# DIFFUSION LMS LOCALIZATION AND TRACKING ALGORITHM FOR WIRELESS CELLULAR NETWORKS

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**ABSTRACT**

We propose a distributed least-mean squares (LMS) procedure based on a diffusion strategy for localization and tracking of mobile terminals in cellular networks. In the proposed algorithm, collaborating base stations measure two sets of parameters, namely, the received signal strength (RSS) and the signal propagation time (SPT) to estimate mobile locations. The proposed algorithm has a simple operational structure, offers agile tracking performance and helps the network to save energy and radio resources by benefiting from its decentralized and adaptive signal processing features.

***Index Terms***— distributed localization, mobile tracking, diffusion adaptation, wireless cellular networks.

## 1 Introduction

There are already several useful techniques for the localization and tracking of moving objects over networks. From the signal processing point of view, these techniques are classified into centralized and decentralized algorithms. The centralized approaches assume the existence of a central processing unit (fusion center) that is responsible for all the processing tasks. Examples of centralized approaches are the least squares localization algorithms [1], maximum likelihood estimators [2]; multidimensional scaling algorithms [3]; positioning with imprecise and noisy distance information [4, 5]; and the other centralized techniques introduced in [6–8]. There are also cooperative centralized localization algorithms where the additional distance-related data by assisting nodes (nodes with known locations) are sent to fusion center to enhance the accuracy of the estimation [9–12]. For example, in cooperative centralized localization in cellular networks, mobile terminals with known locations can act as assisting nodes and transmit additional RSS measurements to the fusion center to locate the remaining mobile users more accurately [11].

Centralized algorithms perform well in small network environments where the energy and communication bandwidth are not scarce resources. In large size networks, the multi-hop message passing transmission schemes used by centralized techniques may create a communication bottleneck and incur high communication cost. In addition, centralized localization algorithms are unscalable, and susceptible to failure if their only single-point central processor breaks down. Therefore, distributed algorithms are more desirable in large-size networks. There are currently several works on distributed localization with each solution method customized to address a specific problem in particular network configurations and environment conditions. Among these methods, we can mention distributed iterative gradient algorithms [13], minimum description length algorithms [14], and particle filtering [15]. These decentralized solutions are categorized as non-adaptive and their tracking abilities tend to be limited.

In this paper, we propose a diffusion LMS algorithm for mobile localization that utilizes a distributed mechanism to process the data, and uses a hybrid of RSS and SPT measurements to increase the robustness and the accuracy of the localization. The proposed algorithm has a simple structure and requires low computational resources. Therefore, it is an attractive solution for applications in networked systems where nodes have limited processing power and where the network radio resources are scarce. More importantly, the algorithm operates in an adaptive manner and has an agile tracking ability which makes it particularly attractive in random and time-varying environments.

*Notation:* We use boldface letters to represent random variables, and normal font to represent deterministic quantities. For complex vectors and matrices, $(\cdot)^*$ denotes complex conjugate transposition. $I_M$ denotes the identity matrix of order $M$, and $\mathbb{E}[\cdot]$ is the expectation operator.

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## 2 Problem Statement

We consider a wireless cellular network over a coverage area that is divided into regular hexagonal cells, where at the center of each cell there is a radio tower equipped with three sectorized base stations. Each of these base station (referred to as a node in this work) uses one directional antennas to serve users within a sector of 120 degrees in azimuth angle. The sectorized base stations act as anchors with known locations and they measure two sets of signal parameters, namely, the RSS and the SPT to locate mobile terminals. The mobile terminals use omnidirectional antennas so their transmit signals propagate in all directions.

We use path loss data model to relate mobile locations with the received signal power, and presume that the base stations and mobiles are synchronized such that the SPT can be measured using a time of arrival (TOA) mechanism with low error. The synchronization assumption, however, can be relaxed if we employ a time difference of arrival (TDOA) technique to measure SPT [7].

