MAXIMUM-SNR TRANSMIT ANTENNA SELECTION WITH TWO RECEIVE ANTENNAS IS POLYNOMIALLY SOLVABLE

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ABSTRACT

The recent increased interest in massive multiple-input multiple-output systems, combined with the cost of the analog RF chains, necessitates the use of efficient antenna selection (AS) schemes. Capacity or SNR optimal AS has been considered to require an exhaustive search among all possible antenna subsets. In this work, we prove that the maximum-SNR transmit AS problem with two receive antennas is polynomially solvable and develop an algorithm that solves it with quartic complexity, independently of the number of selected antennas. Our method also applies to receive AS with two transmit antennas.

1. INTRODUCTION

The cost and complexity of the analog RF chains connected to the inexpensive antenna elements at both sides of multiple-input multiple-output (MIMO) systems [1], [2] is a limiting factor on the number of the antennas that may operate in practice. A low-cost engineering technique that reduces the number of analog chains required is antenna selection (AS) where a number of limited transmit/receive RF chains are multiplexed between a selected set of transmit/receive antennas. AS for MIMO has been studied extensively in literature and research works have dealt with either transmit AS (TAS), receive AS (RAS), or joint transmit-receive AS [3]. However, since there has recently been an increased interest in large-scale multiple-antenna wireless systems [4], [5], often called massive MIMO [6], the need for AS algorithms that efficiently select the antennas is of major importance.

In the past, AS has been considered to maximize either channel capacity [7]-[12], minimum post-processing SNR [13], statistical quantities such as average throughput [14], or effective SNR [1], [2], [15]. All these works present suboptimal algorithms, since the optimal solution is considered to require the evaluation of the metric of interest over all $\binom{N}{K}$ possible combinations of AS sets, where N and K are the numbers of available and selected, respectively, antennas. Due to the exponential complexity of the exhaustive-search maximum-predetection-SNR AS, a suboptimal hybrid algorithm that performs RAS to maximize the receiver predetection SNR with complexity $O(N)$ was considered in [16] where the authors design the beamforming vector at the transmitter side as the principal right singular vector of the full channel matrix.

In this paper, we prove that the maximum-SNR joint beamforming-AS problem at the transmitter side for two receive antennas and an arbitrary number of K selected transmit antennas is polynomially solvable and develop an algorithm that solves this problem with complexity $\mathcal{O}\left(N^4\right)$. This result is possible after introducing an auxiliary vector that partitions our problem into multiple subproblems, each with complexity $O(N)$. The maximum number (upper bound) of subproblems (hence, candidate solutions) that we obtain is $6\binom{N}{3}$. The optimal selection is then determined by a polynomial-time exhaustive search among those candidate selection sets. Our result also applies to RAS with two transmit antennas.

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2. SIGNAL MODEL AND PROBLEM STATEMENT

We consider a MIMO system that consists of N transmit and M receive antennas. The channel between any transmit and receive antennas is assumed flat fading and the $M \times N$ complex baseband channel matrix is denoted by $\mathbf{H} = [\mathbf{h}_1 \;\; \mathbf{h}_2 \;\; \ldots \;\; \mathbf{h}_M]^H$ where $\mathbf{h}_m^H$ is a complex row vector that contains the channel coefficients between the N transmit antennas and the mth receive antenna, $m = 1, 2, \ldots, M$.

The transmitter selects K (out of the N) antennas and employs beamforming to transmit symbol $x \in \mathbb{C}$. We can equivalently say that it uses a beamforming vector $\mathbf{w} \in \mathbb{C}^N$ subject to the constraint $\|\mathbf{w}\|_0 = K$. We also assume without loss of generality (w.l.o.g.) that all transmitted power is absorbed by the channel matrix, therefore $\|\mathbf{w}\| = 1$ and $E\{|x|^2\} = 1$. We note that per-antenna power constraints and unimodular beamforming were examined in [17].

