ERGODIC ROBUST RATE BALANCING FOR RANK-ONE VECTOR BROADCAST CHANNELS VIA SEQUENTIAL APPROXIMATIONS

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ABSTRACT

We focus on a linear beamformer design in the downlink with statistical *channel state information* (CSI) at the transmitter, where the users' *ergodic* rates are balanced. Simplifying the fading channels to given vectors with random scalar factors, which is a good approximation for rural mobile or *satellite communications* (SatCom), the stochastic model mismatch is kept small albeit the ergodic rate structure now allows for adapting the perfect CSI balancing algorithms. Although there is no equivalent *signal-to-interference-and-noise-ratio* (SINR) reformulation for the ergodic constraints, tight inner approximations with SINR structure are found. Based on this observation, a locally optimal sequential approximation strategy is proposed and a fixed point based implementation is provided that requires only few iterations.

***Index Terms*—** rate balancing; beamforming; statistical CSI; ergodic rates; rank one channels; scalar perturbations

1. INTRODUCTION

The *multi-user* downlink with one *multi-antenna* transmitter is well explored for terrestrial wireless communications. The capacity is known [1] and various optimization criteria were considered for linear and non-linear beamforming techniques. Prominent examples are *quality-of-service* (QoS) optimizations with standard SINR requirements and SINR based balancing that can efficiently be solved via either convex optimization tools or fixed point methods (e.g., [2–4]). Recently, these advances where also considered for other communication scenarios, e.g., in *satellite communications* (SatCom) [5].

We consider the problem of maximizing the throughput via linear beamforming in a *vector broadcast channel* (vector BC) under limited total transmit power and for certain fixed ratios of the different user rates. This problem is called rate balancing in the literature (e.g., [6]) and it is closely related to the standard QoS power minimization and SINR balancing. Actually, it is identical to SINR balancing when all rate targets are equal and can be solved very efficiently using standard power minimization methods (e.g., [7, 8]). Therefore, QoS optimization and balancing methods were also applied to the multiple-input multiple-output broadcast channel and multi-carrier (OFDM) systems (e.g., [7, 9]).

In contrast to these perfect CSI methods, we consider only statistical knowledge of the channel states at the transmitter's side and, therefore, address an ergodic rate balancing formulation for the beamformer design. Whereas recent advances were made for the probabilistic constraint case of QoS optimization (e.g., [10–12]), where rate requirements shall be satisfied with certain probabilities, ergodic robust QoS optimization and balancing problems have only received limited attention in literature (e.g., [13]). The difficulties are the general lack of a duality between the Gaussian *multiple-access channel* (MAC) and BC with statistical transmitter CSI [14] and the missing convex reformulations for minimal requirements on the ergodic rates.

Due to all the difficulties in optimization with ergodic rate expressions, we focus on a 'rank-one' Gaussian channel model that is accurate for a large distance from the transmitter to the receivers. The so obtained ergodic rate balancing problem was also addressed in [8, 15, 16]. While [8] and [15] considered suboptimal solutions based on partial zero-forcing and bounding the ergodic requirements, a globally optimal branch and bound technique was applied in [16] that, however, is computationally unattractive due to its exponential complexity. Here, we propose a *locally optimal* sequential optimization method based on [17] for solving the ergodic rate balancing. With tight inner constraint approximations, we are able to reformulate the problem at hand into SINR structure and adapt the fixed point method from [7] (see Section 5). Moreover, we do not restrict ourselves to the zero-mean case here, as in the previous publications, and recast the formulations for general Gaussian scalar randomness in the channels.

For these purposes, the rest of the work is structured as follows. We present the system model, the channel statistics, and the ergodic rate metric in Section 2. The balancing optimization is introduced in Section 3 and the tight sequential approximation method is shown in Section 4, which also includes a convergence discussion. An algorithmic implementation of the inner problem solution is given afterwards in Section 5, before we finally provide some numerical results.

