JOINT POWER-LOADING AND CYCLIC PREFIX LENGTH OPTIMIZATION FOR OFDM-BASED POWER LINE COMMUNICATION

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ABSTRACT

The use of a cyclic prefix (CP) in orthogonal frequency-division multiplexing (OFDM) allows to avoid inter-carrier interference (ICI), but also diminishes the efficiency of the modulation scheme. We study the problem of jointly optimizing the CP length and the discrete bit and power-loading under ICI in OFDM, exemplified by its application to power line communication (PLC). We derive a low-complexity linear programming (LP) based performance upper-bound which is applicable to analyze the suboptimality of bit-loading heuristics under varying CP lengths. Furthermore, a branch-and-bound heuristic is proposed for low-complexity CP-length selection. While we do not expect the LP bound to be tight in all possible interference scenarios, our simulation results support the hypothesis that it is sufficiently tight in the range of near-optimal CP lengths in order to provide guidelines for optimal as well as heuristic CP-length selection.

Index Terms— OFDM modulation, Intersymbol interference, Mathematical programming, Power line communication

1. INTRODUCTION

Cyclic-prefix (CP) based orthogonal frequency-division multiplexing (OFDM) is widely recognized as a prominent modulation scheme that has been adopted for several state-of-the-art wireless and wireline communication systems. The CP prevents intersymbol and intercarrier interference (ICI) [1] if its length is chosen longer (or equal) than the channel impulse response length. This allows for low-complexity OFDM realizations relying on single-tap frequency domain equalization. However, the CP diminishes the efficiency of the modulation scheme. Reducing the CP length below the channel impulse response length improves the efficiency, but introduces ICI. Spectral bit and power-loading is applicable as a means to partly compensate for the thereby created ICI. We study the problem of joint power-loading and CP-length selection in OFDM-based power line communication (PLC) so as to maximize the transmission rate. The problem is approached by first investigating bounds on the rate under power-loading and ICI for a fixed CP length. In a second step the achieved rate is then the objective for CP-length selection.

Our numerical study mimics a PLC environment. However, we hasten to add that CP-length optimization and hence also the proposed algorithms are equally relevant in various other communication systems such as the currently standardized next generation of digital subscriber lines (DSL) [2]. The influence of ICI on various modulation techniques as well as bit and power-loading schemes to combat ICI has been studied in [1], [3]–[10]. The problem of jointly optimizing the CP length and bit and power-loading for OFDM-based PLC has been studied in [3], focussing on practical low-complexity bit-loading heuristics with on/off power-loading and without performance guarantees. More general continuous power-loading strategies have been presented in [11], however, not regarding practical bit-loading restrictions and again without giving any performance bounds. ICI due to asynchronous crosstalk in DSL access networks has also been approached by multi-user power-loading in [6]–[8], [12]. Differently we will mainly focus on a single OFDM transceiver suffering from ICI due to an insufficient CP length. Single-user power-loading under ICI closely resembles the classical multi-user power control problem [13]. However, corresponding global optimal solutions are highly complex and and hence only applicable to problems with few variables [14]–[16].

The main contributions of the present work are a) the low-complexity implementation of the performance bound under ICI in [12] for the CP-length optimization of a single OFDM transceiver; b) an efficient search heuristic for scenario-dependent CP-length selection; and c) the simulation analysis of the effect of bit-loading restrictions and suboptimal power-loading on transmission rate in PLC. After introducing the optimization model in Section 2 we present and compare three performance bounds for power-loading under ICI in Section 3. In Section 4 a branch-and-bound based search heuristic for CP-length selection is derived. The transmission rate and complexity of the proposed schemes is compared through exhaustive simulations in Section 5. Our conclusions are summarized in Section 6.