The downconverted and pulse-matched filtered received vector of size $M \times 1$ is $\mathbf{y} \triangleq \mathbf{H}\mathbf{w}x + \mathbf{n}$ where $\mathbf{n} \in \mathbb{C}^M$ is a zero-mean additive white complex noise vector with variance 1 w.l.o.g. Since $\mathbf{y}$ represents an unknown vector signal in white vector noise, the maximum-SNR filter is the matched filter $\mathbf{f} \triangleq \mathbf{H}\mathbf{w}$ whose output is given by $\mathbf{f}^H\mathbf{y} = \|\mathbf{H}\mathbf{w}\|^2 x + \mathbf{w}^H\mathbf{H}^H\mathbf{n}$. Then, the filter output SNR is

$$\frac{E\{|x|^2\}\,\|\mathbf{H}\mathbf{w}\|^4}{\|\mathbf{H}\mathbf{w}\|^2} = \|\mathbf{H}\mathbf{w}\|^2 . \tag{1}$$

Eq. (1) shows how the predetection SNR is related to the beamforming vector.

Our objective is to jointly select K antennas and optimize the beamforming vector $\mathbf{w}$ to maximize the predetection SNR in (1). That is, we seek $\mathbf{w}$ that solves the problem

$$\max_{\substack{\mathbf{w} \in \mathbb{C}^N \\ \|\mathbf{w}\|=1, \|\mathbf{w}\|_0 = K}} \|\mathbf{H}\mathbf{w}\| = \max_{\substack{\mathcal{I} \subset [N] \\ |\mathcal{I}| = K}} \max_{\substack{\mathbf{w} \in \mathbb{C}^K \\ \|\mathbf{w}\|=1}} \|\mathbf{H}_{:,\mathcal{I}}\mathbf{w}\| , \tag{2}$$

where $[N] \triangleq \{1, 2, \ldots, N\}$. In (2), the optimization problem has been rewritten as two nested problems; the outer one is the antenna selection problem where set $\mathcal{I}$ contains the indices of the K selected antennas and the inner one is the beamforming problem for the particular antenna selection where $\mathbf{w} \in \mathbb{C}^K$ is the "pruned" beamforming vector that consists of the K nonzero loadings of $\mathbf{w} \in \mathbb{C}^N$. For a fixed antenna selection $\mathcal{I}$, we denote the optimal beamforming vector of the inner maximization as

$$\mathbf{w}(\mathcal{I}) \triangleq \underset{\substack{\mathbf{w}\in\mathbb{C}^K \\ \|\mathbf{w}\|=1}}{\arg\max} \|\mathbf{H}_{:,\mathcal{I}}\mathbf{w}\| . \quad (3)$$

It is straightforward to see that $\mathbf{w}(\mathcal{I})$ is given by the right singular vector of $\mathbf{H}_{:,\mathcal{I}}$ that corresponds to its principal singular value $\sigma_{\max}(\mathbf{H}_{:,\mathcal{I}})$, simply called the "principal right singular vector," and leads to a maximum SNR value (for a fixed selection $\mathcal{I}$) equal to $\sigma^2_{\max}(\mathbf{H}_{:,\mathcal{I}})$. That is, (2) is rewritten as

$$\max_{\substack{\mathcal{I}\subset[N] \\ |\mathcal{I}|=K}} \sigma_{\max}(\mathbf{H}_{:,\mathcal{I}}) . \quad (4)$$

To optimally select the K transmit antennas, according to (4), we need to find the K columns of $\mathbf{H}$ that form the $M \times K$ submatrix with the maximum principal singular value. The elements of the optimal set $\mathcal{I}$ that solves (4) are the indices of the optimally selected antennas. Then, for these optimal indices, the optimal "pruned" beamforming vector is given by the principal right singular vector of $\mathbf{H}_{:,\mathcal{I}}$.

A straightforward approach to solve (4) and identify the optimal set $\mathcal{I}$ would be an exhaustive search among all $\binom{N}{K}$ cardinality-K subsets of $[N]$. However, if the number of selected transmit antennas K is a linear function of the total number of transmit antennas N, then such a solver would be impractical even for moderate values of N, since its complexity grows exponentially with N. Even in the case where K is not a function of N, if N is large enough (e.g. $N = 100$ as in massive MIMO [5]) and K is, for example, equal to or greater than 5, then the complexity of the exhaustive search would still be too large to consider for practical implementation. In the next section, we present an efficient algorithm that solves (4) in time that is polynomial in N for any fixed number of receive antennas $M = 1$ or $M = 2$ and any unrestricted number of selected transmit antennas K (that is, even if K grows linearly with N). In fact, the complexity of the proposed algorithm is independent of K and depends only on N and M. Although the case $M = 1$ is straightforward, the case $M = 2$ is challenging and our solution is intuitive in the sense that it may lead to a general polynomial-complexity solution for any fixed number of receive antennas M.