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2. SYSTEM AND CHANNEL MODEL

We consider a vector BC, where an N-antenna transmitter serves K mobiles in the same frequency band. At the transmitter, independent unit-variance data signals $s_k \sim \mathcal{N}_{\mathbb{C}}(0,1)$ are linearly precoded with the beamforming vectors $\boldsymbol{t}_k \in \mathbb{C}^N$, $k \in \{1,\dots,K\}$. The superimposed signal $\boldsymbol{x} = \sum_{i=1}^K \boldsymbol{t}_i s_i$ is transmitted over the channels to the receivers, which suffer from additive Gaussian noise $n_k \sim \mathcal{N}_{\mathbb{C}}(0,1)$. Therefore, the received signal of user k is $y_k = \boldsymbol{h}_k^{\mathrm{H}} \boldsymbol{t}_k s_k + \boldsymbol{h}_k^{\mathrm{H}} \sum_{i \neq k} \boldsymbol{t}_i s_i + n_k$, where $\boldsymbol{h}_k^{\mathrm{H}} \in \mathbb{C}^{1\times N}$ denotes the frequency flat fading channel vector corresponding to user k. This leads to the following expression for the achievable rate:

$$r_k = \log_2(1 + \mathrm{SINR}_k), \tag{1}$$

with the *signal-to-interference-plus-noise-ratio* (SINR)

$$\mathrm{SINR}_k = \frac{|\boldsymbol{h}_k^{\mathrm{H}} \boldsymbol{t}_k|^2}{1 + \sum_{i\neq k} |\boldsymbol{h}_k^{\mathrm{H}} \boldsymbol{t}_i|^2}. \tag{2}$$

Since above rate metric strongly depends on the state $\boldsymbol{h}_k$ of the channel, accurate information is required at the transmitter for the beamformer design. This information may be obtained via pilot based training and feedback or via uplink training in time-division-duplex systems. However, in fast fading scenarios and for long round trip times, only the statistics of the channels $\boldsymbol{h}_k \sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{m}_k, \boldsymbol{C}_k)$ can be obtained. For example, consider the data transmission to moving receivers that are surrounded by scatterers. Thus, we switch to the ergodic rates $\mathrm{E}[r_k]$, $k \in \{1,\dots,K\}$ as performance measures.

The problem with the ergodic rates is that their closed form expressions are involved functions of the precoders containing integrals over the product of hypergeometric and exponential functions that require numerical computations [18]. This makes it intractable for beamformer optimizations, especially when K and N are large. To overcome these drawbacks, we consider a special CSI model that allows for a tight approximation of $\mathrm{E}[r_k]$. Especially, we assume that the transmitter is far distant from the receivers and shadowing and scattering effects are solely close to the mobiles, e.g., as in rural terrestrial environments or SatCom. Then, the spatial channel characteristics can be estimated very well and the randomness is merely captured by the gain. That is, we model the channels as $\boldsymbol{h}_k \approx \tilde{\boldsymbol{h}}_k = \boldsymbol{v}_k w_k(\boldsymbol{h}_k)$, where $\boldsymbol{v}_k$ and the random scalar $w_k(\boldsymbol{h}_k)$ minimize the mean square error $\mathrm{E}\big[\|\boldsymbol{h}_k - \tilde{\boldsymbol{h}}_k\|_2^2\big]$, i.e., $w_k(\boldsymbol{h}_k) = \boldsymbol{v}_k^{\mathrm{H}} \boldsymbol{h}_k$ and $\boldsymbol{v}_k$ is the unit-norm dominant eigenvector of $\boldsymbol{m}_k \boldsymbol{m}_k^{\mathrm{H}} + \boldsymbol{C}_k$ [19].