2. SYSTEM AND OPTIMIZATION MODEL

We assume an OFDM transceiver that splits the bandwidth $\hat{f}$ [Hz] into C subcarriers on complex baseband frequencies $f_c = \hat{f} \cdot c/C$ and applies a CP of length μ [samples], resulting in a symbol rate of $\eta(\mu) = \hat{f}/(C + \mu)$. ICI appears when the CP length μ is smaller than the channel impulse response length ν [samples]. We denote the index-set of usable subcarriers by $\mathcal{C} \subseteq \{1, \ldots, C\}$, and write the index-set excluding subcarrier c as $\mathcal{C} \setminus c$. The number of bits that

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can be carried on subcarrier $c \in \mathcal{C}$ is approximated by [17]

$$r_c^{(\mu)}(p_c, X_c^{(\mu)}(\mathbf{p}^{-c})) = \log_2\Big(1 + \frac{H_c^{(\mu)} p_c}{\Gamma(H_{cc}^{(\mu)} p_c + X_c^{(\mu)}(\mathbf{p}^{-c}) + N_c)}\Big), \tag{1}$$

where p_c and $H_c^{(\mu)}$ denote the transmit power and direct channel gain on subcarrier c, $H_{cc}^{(\mu)} p_c$ is the inter-symbol interference (ISI) noise caused on subcarrier c by the transmission on the same subcarrier, $X_c^{(\mu)}(\mathbf{p}^{-c}) = \sum_{k \in \mathcal{C} \setminus c} H_{kc}^{(\mu)} p_k$ is the total ICI noise originating from all subcarriers $k \in \mathcal{C} \setminus c$, with $H_{kc}^{(\mu)}$ denoting the ICI channel gain [3] from subcarrier k to subcarrier c, $\mathbf{p} \in \mathbb{R}^{|\mathcal{C}|}$ and $\mathbf{p}^{-c} \in \mathbb{R}^{|\mathcal{C}|-1}$ are the vectors of transmit powers on subcarriers $k \in \mathcal{C}$ and $k \in \mathcal{C} \setminus c$, respectively, N_c is the additive white Gaussian background noise power, and Γ represents the SNR gap that models the practical modulation and coding scheme for a targeted bit-error rate. The problem of maximizing the transmission rate by varying the transmit power and (discrete) bit-load over subcarriers for a given CP length is posed as

$$\begin{aligned}
\underset{\{p_c, x_c, \forall c \in \mathcal{C}\}}{\text{maximize}} \quad & \eta(\mu) \cdot \sum_{c \in \mathcal{C}} r_c^{(\mu)}(\, p_c,\, x_c) && && \text{(2a)}\\
\text{subject to} \quad & r_c^{(\mu)}(p_c, x_c) \in \mathcal{B}, && \forall c \in \mathcal{C}, && \text{(2b)}\\
& 0 \le p_c \le \hat{p}_c, && \forall c \in \mathcal{C}, && \text{(2c)}\\
& 0 \le x_c \le X_c^{(\mu)}(\hat{\mathbf{p}}^{-c}), && \forall c \in \mathcal{C}, && \text{(2d)}\\
& X_c^{(\mu)}(\mathbf{p}^{-c}) \ \le \ x_c, && \forall c \in \mathcal{C}, && \text{(2e)}\\
& \sum_{c \in \mathcal{C}} p_c \ \le \ P, && && \text{(2f)}
\end{aligned}$$

where $\mathcal{B}$ is the set of loadable bits corresponding to the finite number of constellations supported in practice, $\hat{\mathbf{p}} \in \mathbb{R}^{|\mathcal{C}|}$ is the vector of transmit-power mask values $\hat{p}_c, c \in \mathcal{C}$, and P denotes a maximum aggregate transmit-power. While in our simulation examples we will assume that P equals the integral of the spectral mask, the constraint in (2f) is added to show how additional constraints can easily be captured by the optimization approaches presented in the following sections. In fact, restricting the aggregate transmit-power allows to operate the transceiver in a more energy-efficient manner. Based on the comparable aggregate transmit-power values in DSL the energy-saving potential by transmit-power reduction [18] is expected to be considerable in PLC as well.[1]

The introduced artificial variables $x_c, c \in \mathcal{C}$, and constraints in (2d) and (2e) are redundant at this point, but serve the latter problem decomposition detailed in Section 3.3.