3. POLYNOMIAL-COMPLEXITY OPTIMAL TRANSMIT ANTENNA SELECTION

3.1. $M = 1$ receive antenna

We consider the trivial case of transmit antenna selection with one receive antenna ($M = 1$). We do so to identify some interesting properties that will be useful in the design of the algorithm for optimal transmit antenna selection with two receive antennas ($M = 2$) in the next subsection.

Since $M = 1$, the channel matrix becomes $\mathbf{H} = \mathbf{h}^H$ and, for a fixed selection set $\mathcal{I}$, we have $\mathbf{H}_{:,\mathcal{I}} = \mathbf{h}^H_{\mathcal{I}}$. Then, the optimal pruned beamforming vector in (3) is the maximal-ratio combining (MRC) vector $\mathbf{w}(\mathcal{I}) = \frac{\mathbf{h}_{\mathcal{I}}}{\|\mathbf{h}_{\mathcal{I}}\|}$ and (2) becomes

$$\max_{\substack{\mathcal{I}\subset[N] \\ |\mathcal{I}|=K}} \left| \mathbf{h}^H_{\mathcal{I}} \frac{\mathbf{h}_{\mathcal{I}}}{\|\mathbf{h}_{\mathcal{I}}\|} \right| = \max_{\substack{\mathcal{I}\subset[N] \\ |\mathcal{I}|=K}} \|\mathbf{h}_{\mathcal{I}}\| \quad (5)$$

which is equivalent to the maximization

$$\max_{\substack{\mathcal{I}\subset[N] \\ |\mathcal{I}|=K}} \|\mathbf{h}_{\mathcal{I}}\|_1 = \max_{\substack{\mathcal{I}\subset[N] \\ |\mathcal{I}|=K}} \sum_{i\in\mathcal{I}} |h_i| , \quad (6)$$

since the optimal set $\mathcal{I}$ in both (5) and (6) is the one that consists of the indices of the K largest elements of $|\mathbf{h}|$.

To describe the latter step, we introduce function *select* which selects the k largest (in magnitude) elements of a vector, as follows.

$$\text{select}(\mathbf{u}; k) \triangleq \underset{\substack{\mathcal{I}\subset[N] \\ |\mathcal{I}|=k}}{\arg\max} \|\mathbf{u}_{\mathcal{I}}\| = \underset{\substack{\mathcal{I}\subset[N] \\ |\mathcal{I}|=k}}{\arg\max} \|\mathbf{u}_{\mathcal{I}}\|_1 . \quad (7)$$

That is, select$(\mathbf{u}; k)$ computes $|h_1|, |h_2|, \ldots, |h_N|$ and returns the indices of the largest k values. It turns out that the outcome of select$(\mathbf{u}; k)$ is a set $\mathcal{I}$ such that $|h_i| \geq |h_j|$ for any $i \in \mathcal{I}$ and $j \in [N] - \mathcal{I}$. The computational cost of select$(\mathbf{u}; k)$ is $\mathcal{O}(N)$ [18].

We conclude that the maximum-SNR transmit antenna selection when one receive antenna is occupied by the receiver is given by $\mathcal{I} = \text{select}(\mathbf{h}; K)$ whose complexity is linear in the number of available transmit antennas N. In the developments that follow, *select* is critical in proving that optimal antenna selection can be solved in polynomial time when $M = 2$.

3.2. $M = 2$ receive antennas

When two receive antennas are utilized ($M = 2$), the channel matrix is $\mathbf{H} = [\mathbf{h}_1 \ \ \mathbf{h}_2]^H$, and the problem of selecting the optimal subset of K transmit antennas in (2) becomes more challenging. In this subsection, we show that this problem can be solved with complexity $\mathcal{O}(N^4)$ for any number of selected transmit antennas K.