This leads to the ergodic rate expression [18]

$$R_k = \mathrm{E}[\tilde{r}_k] = g_k\Big(\sum_{i=1}^K |\boldsymbol{v}_k^{\mathrm{H}} \boldsymbol{t}_i|^2\Big) - g_k\Big(\sum_{i\neq k} |\boldsymbol{v}_k^{\mathrm{H}} \boldsymbol{t}_i|^2\Big). \tag{3}$$

The scalar functions $g_k : \mathbb{R}_+ \to \mathbb{R}_+$, $x \mapsto g_k(x)$ read as

$$\begin{aligned} g_k(x) &= \mathrm{E}\left[\log_2(1 + |w_k(\boldsymbol{h}_k)|^2 x)\right] \qquad (4)\\ &= \mathrm{e}^{-\frac{|m_k|^2}{\sigma_k^2}} \int_0^\infty \log_2(1 + x\sigma_k^2 t)\, \mathrm{e}^{-t}\, {}_0F_1\big(;1; t\tfrac{|m_k|^2}{\sigma_k^2}\big)\, \mathrm{d}t, \end{aligned}$$

with mean $m_k = \mathrm{E}[w_k(\boldsymbol{h}_k)] = \boldsymbol{v}_k^{\mathrm{H}} \boldsymbol{m}_k$, variance $\sigma_k^2 = \mathrm{E}[|w_k(\boldsymbol{h}_k) - m_k|^2] = \boldsymbol{v}_k^{\mathrm{H}} \boldsymbol{C}_k \boldsymbol{v}_k$, and the generalized hypergeometric function ${}_0F_1(;\cdot;\cdot)$ [20]. The evaluation of (4) requires numerical integration, but the outcomes can be tabulated for m_k, σ_k^2, and x. For $m_k = 0$, (4) simplifies to [20]

$$g_k(x) = \tfrac{1}{\ln(2)}\, \mathrm{e}^{\frac{1}{\sigma_k^2 x}}\, \mathrm{E}_1\left(\tfrac{1}{\sigma_k^2 x}\right) \tag{5}$$

with the exponential integral $\mathrm{E}_1(z) = \int_z^\infty \frac{\mathrm{e}^{-t}}{t}\,\mathrm{d}t$.

3. RATE BALANCING PROBLEM

For limited total transmit power P_{tx}, we want to maximize the common factor ρ_0 that balances the ergodic rates R_k regarding their relative requirements ρ_k, $k \in \{1,\dots,K\}$, i.e.,

$$\max_{\rho_0, \{\boldsymbol{t}_k\}} \rho_0 \quad \text{s.t.} \quad \sum_{k=1}^K \|\boldsymbol{t}_k\|_2^2 \le P_{\mathrm{tx}}, \qquad R_k \ge \rho_0 \rho_k \ \forall k \in \{1,\dots,K\}. \tag{6}$$

Even though no tractable reformulation of (6) is known for ergodic constraints, we are aware of some basic properties.

Since the rates are increasing in the used transmit power, it is P_{tx} at the optimum and the resulting rates are balanced, i.e., $R_k = \rho_0 \rho_k$. Moreover, (6) is the inverse problem to the power minimization in [21] with only ergodic rate requirements. Expressing the power minimization as

$$P(\rho_0) = \min_{P_{\mathrm{tx}}} \big\{ P_{\mathrm{tx}} : P_{\mathrm{tx}} \ge \sum_{i=1}^K \|\boldsymbol{t}_i\|_2^2,\ R_k \ge \rho_0 \rho_k \forall k \big\} \tag{7}$$

and the optimum of (6) as $\rho(P_{\mathrm{tx}})$, we can verify that (cf. [22])

$$P\big(\rho(P_{\mathrm{tx}})\big) = P_{\mathrm{tx}}.$$

This means, (6) may be solved via a series of power minimizations (7) until the optimum equals P_{tx}, e.g., using bisection. This procedure, however, leads to a high computational complexity as the already complex locally optimal power minimization from [21] or the even more complex globally optimal branch-and-bound method from [16] has to be applied repeatedly. Therefore, a direct inner approximation method and a suitable implementation is discussed in the next sections.