Let $g_k(\theta)$ denote the gain of directional antenna of node $k$ at angle $\theta$. For a directional antenna with 3dB beamwidth denoted by $\theta_{3dB}$, this gain can be well approximated by [16]:

$$g_k(\theta) = \max\left\{10\log\left(\frac{2\pi}{\theta_{3dB}}\right) - 12\left(\frac{\theta-\theta_p}{\theta_{3dB}}\right)^2, g_{\min}\right\} \quad (1)$$

where $-\pi \leq \theta \leq \pi$, $\theta_p$ is the pointing angle of the antenna, and $g_{\min}$ is the minimum antenna gain. We use $\boldsymbol{p}_k(i)$ to denote the RSS of the mobile terminal in dB at node $k$ and time instant $i$. The RSS of the mobile is related to its distance from node $k$ by the following path loss model [17, page 82]:

$$\boldsymbol{p}_k(i) = P_t + g_k\big(\theta_u(i)\big) - 10\log\left(\frac{\boldsymbol{d}_k(i)}{d_o}\right)^{\alpha} + \boldsymbol{l}_k(i) + \boldsymbol{n}_k^{(p)}(i) \quad (2)$$

where $P_t$ is the transmit power of the mobile and $g_k\big(\theta_u(i)\big)$ is the antenna gain in the direction of the mobile terminal $\theta_u$ at time instant $i$, $d_o$ is the antenna far-field distance, and $\boldsymbol{d}_k(i)$ is the distance between the mobile and node $k$ at time instant $i$. In the above expression, $\alpha$ is the path loss exponent, $\boldsymbol{l}_k(i)$ is the loss caused by obstructions between the mobile and the sectorized base station due to non-line-of-sight (NLOS) and $\boldsymbol{n}_k^{(p)}(i)$ is a zero mean Gaussian variable that represents the loss caused by shadowing. The time varying Euclidean distance between node $k$ and the mobile terminal is given by $\boldsymbol{d}_k(i) = \|\boldsymbol{w}_i^{\mathrm{o}} - s_k\|$, where $s_k$ is the known location of node $k$ in two dimensional space and $\boldsymbol{w}_i^{\mathrm{o}}$ is the location of the mobile user at time instant $i$.

In addition to RSS measurements, each node records the SPT, $\boldsymbol{t}_k(i)$, which is the signal prorogation time from the mobile to node $k$. If we denote the speed of light by $c$, $\boldsymbol{t}_k(i)$ can be expressed as [7]:

$$\boldsymbol{t}_k(i) = \frac{\boldsymbol{d}_k(i)}{c} + \boldsymbol{b}_k(i) + \boldsymbol{n}_k^{(t)}(i) \quad (3)$$

where $\boldsymbol{b}_k(i)$ is a random error with exponential distribution caused by NLOS, and $\boldsymbol{n}_k^{(t)}(i)$ is zero mean measurement noise. To model the mobile motion over time, we consider the following nonlinear equation:

$$\boldsymbol{w}_i^{\mathrm{o}} = \boldsymbol{w}_{i-1}^{\mathrm{o}} + v\big[\cos(\boldsymbol{\phi}(i)) \ \sin(\boldsymbol{\phi}(i))\big]^T \Delta T \quad (4)$$

where $v$ denotes the mobile speed, $\boldsymbol{\phi}(i)$ represents the mobile direction at time $i$ and $\Delta T$ is the sampling time. The mobility function (4) is, in fact, the Gauss–Markov motion model with constant velocity [17]. The time-varying mobile direction, $\boldsymbol{\phi}(i)$, is random and changes according to:

$$\boldsymbol{\phi}(i) = \beta\boldsymbol{\phi}(i-1) + (1-\beta)\bar{\boldsymbol{\phi}} + 2\pi\sqrt{1-\beta^2}\,\boldsymbol{u}(i) \quad (5)$$

where $\bar{\boldsymbol{\phi}}$ is the average direction angle and $\boldsymbol{u}(i)$ is a zero mean random Gaussian variable with variance $\sigma_u^2$.