First, we redefine the problem space by introducing the auxiliary angles $\phi \in \left[0, \frac{\pi}{2}\right]$ and $\theta \in (-\pi, \pi]$ and defining the unit-norm 2×1 vector

$$\mathbf{c}(\phi, \theta) \triangleq \begin{bmatrix} \sin(\phi) \\ e^{j\theta}\cos(\phi) \end{bmatrix} . \quad (8)$$

In the following, we will see that ϕ and θ help us identify a polynomial number of *locally optimal* candidate selection sets $\mathcal{I}$. The optimal solution of (4) will be among the locally optimal ones.

Due to the unity of the norm of $\mathbf{c}(\phi, \theta)$, from Cauchy-Schwarz Inequality, we obtain, for any vector $\mathbf{a} \in \mathbb{C}^2$,

$$\left| \mathbf{a}^H \mathbf{c}(\phi, \theta) \right| \leq \|\mathbf{a}\| \|\mathbf{c}(\phi, \theta)\| = \|\mathbf{a}\|. \quad (9)$$

The equality above is achieved if and only if $\mathbf{c}(\phi, \theta)$ is collinear with $\mathbf{a}$ within a phase rotation such that its first element is real positive, i.e., if and only if

$$\mathbf{c}(\phi, \theta) = \frac{\mathbf{a}}{\|\mathbf{a}\|} e^{-j\arg(a_1)} \quad (10)$$

where a_1 is the first element of $\mathbf{a}$. We note that, for any $\mathbf{a} \in \mathbb{C}^2$, there always exists a pair of angles $(\phi, \theta) \in \Phi$, where $\Phi \triangleq \left[0, \frac{\pi}{2}\right] \times (-\pi, \pi]$, such that (10) is satisfied. Therefore, from (9), we obtain $\|\mathbf{a}\| = \max_{(\phi,\theta)\in\Phi} \left| \mathbf{a}^H \mathbf{c}(\phi, \theta) \right|$ for any $\mathbf{a} \in \mathbb{C}^2$.

If we substitute $\mathbf{a}$ with $\mathbf{H}_{:,\mathcal{I}}\mathbf{w}$ in (2), then our optimization problem becomes

$$\begin{aligned} &\max_{\substack{\mathcal{I}\subset[N] \\ |\mathcal{I}|=K}} \max_{\substack{\mathbf{w}\in\mathbb{C}^K \\ \|\mathbf{w}\|=1}} \|\mathbf{H}_{:,\mathcal{I}}\mathbf{w}\| \quad (11) \\ &= \max_{\substack{\mathcal{I}\subset[N] \\ |\mathcal{I}|=K}} \max_{\substack{\mathbf{w}\in\mathbb{C}^K \\ \|\mathbf{w}\|=1}} \max_{(\phi,\theta)\in\Phi} \left| \mathbf{w}^H \mathbf{H}^H_{:,\mathcal{I}} \mathbf{c}(\phi, \theta) \right| . \end{aligned}$$

We define $\mathbf{u}(\phi,\theta) \stackrel{\triangle}{=} \mathbf{H}^H\mathbf{c}(\phi,\theta)$ and change the order of the maximizations in (11) to obtain

$$\max_{(\phi,\theta)\in\Phi}\ \max_{\substack{\mathcal{I}\subset[N]\\|\mathcal{I}|=K}}\ \max_{\substack{\mathbf{w}\in\mathbb{C}^K\\\|\mathbf{w}\|=1}} \left|\mathbf{w}^H\mathbf{u}_{\mathcal{I}}(\phi,\theta)\right|. \tag{12}$$