4. INNER APPROXIMATION TECHNIQUE

With (4) and $I_k = \sum_{i\neq k} |\boldsymbol{v}_k^{\mathrm{H}} \boldsymbol{t}_i|^2$ for the interference power, we recast the ergodic rate requirements in (6) as

$$g_k\left(|\boldsymbol{v}_k^{\mathrm{H}} \boldsymbol{t}_k|^2 + I_k\right) - g_k\left(I_k\right) \ge \rho_0 \rho_k. \tag{8}$$

Similar to [21], we rewrite (8) as a minimum requirement for the useful signal power. To this end, we add $g_k(I_k)$ on both sides, take the inverse of g_k, and subtract I_k to obtain

$$|\boldsymbol{v}_k^{\mathrm{H}} \boldsymbol{t}_k|^2 \ge f_k(\rho_0 \rho_k, I_k) \tag{9}$$

with the non-linear continuous function $f_k : \mathbb{R}_+^2 \to \mathbb{R}$,

$$f_k(c, x) = g_k^{-1}\big(c + g_k(x)\big) - x,$$

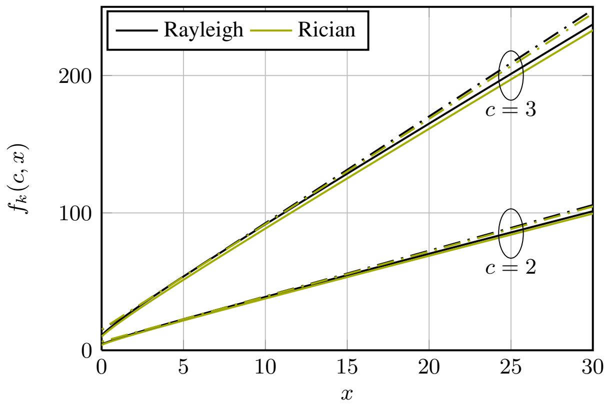

Fig. 1. Plot of $f_k(c, x)$ and the linearization at $x = 8$ (dash-dotted) for Rayleigh fading with $m_k = 0$ and $\sigma_k^2 = 1$ and for Rician fading with $m_k = \frac{1}{\sqrt{2}}$ and $\sigma_k^2 = \frac{1}{2}$

which is non-linearly increasing with x and c. In fact, $f_k(c, x)$ is exponentially increasing in c for fixed x since $g_k(y)$ is logarithmically increasing with y. Moreover, $f_k(c, x)$ appears to be concave and close to linear in x for fixed c (see Fig. 1).

Due to this property, an approximation of (9) that is tight at $\hat{I}_k$ can be obtained via the linearization (cf. [21])

$$f_k(\rho_0\rho_k, I_k) \approx \alpha_k(\rho_0, \hat{I}_k) I_k + \beta_k(\rho_0, \hat{I}_k) \tag{10}$$

where the slope α_k reads as

$$\alpha_k(\rho_0, \hat{x}) = \frac{\partial f_k(\rho_0\rho_k, x)}{\partial x}\Big|_{x=\hat{x}} = \frac{g_k'(\hat{x})}{g_k'\left(g_k^{-1}(\rho_0\rho_k + g_k(\hat{x}))\right)} - 1,$$

with the derivative

$$g_k'(x) = \mathrm{e}^{-\frac{|m_k|^2}{\sigma_k^2}} \int_0^\infty \frac{t\sigma_k^2}{\ln(2)(1+x\sigma_k^2 t)}\, \mathrm{e}^{-t}\, {}_0F_1\big(;1;t\tfrac{|m_k|^2}{\sigma_k^2}\big)\, \mathrm{d}t,$$

which requires numerical integration, and the offset is

$$\beta_k(\rho_0, \hat{x}) = f_k(\rho_0\rho_k, \hat{x}) - \alpha_k(\rho_0, \hat{x})\hat{x}.$$

The linearization for $m_k = 0$ can be simplified as in [21].