3. BOUNDING TECHNIQUES

Unless stated otherwise we will assume that the problem in (2) is evaluated for various CP lengths μ, leaving us with the problem of bit and power-loading that is studied in this section. Various bit-loading heuristics [3], [22] have been proposed to solve the problem in (2). In the following we use two approaches to judge the quality of any potentially suboptimal bit-loading solution. The first is to apply a certificate of optimality that proves that no better bit and power-loading than a given one exists (cf. Section 3.1). In case this fails we need to resort to the second approach, namely a low-complexity problem relaxation that upper-bounds the achievable transmission rate by the primal problem in (2) and thereby allows to infer bounds on the suboptimality of CP-length optimization under heuristic bit-loading schemes. For the purpose of computing upper-bounds we compare two efficiently computable alternatives, where the first in Section 3.2 is based on the continuous sum-power-constrained relaxation in [23] and the second one in Section 3.3 is a specialization of the Lagrange relaxation in [12].

3.1. Optimality Certificate for Discrete Power Control

As already noted in Section 1 the problem in (2) over a single user and multiple coupled subcarriers is equivalent to one over a single subcarrier and multiple coupled users. The latter problem is known as power control [13]. More precisely, our problem corresponds to sum-rate maximizing power control over C users with discrete rate or transmit-power levels and a sum-power constraint over all users. For discrete power control without sum-power constraint a simple "search-space reduction" technique is proposed in [24] which relies on the analytically solvable continuously relaxed problem without interference. If this method returns a singleton as the reduced search-space then this certifies that a given feasible solution to the problem in (2) is optimal. We refer to [24] for further details on the method.

3.2. Geometric Programming Relaxation

One relaxation of the problem in (2) is obtained by neglecting the constraints in (2b) and (2c) but considering a constraint in (2f) with maximum aggregate transmit-power $\tilde{P} = \min\{P, \sum_{c \in \mathcal{C}} \hat{p}_c\}$ instead. Furthermore, we define matrices $B_{ck} \triangleq \frac{H_{kc}^{(\mu)}}{H_c^{(\mu)}} + \frac{\Gamma N_c}{\tilde{P} H_c^{(\mu)}} + \mathbb{1}_{\{c\}}(k)$, $c, k \in \mathcal{C}$, and $\mathbf{C} \triangleq \mathbf{I} - \mathbf{B}^{-1}$, where $\mathbb{1}_{\{c\}}(k)$ equals 1 if k equals c and zero otherwise. As proven in [23], if $\mathbf{B}$ is non-singular and all elements of $\mathbf{C}$ are non-negative this relaxation is efficiently globally solvable by the geometric program (GP)

$$\begin{aligned}
\underset{\{t_c, y_c, \forall c \in \mathcal{C}\}}{\text{minimize}} \quad & \prod_{c \in \mathcal{C}} t_c && && \text{(3a)}\\
\text{subject to} \quad & y_c^{-1} t_c^{-1} \sum_{k \in \mathcal{C}} C_{ck} y_k \le 1, && \forall c \in \mathcal{C}, && \text{(3b)}\\
& y_c^{-1} \sum_{k \in \mathcal{C}} C_{ck} y_k \le 1, && \forall c \in \mathcal{C}, && \text{(3c)}\\
& \prod_{c \in \mathcal{C}} y_c = 1. && && \text{(3d)}
\end{aligned}$$

The actual power-loading is computable based on the optimal variables $y_c, c \in \mathcal{C}$, as shown in [23]. Here we are only interested in an objective upper-bound to the primal problem in (2), given by $-\eta(\mu) \cdot \log_2(\prod_{c \in \mathcal{C}} t_c)$ for optimal variables $t_c, c \in \mathcal{C}$, in (3).