In this paper, the objective is to develop a distributed adaptive algorithm to estimate mobile trajectory $\boldsymbol{w}_i^o$ for $i \in \{0, 1, \cdots\}$, given noisy measurements $\{\boldsymbol{p}_k(i), \boldsymbol{t}_k(i)\}$. To maintain simplicity in the derivation of the algorithm, we omit the index $i$ from $\boldsymbol{w}_i^o$ and work with $w^o$ instead. Let us first assume there exists a fusion center where the measurements by $N$ sectorized base stations (nodes) are sent to for localization. Then, $w^o$ can be found by minimizing the following hybrid global cost function over $w$:

$$J^{\mathrm{ctrl}}(w) = \sum_{k=1}^{N}\Big((1-\eta)\,J_k^{(p)}(w) + \eta\,\nu\,J_k^{(t)}(w)\Big) \quad (6)$$

where $J_k^{(p)}(w)$ and $J_k^{(t)}(w)$ are the local costs associated with node $k$ and related to RSS and time interval measurements, respectively. The variable $\eta \in [0,1]$ specifies the amount of the participation of RSS and SPT measurements in locating the mobile terminal. Parameter $\nu$ magnifies $J_k^{(t)}(w)$ to be approximately in the same numerical range as $J_k^{(p)}(w)$. The local cost functions are defined as:

$$J_k^{(p)}(w) = \mathbb{E}\big|\boldsymbol{p}_k(i) + 10\alpha\log\|w - s_k\| - h_k(i)\big|^2 \quad (7)$$

$$J_k^{(t)}(w) = \mathbb{E}\big|\boldsymbol{t}_k(i) - \|w - s_k\|/c\big|^2 \quad (8)$$

where $h_k(i) = P_t + g_k(\theta_u(i)) + 10\alpha\log(d_o)$. Next, we elaborate on how the global cost (6) can be optimized over the network in a distributed and adaptive manner.

## 3 Adaptive Mobile Localization

In this section, we first sketch the development of a centralized LMS algorithm and then present the proposed distributed solution. We use the centralized LMS algorithm as a benchmark in our computer experiments in Section 4.

### 3.1 Centralized LMS Solution

The gradient vector of the global objective function (6) can be expressed as:

$$\nabla_w J^{\text{ctrl}}(w) = \sum_{k=1}^{N} \Big( (1-\eta)\nabla_w J_k^{(p)} + \eta\,\nu \nabla_w J_k^{(t)} \Big) \quad (9)$$

where
$$\nabla_w J_k^{(p)} = \frac{20\alpha}{\ln 10}\,\mathbb{E}\Big\{ \frac{w - s_k}{\|w - s_k\|^2} \boldsymbol{e}_k^{(p)}(i) \Big\} \quad (10)$$

$$\nabla_w J_k^{(t)} = -\frac{2}{c}\,\mathbb{E}\Big\{ \frac{w - s_k}{\|w - s_k\|} \boldsymbol{e}_k^{(t)}(i) \Big\} \quad (11)$$

with the error functions:

$$\boldsymbol{e}_k^{(p)}(i) = \boldsymbol{p}_k(i) + 10\alpha \log \|w - s_k\| - h_k(i) \quad (12)$$

$$\boldsymbol{e}_k^{(t)}(i) = \boldsymbol{t}_k(i) - \|w - s_k\|/c \quad (13)$$

For minimization of (6), the centralized steepest descent algorithm takes the form:

$$\boldsymbol{w}_i = \boldsymbol{w}_{i-1} - \mu \nabla_w J^{\text{ctrl}}(\boldsymbol{w}_{i-1}) \quad (14)$$

where parameter $\mu > 0$ is the step size, and $\boldsymbol{w}_i$ is the estimate of the mobile location at iteration $i$. This iterative approach may show an abrupt behavior when the mobile terminal gets very close to a sectorized base station. This is because under these circumstances the gradients (10) and (11) become very large. This problem can be alleviated by multiplying $\nabla_w J_k^{(p)}$ by $\|w - s_k\|^2 \ln 10/20$ and scaling $\nabla_w J_k^{(t)}$ with $c\|w - s_k\|/2$. Doing so and approximating the gradient (9) with the instantaneous data at time $i$, we arrive at the following centralized LMS solution for mobile localization.