Replacing $f_k(\rho_0\rho_k, I_k)$ by its linearization in (9), the resulting demand for the useful signal power is still exponentially increasing with ρ_0, but linear in the interference power on the right hand side, i.e.,

$$|\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{t}_k|^2 \geq \alpha_k(\rho_0, \hat{I}_k) \textstyle\sum_{i\neq k} |\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{t}_i|^2 + \beta_k(\rho_0, \hat{I}_k). \tag{11}$$

This allows for an approximate reformulation of (6) in SINR constrained form, where efficient algorithmic solution methods based on SINR constrained QoS optimization are known (e.g., see [7] and [8]). That is, defining the ergodic 'signal-to-interference-and-noise-ratio' as

$$\underline{\mathrm{SINR}}_k = \frac{|\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{t}_k|^2}{\frac{\beta_k(\rho_0, \hat{I}_k)}{\alpha_k(\rho_0, \hat{I}_k)} + \sum_{i\neq k} |\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{t}_i|^2}, \tag{12}$$

we can approximate (6) via

$$\max_{\rho_0, \{\boldsymbol{t}_k\}} \rho_0 \quad \text{s.t.} \quad \textstyle\sum_{k=1}^K \|\boldsymbol{t}_k\|_2^2 \leq P_{\text{tx}}, \qquad \underline{\mathrm{SINR}}_k \geq \alpha_k(\rho_0, \hat{I}_k)\ \forall k \in \{1, \ldots, K\}. \tag{13}$$

Note that the solution $\{\boldsymbol{t}_k^\star\}$ of (13) is feasible for (6), but in general suboptimal and only accurate if the approximation is tight, i.e., $\hat{I}_k = \sum_{i\neq k} |\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{t}_i^\star|^2$ for all $k \in \{1, \ldots, K\}$. This motivates an iterative procedure to solve (6), where the following two steps are performed in the n-th iteration:

1. $\hat{I}_k^{(n)} = \sum_{i\neq k} |\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{t}_i^{(n-1)}|^2$ for all $k \in \{1, \ldots, K\}$
2. and approximate (6) with (13) at $\hat{I}_k^{(n)}$ to obtain $\{\boldsymbol{t}_k^{(n)}\}$.

As an initialization, we use the maximum-ratio-combining beamformers with equal power allocation, i.e., $\boldsymbol{t}_k^{(0)} = \frac{P_{\text{tx}}}{K}\boldsymbol{v}_k$.

4.1. Convergence of Sequential Strategy

We remark that above sequential optimization procedure globally converges to a locally optimal point if the constraint set in (13) is a *tight inner approximation* of that in (6) within each iteration [17]. That is, the linearization has to satisfy the following three properties (cf. [17]):

(i) $f_k(\rho_0\rho_k, I_k) \leq \alpha_k(\rho_0, \hat{I}_k^{(n)}) I_k + \beta_k(\rho_0, \hat{I}_k^{(n)})$,

(ii) $f_k(\rho_0\rho_k, \hat{I}_k^{(n)}) = \alpha_k(\rho_0, \hat{I}_k^{(n)}) \hat{I}_k^{(n)} + \beta_k(\rho_0, \hat{I}_k^{(n)})$,

(iii) $\frac{\partial}{\partial I_k} f_k(\rho_0\rho_k, I_k)\big|_{I_k = \hat{I}_k^{(n)}} = \alpha_k(\rho_0, \hat{I}_k^{(n)})$.

Items (ii) and (iii) are clearly satisfied for the linearization of a continuous differentiable function. Property (i) is satisfied if $f_k(c, x)$ is concave in x, which happened in all cases that we investigated in our numerical simulations (e.g., see Fig. 1). We leave a rigorous proof of this property for future research.

5. ALGORITHMIC IMPLEMENTATIONS

For solving (13), we could apply a bisection over ρ_0 as proposed in [8], where in each bisection step it is tested whether the current SINR requirements are achievable with the given transmit power. Next, we present an alternative algorithmic approach that has faster convergence speed.