3.3. Linear Programming based Lagrange Relaxation

The bounds in Sections 3.1 and 3.2 only hold when certain interference related criteria hold. Differently, the weak duality relation [25] guarantees that the optimal objective of the primal problem in (2) is in any scenario upper-bounded by that of the Lagrange-dual problem. Lagrange relaxation of the constraints in (2e) and (2f) with

[1] The aggregate transmit-power under the spectral mask [19] defined for communication over power lines up to 100 MHz by the home networking standard in [20] is over 19.48 dBm, and for home-plug AV (HPAV) PLC systems operating between 2 and 28 MHz over 24 dBm [3]. For comparison, the maximum transmit-power in VDSL2 [21] under band plan 8b is 20.5 dBm.

associated dual multipliers $\boldsymbol{\kappa} \in \mathbb{R}^C$ and $\lambda \in \mathbb{R}$ results in the decomposed per-subcarrier problems

$$\underset{r_c, p_c, x_c}{\text{maximize}} \;\; \eta(\mu) r_c - w_c p_c + \kappa_c x_c \tag{4a}$$

$$\text{subject to} \;\; r_c \in \mathcal{B}, \tag{4b}$$

$$0 \le p_c \le \hat{p}_c, \tag{4c}$$

$$0 \le x_c \le X_c^{(\mu)}(\hat{\mathbf{p}}^{-c}), \tag{4d}$$

$$r_c \le r_c^{(\mu)}(p_c, x_c), \tag{4e}$$

where we make the discrete bit-loading variables $r_c, c \in \mathcal{C}$, explicit, and $w_c = \lambda + \sum_{k \in \mathcal{C} \setminus c} H_{ck}^{(\mu)} \kappa_k$. For fixed rate r_c we can solve the remaining problem in p_c and x_c explicitly. More precisely, both weights w_c and κ_c are non-negative, which is why we can assume that p_c and x_c will take the lowest and highest values according to the constraint set in (4c)–(4e), respectively. Furthermore, the right-hand-side in (4e) is increasing in p_c and decreasing in x_c. Hence, neglecting the box-constraints in (4c) and (4d) for now, we see that the constraint in (4e) holds with equality, and that we can replace x_c in the objective to obtain one that is linear in the transmit power p_c with weight $\tilde{w}_c = \kappa_c(\frac{H_c^{(\mu)}}{(2^{r_c}-1)\Gamma} - H_{cc}^{(\mu)}) - w_c$. Depending on the weight $\tilde{w}_c$ we would opt for minimizing or maximizing the transmit power, subject to the mentioned box constraints. Minimizing transmit power implies $x_c = 0$, while maximizing it either leads to $p_c = \hat{p}_c$ or $x_c = X_c^{(\mu)}(\hat{\mathbf{p}}^{-c})$, depending on which constraint is more restrictive. In any case the constraint in (4e) will hold with equality due to the mentioned tendency to minimize transmit power.

This analysis allows us to compactly write the set of all possible solutions, indexed by $i \in \mathcal{I}_c$ and consisting of two subsets: (i) at most $|\mathcal{B}|$ solutions (those that are feasible in (4)) where r_c takes each value in $\mathcal{B}$, $p_c = N_c/(\frac{H_c^{(\mu)}}{(2^{r_c}-1)\Gamma} - H_{cc}^{(\mu)})$, and $x_c = 0$, and (ii) at most $|\mathcal{B}|$ solutions where r_c takes each value in $\mathcal{B}$, and either $p_c = \hat{p}_c$ (if the x_c according to (4e) is less or equal to $X_c^{(\mu)}(\hat{\mathbf{p}}^{-c})$) or $x_c = X_c^{(\mu)}(\hat{\mathbf{p}}^{-c})$ (with p_c taking the minimal value according to the constraint in (4e)). This results in a set of transmit power and ICI-noise solutions $\{(p_c^{(i)}, x_c^{(i)}), i \in \mathcal{I}_c, c \in \mathcal{C}\}$. The equivalent LP form [22] of the dual master problem for the primal problem in (2) is stated as

