The small cell is located in the centre of an area of radius 25 m. Guest users are randomly located within the coverage of small cell. The noise power is $\sigma^2 = 10^{-3}$ W and the aggregate interference constraint is $Q = 5 \times 10^{-3}$ W. The transmit power and SINR threshold of the home user are $p_0 = 1$ W and $\gamma_0 = 1$ dB, respectively. We model g_i as $K10^{\mu_i/10}d_i^{-\alpha}$, where $K = 10^3$ is the antenna gain, μ_i is a Gaussian random variable with zero mean and standard deviation of 3 dB to account for log-normal shadowing effect and $\alpha = 3$. The maximum transmission power for guest users is $p^{\mathrm{MAX}} = 2.5$ W.

In Fig.1, consider typical small cell with 3 guest users in algorithm 1, we use the fast/slow gain and SINRs: $\boldsymbol{h} = [0.6701, 1.0742, 0.4524]$, $\boldsymbol{g} = [2.4456, 0.1721, 0.6436]$, and $\boldsymbol{\gamma} = [0.2287, 0.7581, 0.3426]$. For fixed refunding, the SH's utility increases with backhaul capacity. For fixed backhaul connection, a higher refunding can increase SH's utility and encourages SH to admit more guest users. If the backhaul is available to support more guest users, then I_m is the constraint to SH's utility.

In Fig.2, we consider two small cells, where small cell 1 has a higher backhaul capacity $C_1 = 10$ bit/Hz and the small cell 2 has a smaller backhaul capacity $C_2 = 1$ bit/Hz. $\pi = 4$, $\Delta\pi = 0.5$, and $\Delta I = 10^{-3}$. From Fig. 2, we know that a higher refunding to SHs can actually decrease MNO's revenue, hence, the MNO only motivates SHs to admit limited number of guest users, in some cases, far less than their maximum capability. In this example, small cell 1 admits 1 guest users instead of its maximum capability of 3. Because $\pi|\Omega_m|$ increases slower than $B_m|\Omega_m|$. At the optimal refunding point, the MNO can also adjust I_m to further maximize its revenue.

6. CONCLUSIONS

We show that backhaul is a key constraint which significantly affects both the MNO and SHs' utilities. The MNO refunds SHs differently to achieve the maximum revenue. In practical implementation, the MNO can calculate the lookup table offline and then the SHs only need to feedback guest users' information. In some cases, $\mathcal{P}_2^m$ can be approximated by LP and we can apply iterative guest user removal algorithm.

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