This method requires a transformation to the dual MAC according to [22]. Even though such a duality does not hold for the ergodic rates, we can apply duality for the approximated constraints since we could rewrite the approximation in terms of an ergodic SINR (13), which has the same structure as the conventional SINR (2). In the dual MAC, the requirements read as

$$\underline{\mathrm{SINR}}_k^{\mathrm{MAC}} = \frac{\frac{\alpha_k(\rho_0, \hat{I}_k)}{\beta_k(\rho_0, \hat{I}_k)} |\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{u}_k|^2 p_k}{\|\boldsymbol{u}_k\|_2^2 + \sum_{i\neq k} \frac{\alpha_i(\rho_0, \hat{I}_i)}{\beta_i(\rho_0, \hat{I}_i)} |\boldsymbol{v}_i^{\mathrm{H}}\boldsymbol{u}_k|^2 p_i} \geq \alpha_k(\rho_0, \hat{I}_k), \tag{14}$$

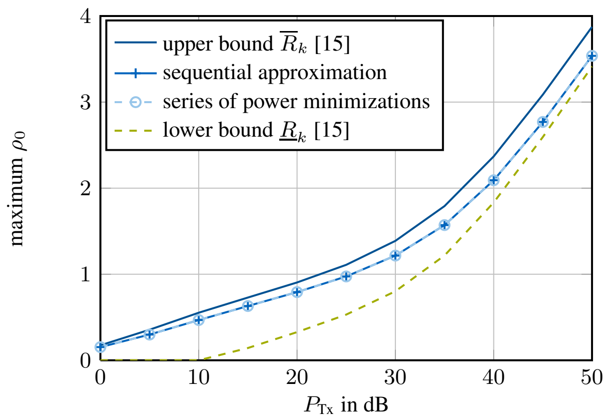

Fig. 2. Plot of maximum ρ_0 over P_{tx} in [dB] for a system with $K = N = 4$ in an satellite system setup.

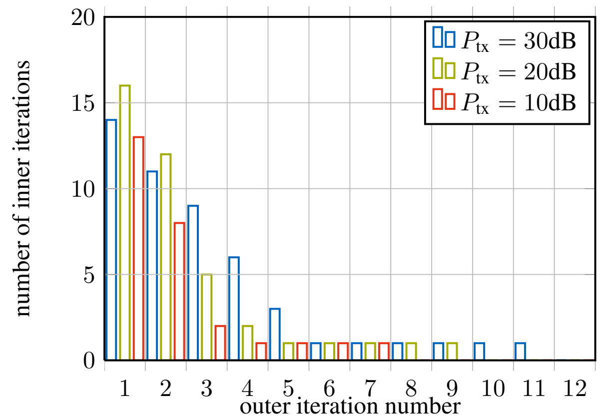

Fig. 3. Required number of inner iterations over the outer iteration number for $K = N = 4$ and various transmit powers.

where the receive filters and transmit powers in the MAC are denoted by $\boldsymbol{u}_k$ and p_k, $k \in \{1, \ldots, K\}$, respectively, and the sum transmit power constraint is

$$\textstyle\sum_{i=1}^{K} p_i \leq P_{\text{tx}}. \quad (15)$$

Note that given a set $\{\boldsymbol{t}_k\}$ in the BC, the corresponding MAC filters $\{\boldsymbol{u}_k\}$ and powers $\{p_k\}$, i.e., those that achieve the same SINRs with the same sum transmit power, can be obtained via solving a K-dimensional linear equation system (cf. [22]), and vice versa. The BC-to-MAC transformation is used to iteratively solve (13) in the MAC, starting from the current BC operation point. The results are delivered to the BC via the MAC-to-BC transformation for the subsequent sequential approximation in the inner approximation strategy.

The solution approach for (13) in the MAC is from [7, Section III.] and adapted to the given system. Based on the fact that the optimal filters and powers in the MAC satisfy (14), we write a power update from [23] as

$$p_k^{(j+1)} \leftarrow \frac{\alpha_k(\rho_0^{(j+1)}, \hat{I}_k) p_k^{(j)}}{\text{SINR}_k^{\text{MAC}}}. \quad (16)$$