$$\underset{\xi_c^{(i)} \ge 0, i \in \mathcal{I}_c, c \in \mathcal{C}}{\text{maximize}} \;\; \eta(\mu) \cdot \sum_{c \in \mathcal{C}} \sum_{i \in \mathcal{I}_c} r_c^{(\mu)}(p_c^{(i)}, x_c^{(i)}) \xi_c^{(i)} \tag{5a}$$

subject to

$$\sum_{i \in \mathcal{I}_c} \xi_c^{(i)} = 1, \qquad \forall c \in \mathcal{C}, \tag{5b}$$

$$\sum_{k \in \mathcal{C} \setminus c} \sum_{i \in \mathcal{I}_k} H_{kc}^{(\mu)} p_k^{(i)} \xi_k^{(i)} \le \sum_{i \in \mathcal{I}_c} x_c^{(i)} \xi_c^{(i)}, \quad \forall c \in \mathcal{C}, \tag{5c}$$

$$\sum_{c \in \mathcal{C}} \sum_{i \in \mathcal{I}_c} p_c^{(i)} \xi_c^{(i)} \le P, \tag{5d}$$

where the ICI noise and aggregate transmit-power constraints in (2e) and (2f) are relaxed by their averages in (5c) and (5d) over solutions to the dual subproblems in (4). Furthermore, the constraint in (5b) enforces a convex combination of those solutions. We emphasize that in the more general multi-user and multi-carrier ICI channel [12] we need to resort to an iterative scheme to fill the LP with various per-subcarrier bit and power-loadings due to the large number of possible solutions. However, for the single-user problem in (2) we have found a restricted set of bit and power-loadings that contains optimal solutions to all possible per-subcarrier dual subproblems in (4). Also note that the presented bound can easily be extended to various bit-loading restrictions, such as an on-off type of power-loading (that is, notching of subcarriers) as used in homeplug AV (HPAV) [3] or discrete power loading steps. For example, the home networking standard in [20] defines a transmit-spectrum shaping that can be varied in steps of 0.5 dB. While we focus on a single-user system, the extension of the derived performance bound to the downlink of an orthogonal frequency-division multiple access (OFDMA) system realized by a single transceiver serving multiple users [3] is straightforward. In fact, the LP relaxation approach for bit-loading in this OFDMA case presented in [3] corresponds to a special case of the problem in (5) where transmit powers are constant, the number of bits that can be transmitted by each user on any subcarrier is therefore known, and hence the only open task is to allocate subcarriers to users.

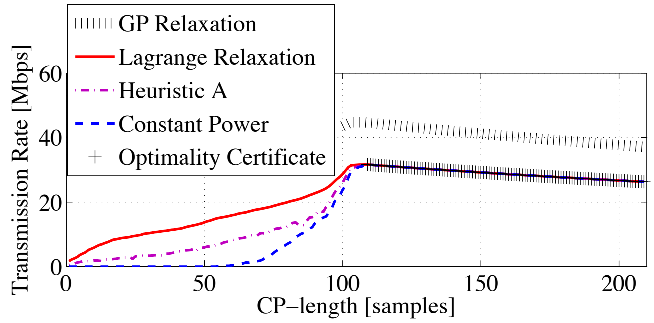

Fig. 1. Comparison of two bit-loading heuristics, the optimality certificate of Section 3.1, and the low-complexity upper-bounds of Sections 3.2 and 3.3 based on an exemplary power-line channel instance.