**Algorithm 1** : Centralized LMS for mobile localization

$$\widehat{\nabla}_w J_k(\boldsymbol{w}_{i-1}) = \big[\alpha(1-\eta)\boldsymbol{e}_k^{(p)}(i) - \nu\,\eta\ \boldsymbol{e}_k^{(t)}(i)\big](\boldsymbol{w}_{i-1} - s_k)$$

$$\boldsymbol{w}_i = \boldsymbol{w}_{i-1} - \mu \sum_{k=1}^{N} \widehat{\nabla}_w J_k(\boldsymbol{w}_{i-1})$$

In this algorithm, the errors $\boldsymbol{e}_k^{(p)}(i)$ and $\boldsymbol{e}_k^{(t)}(i)$ are evaluated using (12) and (13) at $w = \boldsymbol{w}_{i-1}$.

### 3.2 Diffusion LMS Solution

In the proposed algorithm, two nodes are said to be neighbors if their distance is less than a threshold $r^o$. A nominal value for the threshold is $r^o = 2r$ where $r$ is the radius of each hexagon in the network. There are different distributed optimization techniques that can be applied on (6) to find $w^o$. One possible strategy is the alternating directions method of multipliers [18, 19] in which the global cost (6) is decoupled and written as a group of local constrained optimization problems. In this method, the constraints force nodes to align their estimates with that of their neighbors. Therefore, nodes may not be able to respond quickly to the data without being critically constrained by agreement with their neighbor. A technique that does not suffer from such difficulty and endows networks with adaptation and learning abilities in real-time is the diffusion strategy [20–23]. In this technique, minimizing the global cost (6) can be pursued by solving the following unconstrained local optimization problems for $k \in \{1, \cdots, N\}$:

$$\min_w \Big\{ \sum_{\ell \in \mathcal{N}_k} c_{\ell,k}\big[(1-\eta)\,J_k^{(p)}(w) + \eta\,\nu\,J_k^{(t)}(w)\big] + \sum_{\ell \in \mathcal{N}_k \setminus \{k\}} p_{\ell,k}\|w - \psi_\ell\|^2 \Big\} \quad (15)$$

where $\mathcal{N}_k$ denotes the set of neighbors of node $k$, including $k$ itself, $\psi_\ell$ is a local variable that represents the global parameter at node $\ell$ and $\mathcal{N}_k \setminus \{k\}$ denotes the set $\mathcal{N}_k$ excluding node $k$. In this formulation, $\{p_{\ell,k}\}$ are nonnegative parameters and the scalars $\{c_{\ell,k}\}$ denote nonnegative entries of a right-stochastic matrix $C$ satisfying, $c_{\ell,k} = 0$ if $\ell \notin \mathcal{N}_k$ and $\sum_{k=1}^{N} c_{\ell,k} = 1$. Following the arguments in [20–22] and performing a similar normalization as in Algorithm 1, we arrive at the following normalized diffusion LMS algorithm for minimizing (6) in a distributed and adaptive manner.

**Algorithm 2** : Diffusion LMS for mobile localization

$$\boldsymbol{\psi}_{k,i} = \boldsymbol{w}_{k,i-1} - \mu_k \sum_{\ell \in \mathcal{N}_k} c_{\ell,k} \widehat{\nabla}_w J_\ell(\boldsymbol{w}_{k,i-1})$$

$$\boldsymbol{w}_{k,i} = \sum_{\ell \in \mathcal{N}_k} a_{\ell,k} \boldsymbol{\psi}_{\ell,i}$$