For any given pair of angles $(\phi,\theta)\in\Phi$ and any given selection subset $\mathcal{I}\subset[N]$, the inner maximization in (12) is achieved by the pruned beamforming vector

$$\mathbf{w}(\phi,\theta;\mathcal{I}) \stackrel{\triangle}{=} \frac{\mathbf{u}_{\mathcal{I}}(\phi,\theta)}{\|\mathbf{u}_{\mathcal{I}}(\phi,\theta)\|}, \tag{13}$$

resulting in the value $\max_{\substack{\mathbf{w}\in\mathbb{C}^K\\\|\mathbf{w}\|=1}} \left|\mathbf{w}^H\mathbf{u}_{\mathcal{I}}(\phi,\theta)\right| = \|\mathbf{u}_{\mathcal{I}}(\phi,\theta)\|$. Then, the optimization problem in (12) becomes

$$\max_{(\phi,\theta)\in\Phi}\ \max_{\substack{\mathcal{I}\subset[N]\\|\mathcal{I}|=K}} \|\mathbf{u}_{\mathcal{I}}(\phi,\theta)\| \tag{14}$$

where, for any given $(\phi,\theta)\in\Phi$, the inner maximization is achieved by the subset that consists of the indices of the K largest elements of $|\mathbf{u}(\phi,\theta)|$, i.e.,

$$\mathcal{I}(\phi,\theta) \stackrel{\triangle}{=} \arg\max_{\substack{\mathcal{I}\subset[N]\\|\mathcal{I}|=K}} \|\mathbf{u}_{\mathcal{I}}(\phi,\theta)\| = \mathrm{select}(\mathbf{u}(\phi,\theta);K). \tag{15}$$

Notice that, for any $n\in[N]$, $|u_n(\phi,\theta)| = \left|\mathbf{H}_{:,n}^H\mathbf{c}(\phi,\theta)\right| = \left|H_{1,n}^*\sin\phi + H_{2,n}^*e^{j\theta}\cos\phi\right|$, i.e., every element of $|\mathbf{u}(\phi,\theta)|$ is a continuous function, or a surface, of (ϕ,θ). When, due to (15), we select the K largest elements of $|\mathbf{u}(\phi,\theta)|$ at a given point (ϕ,θ) as function *select* requires, we actually compare the surfaces $|u_1(\phi,\theta)|$, $|u_2(\phi,\theta)|$, …, $|u_N(\phi,\theta)|$ at point (ϕ,θ). The optimal selection $\mathcal{I}\subset[N]$ in (12), i.e., the solution of (4), is met if we scan the entire space Φ and collect the locally optimal selection $\mathcal{I}(\phi,\theta)$ for any point $(\phi,\theta)\in\Phi$.

A natural question that arises is the following. How many selection subproblems are induced if we scan all values of $(\phi,\theta)\in\Phi$? An answer to the previous question identifies exactly the number of locally optimal solutions for (4).

The auxiliary angles ϕ,θ now become relevant in answering the above question. Due to the continuity of the surfaces $|u_n(\phi,\theta)|$, we expect that in an area around (ϕ,θ) the selection subset $\mathcal{I}(\phi,\theta)$ will be retained because either the sorting of the surfaces does not change (although the surfaces vary) or the sorting of the surfaces changes but the group of the K surfaces with the higher value is retained (that is, the change of the sorting occurs either within the higher K surfaces or within the lower $N-K$ surfaces). Hence, we expect the formation of regions in Φ within which the locally optimal selection subset $\mathcal{I}$ is unique. In the sequel, we determine all these regions, show that their number is less than or equal to $6\binom{N}{3}$, and present a polynomial-time algorithm that identifies the selection subsets $\mathcal{I}$ that are associated with these intervals. Once we have collected all candidate subsets, the solution of (4) is determined through a polynomial-time exhaustive search among them.

We begin by noting that, as we scan the space Φ, the selection subset $\mathcal{I}$ does not change unless two surfaces intersect (which implies that the sorting of the surfaces changes). Therefore, to identify all regions that retain their selection subset $\mathcal{I}$, it suffices to examine when two surfaces intersect. We note that this is a necessary, but not sufficient, condition for a change of $\mathcal{I}$, since the intersecting surfaces may correspond to elements of $|\mathbf{u}(\phi,\theta)|$ that, before they intersect, both belong to $\mathcal{I}$ or neither belongs to $\mathcal{I}$. In the latter case, although the magnitude sorting of the surfaces changes, the selection subset $\mathcal{I}$ does not.