Based on the solution of the LP in (5) the following feasible bit-loading heuristic, referred to as heuristic A, may be derived. The bit-load according to the average power-load per subcarrier (which obeys the maximum sum-power due to the constraint in (5d)) is on each subcarrier floored to the next-lower element in $\mathcal{B}$ and used as an initialization for the bit-adding scheme in [6], adapted to cover an arbitrary set $\mathcal{B}$ of feasible bit-loads.

In Figure 1 we compare the three bounding techniques presented in this section to heuristic A and the reference performance obtained by constant power-loading at spectral mask under the simulation setup described in Section 5 (a realization of a class-1 power line channel [26]). As this example already indicates, the LP bound is sufficiently tight to allow for global performance guarantees, differently to the GP bound or the optimality certificate which only hold for high CP lengths. More exhaustive results follow in Section 5.

4. CP-LENGTH OPTIMIZATION HEURISTIC

The goal in this section is to reduce the computational complexity in evaluating the sum-rate for every CP length $\mu, 0 \le \mu < \nu$, ν denoting the channel impulse response length. A heuristic metric proposed in [3] is based on the root-mean-square delay spread for a specific channel realization and a multiplicative factor derived from channel statistics. Differently, we propose an iterative branch-and-bound (BnB) search for scenario-specific CP-length selection that is optimal whenever the following assumption holds:

Assumption 1 *The total bit-load $R(\mu)$ under a specific bit-loading scheme that determines the transmit power* $\mathbf{p}$ *is non-decreasing with*

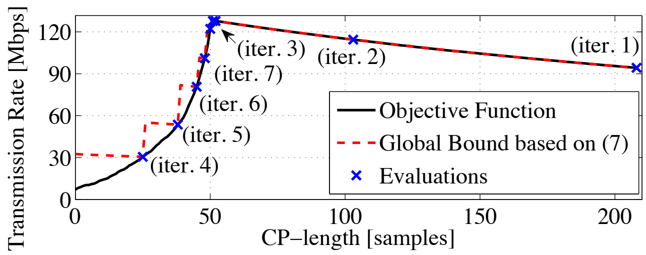

Fig. 2. Example demonstrating the BnB heuristic for CP-length optimization under constant power-loading (iterations 1 to 7 are labeled).

an increasing CP length μ, where

$$R(\mu) = \sum_{c \in \mathcal{C}} r_c^{(\mu)} \left(p_c, X_c^{(\mu)}(\mathbf{p}^{-c}) \right). \tag{6}$$

Having evaluated the bit-load $R(\bar{\mu})$ for a specific CP length $\bar{\mu}$, from Assumption 1 it follows that a performance bound is obtained as

$$f^{\bar{\mu}}(\mu) = \eta(\mu) R(\bar{\mu}), \quad 0 \leq \mu \leq \bar{\mu}, \tag{7}$$

where the restricted domain follows from our assumption. This assumption was seen to hold in over 99 % of the tested CP-length instances according to the channel realizations and simulation setup of Section 5, which motivates the following low-complexity heuristic.

The proposed BnB scheme is illustrated in Figure 2 by an example (a realization of a class-5 power line channel [26]) using the simulation setup of Section 5. It starts by evaluating $\bar{\mu} = \nu - 1$, the objective value $\eta(\bar{\mu})R(\bar{\mu})$, and the corresponding upper-bound $f^{\bar{\mu}}(\mu)$ over the whole search space $\{0, 1, \ldots, \nu - 1\}$. Excluding the evaluated $\bar{\mu}$ from the search space we obtain an open search-list. In each iteration we pick the median over this search-list as our next evaluation point. Besides excluding the already visited values of the CP length from the search-list, at every iteration we also exclude values for which the lowest found upper-bound based on (7) is below or equal to the best objective value found so far. For the given example in Figure 2 we find the same optimal CP length after only 9 CP-length evaluations out of the $\nu = 209$ possibilities. More results on the proposed BnB heuristic are presented in the following section.