Since the optimal powers shall also satisfy (15) with equality, we insert this update into (15) to require $\rho_0^{(j+1)}$ to fulfill

$$\textstyle\sum_{k=1}^{K} \frac{\alpha_k(\rho_0^{(j+1)}, \hat{I}_k) p_k^{(j)}}{\text{SINR}_k^{\text{MAC}}} - P_{\text{tx}} = 0, \quad (17)$$

which can be obtained via a fixed point search for $\rho_0^{(j+1)}$, e.g., bisection. Iteratively performing (16) using $\rho_0^{(j+1)}$ from (17), we solve (13) in the MAC. In contrast to [7], inserting the optimal $\boldsymbol{u}_k^{(j)} = (\mathbf{I}_N + \sum_{i \neq k} \frac{\alpha_i(\rho_0^{(j+1)}, \hat{I}_i)}{\beta_i(\rho_0^{(j+1)}, \hat{I}_i)} \boldsymbol{v}_i \boldsymbol{v}_i^{\text{H}} p_i^{(j)})^{-1} \boldsymbol{v}_k$, which maximizes the SINR, into (16) and (17), the denominator in

$$\frac{\alpha_k(\rho_0^{(j+1)}, \hat{I}_k) p_k^{(j)}}{\text{SINR}_k^{\text{MAC}}} = \frac{1/\beta_k(\rho_0^{(j+1)}, \hat{I}_k)}{\boldsymbol{v}_k^{\text{H}} \left(\mathbf{I}_N + \sum_{i \neq k} \frac{\alpha_i(\rho_0^{(j+1)}, \hat{I}_i)}{\beta_i(\rho_0^{(j+1)}, \hat{I}_i)} \boldsymbol{v}_i \boldsymbol{v}_i^{\text{H}} p_i^{(j)} \right)^{-1} \boldsymbol{v}_k}$$

clearly depends on $\rho_0^{(j+1)}$, why the inverse needs to be computed several times for finding $\rho_0^{(j+1)}$. Since the evaluations of $\alpha_k(\rho_0, \hat{I}_k)$ and $\beta_k(\rho_0, \hat{I}_k)$ can be tabulated, iteratively calculating the inverse within update (17) dominates the computational complexity for solving (13) and therewith (6).

6. NUMERICAL COMPARISON

For a numerical performance analysis of the sequential approximation method, we used a similar system setup as in [16], i.e., with a satellite geometry and a free space path loss model for determining $\boldsymbol{v}_k$, $k \in \{1, \ldots, K\}$. However, here, we set the mean to $m_k = \frac{1}{\sqrt{2}}$ and the variance to $\sigma_k^2 = \frac{1}{2}$ for all channels of the system with $K = N = 4$. The users' targets are chosen to be $\rho_1 = \rho_3 = 1$ and $\rho_2 = \rho_4 = 2$.

In Fig. 2, we plotted the (optimal) balancing level ρ_0 versus P_{tx} in dB for four approaches. The lower and upper bound curves are obtained via replacing R_k in (6) with the ergodic rate lower bound and upper bounds $\underline{R}_k$ and $\overline{R}_k$ from [15]. Clearly, the locally optimal sequential approximation strategy of Section 4 lies between these bounds as expected. For comparison, we have plotted the curve that is obtained by a series of power minimizations. We see that the proposed method achieves the same performance while avoiding the problems of power minimizations that are discussed in Section 3.

For evaluating the complexity of the implemented sequential approximation, we plotted the number of inner fixed-point iterations that are required within each sequential approximation update in Fig. 3 for $P_{\text{tx}} \in \{10\text{dB}, 20\text{dB}, 30\text{dB}\}$. As can be seen, within the first approximations, the most inner iterations are required until convergence. That is, the more accurate the outer approximation is, the less inner iterations are required since $\alpha_k(\rho_0, \hat{I}_k)$ and $\beta_k(\rho_0, \hat{I}_k)$ are more accurate.

For a near future work, we leave a strict proof for concavity of $f_k(c, x)$ in x, a complexity analysis of the proposed algorithm, a comparison to a simplified version, where only one iteration step of the inner power and filter update is performed in the MAC, and a comparison of the resulting rates to the actually achieved rates of vector Gaussian channels.

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