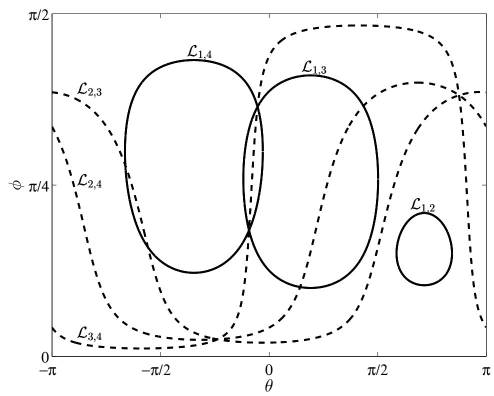

Fig. 1. An illustration of intersection curves $\mathcal{L}_{n,m}$, for $n,m\in\{1,2,3,4\}$ with $n\neq m$, and formed regions, resulting from $N=4$ surfaces.

Two surfaces, say $|u_n(\phi,\theta)|$ and $|u_m(\phi,\theta)|$, intersect when $|u_n(\phi,\theta)| = |u_m(\phi,\theta)|$. We note that, for any $n,m\in[N]$, the intersection of $|u_n(\phi,\theta)|$ and $|u_m(\phi,\theta)|$ always exists, since all surfaces meet 0 for some ϕ,θ.[1] As a result, the intersection of two surfaces determines a curve on the (ϕ,θ)-plane which we define as $\mathcal{L}_{n,m} \stackrel{\triangle}{=} \{(\phi,\theta)\in\Phi:\ |u_n(\phi,\theta)| = |u_m(\phi,\theta)|\}$. To illustrate this, in Fig. 1, we set N to 4, consider an arbitrary 2×4 channel matrix $\mathbf{H}$, and plot curves $\mathcal{L}_{n,m}$, for any $n,m\in[N]$ with $n\neq m$. We observe the regions that are formed; within each region, the selection subset $\mathcal{I}$ remains the same. We also observe that, in most of the cases, each region "touches" an intersection of two or three curves.[2] That is, we can identify these regions by examining intersections between curves $\mathcal{L}_{n,m}$. In addition, we can concentrate only on intersections of three curves and ignore intersections of two curves, since the latter change the sorting of surfaces but do not "generate" a new subset $\mathcal{I}$ that has not been generated by a neighboring three-curve intersection.

To examine all three-curve intersections, we observe that each such intersection corresponds to a three-surface intersection (otherwise, more curves would pass through the intersection point, something that happens w.p.0). Hence, it suffices to find when three surfaces, say $|u_n(\phi,\theta)|$, $|u_m(\phi,\theta)|$, and $|u_l(\phi,\theta)|$, where $n,m,l\in[N]$ with $n\neq m$, $n\neq l$, and $m\neq l$, intersect. That is, we have to find (ϕ,θ) that satisfies $\left|\mathbf{H}_{:,n}^H\mathbf{c}(\phi,\theta)\right| = \left|\mathbf{H}_{:,m}^H\mathbf{c}(\phi,\theta)\right| = \left|\mathbf{H}_{:,l}^H\mathbf{c}(\phi,\theta)\right|$ or, equivalently,

$$\begin{bmatrix} e^{j\lambda}\mathbf{H}_{:,n}^H - \mathbf{H}_{:,m}^H \\ e^{j\mu}\mathbf{H}_{:,n}^H - \mathbf{H}_{:,l}^H \end{bmatrix}\mathbf{c}(\phi,\theta) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{16}$$

for some $\lambda,\mu\in\mathbb{R}$. A solution to (16), i.e., an intersection between the three surfaces, exists if and only if there exist $\lambda,\mu\in\mathbb{R}$ that make the matrix $\mathbf{A} = \begin{bmatrix} e^{j\lambda}\mathbf{H}_{:,n}^H - \mathbf{H}_{:,m}^H \\ e^{j\mu}\mathbf{H}_{:,n}^H - \mathbf{H}_{:,l}^H \end{bmatrix}$ singular. We define the 2×1 vector $\mathbf{d} \stackrel{\triangle}{=} \left[\mathbf{H}_{:,m}\ \ \mathbf{H}_{:,l}\right]^{-1}\mathbf{H}_{:,n}$ and the scalar $D \stackrel{\triangle}{=} \frac{1-\|\mathbf{d}\|^2}{2|d_1 d_2|}$. Then,

[1] For any surface $|u_n(\phi,\theta)| = \left|\mathbf{H}_{:,n}^H\mathbf{c}(\phi,\theta)\right|$, we can always find a vector $\mathbf{c}(\phi,\theta)$ which is orthogonal to $\mathbf{H}_{:,n}$.