5. SIMULATION RESULTS

In this section we compare our three performance bounds of Section 3 to four bit-loading heuristics: the constant power-loading at spectral mask under discrete bit-loading, heuristic A of Section 3.3, and heuristics B and C proposed in [3]. Heuristic B assumes on-off power-loading and variable bit-loading per subcarrier. Heuristic C assumes on-off power-loading as well, but loads the same number of bits on all subcarriers. The simulation setup consists of the ICI model and OFDM parameters used in [3] according to an HPAV transceiver, and the stochastic multipath channel model in [26] with log-normal path-gain-magnitudes.[2] The considered channel classes 1 and 5 [26] represent highly attenuated (mean path-loss of 46.5 dB) and moderately attenuated (mean path-loss of 19.5 dB) PLC channels, respectively, where we provide the results for the latter in bracket. The stated intervals correspond to a 99% confidence according to a student-t test, where over 70 realizations were considered per channel class.

The GP approach provides in only 59±1 % (80±1 %) of the CP-length scenarios a valid performance upper-bound. The proposed optimality certificate applied to heuristic A holds in only 50 ± 1 % (27 ± 7 %) of the CP-length scenarios. Moreover, for those CP-length values where the GP bound is valid, it is on average 78 % (15 %) weaker than the LP bound, cf. Figure 1. Altogether this leads us to the conclusion that the LP bound is indeed outperforming the two previously proposed techniques. Comparison to the LP bound reveals that the suboptimality under heuristic A is (under a CP length optimally chosen for the respective power-loading scheme) on average provably lower than 0.2 ± 0.1% (0.6 ± 0.1 %), while heuristic C (constant bit-loading over subcarriers) provides on average 26.2 ± 2.9 % (26.1 ± 1.3 %) lower rates than heuristic A. The study in [11] shows gains by power-loading heuristics (not obeying any bit-loading restrictions) at low SNR. Differently, under our setup the loss by constant power-loading (on top of CP-length selection) is bounded by 0.5 ± 0.2 % (1.3 ± 0.3 %). Hence, the LP bound is sufficient for proving global near-optimality of constant power-loading under optimal CP-length selection.

Using the LP bound and heuristic A the range of possibly optimal CP lengths can be restricted to 3.6 ± 0.9 (5.1 ± 0.9) samples. The average difference in optimal CP-length μ to that under constant power-loading is 3.2, 1.5, 0.3 and 0.9 (2.0, 1.2, 0, and 1.5) samples under the LP bound and heuristics A,B, and C, respectively. As intuitively expected, the optimal CP length under the LP bound was in our simulations always lower than or equal to that under constant power-loading. However, the performance drops below the optimal CP-length range (cf. Figure 1). Hence, the near-optimal constant power-loading appears to be a practical basis for CP-length selection. The average suboptimality over all tested channel realizations of the proposed BnB heuristic in connection with constant power-loading is below 0.01 % and maximum 0.3 %. The average reduction in CP-length amounts to 49 % (73 %), leading to an improvement in transmission rate by over 20 % (32 %), which confirms the reported achievable gains in [3]. The BnB search takes on average 8.8 (9.3) and maximum 19 CP-length evaluations. This corresponds to an average complexity reduction compared to an exhaustive search by over 95 %.

6. CONCLUSIONS

We have proposed and analyzed upper-bounds on the transmission rate for power and bit-loading in multicarrier communication under inter-carrier interference (ICI) due to an insufficient cyclic prefix (CP) length. Simulation results based on home-plug AV power-line channels suggest that the proposed linear programming based bound provides tight global suboptimality guarantees for low-complexity heuristic bit-loading schemes. In fact, constant power-loading has been found to be sufficient for CP-length selection and the CP length can on average be at least halved compared to the maximum assumed channel length. It has also been demonstrated that the complexity of global CP-length optimization is reducible by an order of magnitude through the proposed problem-specific branch-and-bound heuristic.

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