In this algorithm, $\mu_k > 0$ is the step-size at node $k$, $\{\boldsymbol{w}_{k,i}, \boldsymbol{\psi}_{k,i}\}$ are intermediate estimates of $w^o$, and
$\widehat{\nabla}_w J_\ell(\boldsymbol{w}_{k,i-1}) = \big[\alpha(1-\eta)\boldsymbol{e}_\ell^{(p)}(i) - \nu\,\eta\ \boldsymbol{e}_\ell^{(t)}(i)\big](\boldsymbol{w}_{k,i-1} - s_\ell)$
where $\boldsymbol{e}_\ell^{(p)}(i)$ and $\boldsymbol{e}_\ell^{(t)}(i)$ are evaluated at $\boldsymbol{w}_{k,i-1}$. Moreover, the parameters $\{a_{\ell,k}\}$ are nonnegative entries of a left-stochastic matrix $A \in \mathbb{R}^{N\times N}$ that satisfy $a_{\ell,k} = 0$ if $\ell \notin \mathcal{N}_k$ and $\sum_{\ell \in \mathcal{N}_k} a_{\ell,k} = 1$. We note that the coefficients $\{p_{\ell,k}\}$ in (15) are now replaced by the entries of matrix $A$. In Algorithm 2, the first expression is an adaptation step where base station $k$ updates its intermediate estimate $\boldsymbol{w}_{k,i-1}$ to $\boldsymbol{\psi}_{k,i}$ using measured data $\{\boldsymbol{p}_\ell(i), \boldsymbol{t}_\ell(i)\}_{\ell \in \mathcal{N}_k}$. The second expression is a combination step, in which base station $k$ combines its intermediate estimate $\boldsymbol{\psi}_{k,i}$ with those of its neighbors to obtain $\boldsymbol{w}_{k,i}$.

## 4 Simulation Results

In our computer experiments, we illustrate the performance of the proposed algorithm in tracking a mobile terminal in a two tier cellular network with 19 radio tower, $N = 57$ sectorized base stations, and $r = 2$km. In these experiments, the radiation pattern of the directional antenna can be described using (1) with parameters $\theta_{3\text{dB}} = 70$ degrees and $g_{\min} = -20$dB. The RSS

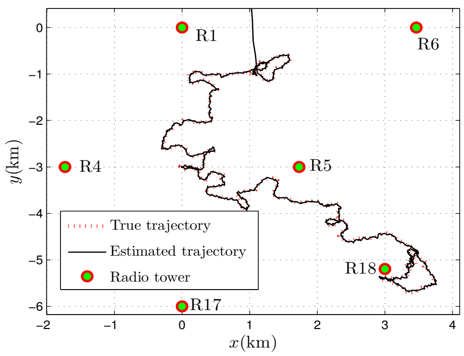


**Fig. 1**: The true and estimated mobile trajectory.

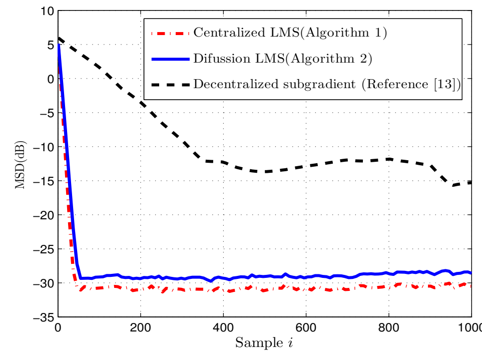


**Fig. 2**: MSD learning curve.

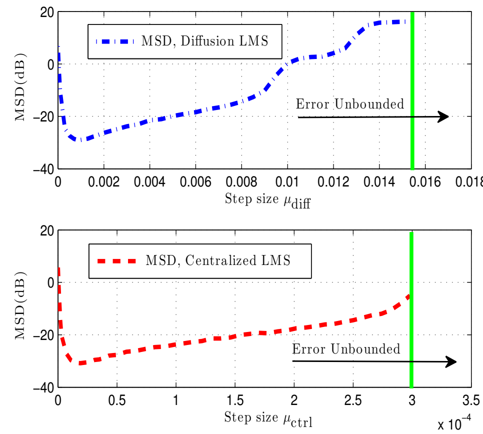


**Fig. 3**: MSD versus step-size.

parameters are chosen as $\alpha = 2.5$, $P_t = 23$dBm, and $d_o = 50m$. The communication between mobile and sectorized base stations takes place under a line-of-sight (LOS) condition where $\boldsymbol{l}_k(i)$ and $\boldsymbol{b}_k(i)$ are zero. The noise terms $\boldsymbol{n}_k^{(p)}(i)$ and $\boldsymbol{n}_k^{(t)}(i)$ are zero mean white, i.i.d over time and independent over space. The RSS and SPT noise variances are chosen uniformly from $[0.05, 3]$ and $[0, 3] \times 10^{-12}$, respectively.