[2] The case where a region does not touch any intersection is examined in the end of this subsection.

after algebraic computations, we can prove that $|D| \leq 1$ is a necessary and sufficient condition to make $\mathbf{A}$ singular. If the condition is not satisfied, then no intersection exists between the three surfaces (and three corresponding curves). If the condition is satisfied, then we set $\psi \triangleq -\text{angle}(d_1 d_2^*) \pm \cos^{-1} D$ and $\omega \triangleq \text{angle}\left(\begin{bmatrix} 1 & e^{-j\psi} \end{bmatrix} \mathbf{d}\right)$ and can prove, after algebraic computations, that $\mathbf{A}$ becomes singular when $\lambda = \omega$ and $\mu = \psi + \omega$. Then, $\mathbf{c}(\phi, \theta)$ is the unit-norm vector in the null space of $\mathbf{A}$. Finally, from (8), we can uniquely determine the intersection point (ϕ, θ). Since there are two values of ψ that make $\mathbf{A}$ singular, we obtain two intersection points.

For any combination of three surfaces, the above procedure, with complexity $\mathcal{O}(1)$, examines if they intersect and, if so, also computes the two intersection points (ϕ, θ). Each intersection point is a vertex of six regions. Then, function $\mathcal{I} = \text{select}(\mathbf{u}(\phi, \theta); K)$ computes, with complexity $\mathcal{O}(N)$, the indices of the K largest surfaces. We recall that, at the intersection point, all three surfaces that intersect have the same value. If $\mathcal{I}$ contains the indices n, m, l of the three surfaces that intersect or it does not contain either of them, then $\mathcal{I}$ is the common optimal selection subset for all six neighboring regions. Otherwise, we have to consider three different cases for the index among n, m, and l that belongs or does not belong to $\mathcal{I}$. Overall, for each combination of three surfaces, we obtain at most 6 candidate sets $\mathcal{I}$ with complexity $\mathcal{O}(N)$. Since the total number of surfaces is N, the overall number of candidates is upper bounded by $6\binom{N}{3}$.

Finally, regarding the case where a region does not touch any intersection, we can see that such a region touches a curve $\mathcal{L}_{n,m}$ which does not intersect with any other curve. For example, in Fig. 1, such a curve is $\mathcal{L}_{1,2}$. Therefore, when we examine the three-surface intersections, as described before, we should mark the corresponding curves and, in the end, if a curve, say $\mathcal{L}_{n,m}$, is unmarked, then we should examine it separately as follows. We just need to pick a point on $\mathcal{L}_{n,m}$ by solving $\left|\mathbf{H}_{:,n}^H \mathbf{c}(\phi, \theta)\right| = \left|\mathbf{H}_{:,m}^H \mathbf{c}(\phi, \theta)\right|$ or, equivalently, $\left(e^{j\lambda} \mathbf{H}_{:,n}^H - \mathbf{H}_{:,m}^H\right) \mathbf{c}(\phi, \theta) = 0$ for some $\lambda \in \mathbb{R}$. Actually, we can set λ to zero, obtain $\mathbf{c}(\phi, \theta)$ as the unit-norm vector in the null space of $\mathbf{H}_{:,n}^H - \mathbf{H}_{:,m}^H$, and, from (8), uniquely determine the intersection point (ϕ, θ). We note that the existence of such unmarked curves does not increase the complexity of our algorithm since, at the same time, we avoid the computation of many candidates that would have been generated if the curve had participated in any three-curve intersection(s).

In the end, we keep at most $6\binom{N}{3} = \mathcal{O}\left(N^3\right)$ candidate selection subsets $\mathcal{I}$ and compare with each other against the metric of interest in (4). This way, we solve the maximum-SNR transmit antenna selection problem with complexity $\mathcal{O}\left(N^4\right)$, independently of K.

4. $M > 2$ RECEIVE ANTENNAS

For simplicity of the presentation, we concentrated our analysis and developments to the two-receive-antenna ($M = 2$) case. If $M > 2$, then we could introduce more auxiliary angles and work on the multidimensional space with the help of the $M \times 1$ vector

$$\mathbf{c}(\boldsymbol{\phi}, \boldsymbol{\theta}) \triangleq \begin{bmatrix} \sin \phi_1 \\ e^{j\theta_1} \cos \phi_1 \sin \phi_2 \\ \vdots \\ e^{j\theta_{M-2}} \cos \phi_1 \ldots \cos \phi_{M-2} \sin \phi_{M-1} \\ e^{j\theta_{M-1}} \cos \phi_1 \ldots \cos \phi_{M-2} \cos \phi_{M-1} \end{bmatrix}. \tag{17}$$

Eventually, we have to work as in the case $M = 2$ to identify all intersection points of hypersurfaces and determine the candidate solution subsets $\mathcal{I}$ of neighboring regions. At this time, we have not finalized the methodology and algorithm for $M > 2$, therefore we can only conjecture that the complexity is still polynomial, following the above steps.

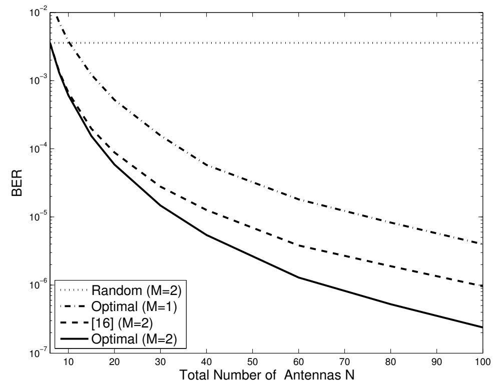

Fig. 2. Bit error rate versus total number of receive antennas N for $M = 2$ transmit antennas and selection of $K = 6$ receive antennas.

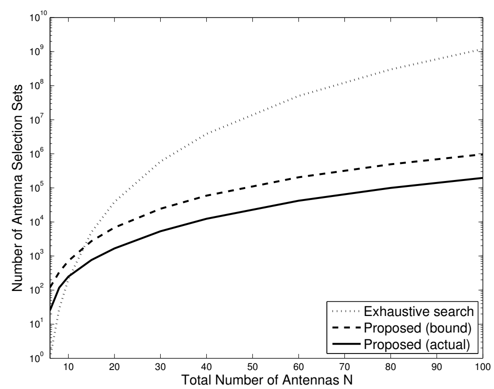

Fig. 3. Complexity versus total number of receive antennas N for $M = 2$ transmit antennas and selection of $K = 6$ receive antennas.

5. SIMULATION RESULTS

To illustrate our developments, although we presented our method for TAS, we consider a MIMO system with 2 transmit and N receive antennas and perform $K = 6$ RAS to allow for comparisons with the RAS method in [16]. The presented results are averages over 1000 i.i.d. Rayleigh channel realizations.

In Fig. 2, we set the total transmit SNR to -3dB and plot the bit error rate (BER) as a function of the available receive antennas N when $K = 6$ receive antennas are selected optimally by our proposed algorithm with complexity $\mathcal{O}\left(N^4\right)$. We compare against the RAS method in [16] (of complexity $\mathcal{O}(N)$) and the random RAS. As a reference, we also include the BER of the optimal RAS when 1 transmit antenna is occupied; the latter has complexity $\mathcal{O}(N)$.

In Fig. 3, we plot the corresponding complexity (in term of number of candidate AS sets that are examined) of our proposed algorithm, together with the corresponding upper bound $6\binom{N}{3}$, and the exhaustive-search complexity. Our algorithm offers significant computation gains in comparison with the exhaustive-search solution, without losing optimality. These gains are offered in conjunction with performance improvement, as indicated in Fig. 2.

6. REFERENCES

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