The objective of the network is to cooperatively estimate the mobile trajectory, $\boldsymbol{w}_i^o$, which follows the motion model (4) with $\boldsymbol{w}_{-1}^o = [-r/2,\, r/2]^T$, $\Delta T = 0.1$s and $v = 20$m/s. The mobile time-varying direction follows (5) with parameters $\beta = 0.99$, $\bar{\boldsymbol{\phi}} = -\pi/2$. For the diffusion algorithm, we choose $\eta = 0.5$ and $\nu = 40c$, and consider an equal step-size $\mu_{\text{diff}} \triangleq \mu_k = 1 \times 10^{-3}$ and $\boldsymbol{w}_{k,-1} = 0$ for all $k$. We use the relative degree criterion [20] to compute $A$ and choose $C$ as the identity matrix. In the centralized set-up, the nodes transmit their data, $\{\boldsymbol{p}_k(i), \boldsymbol{t}_k(i)\}$, to a fusion center located at $(x,\, y) = (0,\, 0)$. The step-size in the centralized LMS algorithm is chosen as $\mu = \mu_{\text{diff}}/N$ to ensure the same convergence rate as that of diffusion LMS. To evaluate the estimation error, we define the mean-square deviation (MSD) performance measure of diffusion and centralized LMS algorithm, respectively, as $\eta_{\text{diff}}(i) = \frac{1}{N}\sum_{k=1}^{N} \mathbb{E}\|\tilde{\boldsymbol{w}}_{k,i}\|^2$ and $\eta_{\text{ctrl}}(i) = \mathbb{E}\|\tilde{\boldsymbol{w}}_i\|^2$, where $\tilde{\boldsymbol{w}}_{k,i} = \boldsymbol{w}_i^o - \boldsymbol{w}_{k,i}$ and $\tilde{\boldsymbol{w}}_i = \boldsymbol{w}_i^o - \boldsymbol{w}_i$.

Fig. 1 shows the true and estimated mobile trajectory by diffusion LMS over 1000 seconds. As it is seen, the proposed diffusion LMS tracks well. Fig. 2 illustrates the convergence of Algorithm 1 and 2 in terms of MSD. These results are drawn from the average of 400 independent runs. In this figure, we also compare the MSD performance of the proposed methods with that of the decentralized subgradient algorithm introduced in [13]. We observe that the proposed diffusion algorithm outperforms decentralized subgradient solution. Moreover, the performance discrepancy between the proposed diffusion LMS and the centralized LMS is not significant.

Fig. 3 shows the network MSD performance in steady-state against the step-size $\mu_{\text{diff}}$ and $\mu_{\text{ctrl}}$. As this figure indicates, the MSD curves reach their minimum at $\mu_{\text{diff}} = 1 \times 10^{-3}$ and $\mu_{\text{ctrl}} = 1.75 \times 10^{-5}$ which are the optimal step-size values. For both algorithms, as the chosen step-sizes move away from their optimal points in either directions, the MSD increases. The MSD of the both algorithms grow unbounded if the step-sizes become larger and pass the solid vertical (green) line. From this figure, it can be observed that the diffusion algorithm can achieve the same MSD performance results as that of the centralized LMS if the step-sizes are chosen appropriately.

## 5 Conclusion

We developed a diffusion LMS algorithm to locate and track mobile terminals in cellular networks. The algorithm uses a hybrid of RSS and SPT measurements to update its estimate at each sampling time. Computer experiments showed that the proposed algorithm has powerful tracking ability and can achieve similar performance as that of the centralized LMS.

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