A COALITIONAL GAME FOR DISTRIBUTED ESTIMATION IN WIRELESS SENSOR NETWORKS

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ABSTRACT

We consider a collaborative estimation problem using dependent observations in a wireless sensor network, where each sensor aims to maximize its estimation performance in terms of Fisher information (FI) by forming coalitions with other sensors and collaborating within a coalition. The energy consumed by the sensors increases with the size of the coalition and hence we prove that grand coalition will not form. We investigate the formation of non-overlapping coalitions such that each sensor's performance is maximized under a specific energy constraint. We decouple marginal and dependent components of FI obtained from the joint distribution by using copula theory. We introduce the concept of diversity gain and redundancy loss and demonstrate how a copula based formulation allows us to characterize these concepts. Distributed estimation problem is formulated as a coalitional game. A merge-and-split algorithm is used for finding an optimal partition. Stability of the proposed algorithm for this game is discussed. Finally, numerical results are discussed.

Index Terms— Distributed estimation, Fisher information, Coalitional game, Dependent observations, Copula theory

1. INTRODUCTION

In a distributed estimation problem, each sensor collects observations regarding a parameter of interest, then shares them with other sensors or transmits to the fusion center (FC). To reduce the energy cost for communication, the observations may be quantized before transmission. The distributed nature of wireless sensor networks indicates a tradeoff between minimizing the communication cost and maintaining acceptable performance levels. Although there has been a lot of work on distributed estimation with conditionally independent observations (see e.g. [1] and references therein), much less has been done for the dependent observations case. Parameter estimation with dependent observations in a variety of communication scenarios was considered in [2], but was limited to the case of "geometric" dependent Gaussian noise.

Dependence among observations may make some sensors' measurements redundant at the FC. An extreme case is that when two sensors' observations are identically distributed and highly positively correlated, the second sensor will contribute little to the overall performance, and will become "redundant". Since transmitting "redundant" observations from battery powered sensors is energy inefficient, we have an opportunity to conserve energy via collaboration.

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We formulate a novel distributed estimation framework where each individual sensor is capable of *sensing* and *estimating* and there is no FC. Sensors form coalitions and collaborate within a coalition by sharing their observations, such that certain estimation performance is maintained and energy efficiency is increased. In our framework, each sensor aims at maximizing its own performance, and thus that of the coalition to which it belongs, through cooperation. The "redundant" observations in the parallel framework, will be fully exploited in our framework.

Since both, estimation performance and energy cost, are increasing functions of the coalition size, there is a tradeoff between the performance and energy efficiency. Thus, the problem is to find a set of non-overlapping coalitions to maximize each coalition's estimation performance under certain energy efficiency constraints. We employ a game theory based approach and formulate our collaborative distributed estimation problem as a coalition formation game. In our framework, each sensor is characterized not only by its individual estimation performance, which is achieved with its own observation, but also by its dependence with other sensors. We use *copula theory* to model and analyze the dependence among sensor observations.

1.1. Related work

The effect of dependent noise and hence dependent observations on FI is studied by Yoon and Sompolinsky in [3]. The authors show that, in the biologically relevant regime of parameters, positive correlations decrease the estimation performance compared with uncorrelated population. Sundaresan et al. [4] consider location estimation of a random signal source where they focus on improving system performance by exploiting the spatial dependence of sensor observations. Copula based approaches for centralized and decentralized detection using dependent observations have been considered by Iyengar et al. [5] and Sundaresan et al. [4, 6].

Several authors have used game theory for statistical inference; such as measurement allocation [7], communication [8], and spectrum sensing [9].

1.2. Preliminaries

In this paper, dependence is characterized using a copula based approach, copula theory allows one to construct a valid joint distribution from a variety of marginal distributions. According to Sklar's Theorem [10], for continuous distributions, the joint probability density function (pdf) is expressed as

$$f(x_1, \dots, x_m) = \left(\prod_{i=1}^{m} f_i(x_i) \right) c(F_1(x_1), \dots, F_m(x_m)|\mathbf{\Phi}) \quad (1)$$

where c is termed as the copula density and $\mathbf{\Phi}$ is the copula parameter which captures the dependence. Several copula functions are defined in the literature, and are constructed to characterize different types of dependence [10].

To facilitate the formulation of our problem, we introduce basic concepts in coalitional game theory. Let $\mathcal{N} = \{1, 2, \ldots, N\}$ be a set of fixed players called the *grand coaltion*. Nonempty subsets of $\mathcal{N}$ are called *coalitions*. A *collection* is any family $\mathcal{S} := \{S_1, \ldots, S_m\}$ of mutually disjoint coalitions. If additionally $\cup_{j=1}^{m} S_j = \mathcal{N}$, the collection $\mathcal{S}$ is called a *partition* of $\mathcal{N}$.

Assuming a comparison relation $\triangleright$, $\mathcal{R} = \{R_1, \ldots, R_k\} \triangleright \mathcal{S} = \{S_1, \ldots, S_m\}$ means that the way $\mathcal{R}$ partitions A, where $A = \cup_{i=1}^{k} R_i = \cup_{j=1}^{m} S_j$, is preferred over the way $\mathcal{S}$ partitions A. For a non-transferable $(\mathcal{N}, v)$ coalitional game, Pareto order can be used as a comparison relation $\triangleright$. For a collection $\mathcal{R} = \{R_1, \ldots, R_k\}$, the utility of a player j in a coalition $R_j \in \mathcal{R}$ is denoted by $\phi_j(\mathcal{R})$, and the Pareto order is defined as follows

$$\mathcal{R} \triangleright \mathcal{S} \iff \{\phi_j(\mathcal{R}) \geq \phi_j(\mathcal{S}), \forall j \in \mathcal{R}, \mathcal{S}\} \tag{2}$$

with at least one strict inequality for a player k.

Apt and Witzel [11] proposed an abstract approach to coalition formation that focuses on simple merge-and-split rules transforming partitions of a group of players. Details of coalition formation will be introduced in detail in Section 3. In this paper, we formulate the energy constrained collaborative estimation problem as a coalitional game. We use copula theory to characterize inter-sensor dependence. By introducing the concept of diversity gain and redundancy loss, we provide some insights on the effect of dependent observations on our coalitional game. A merge-and-split algorithm is proposed and its stability is discussed.

2. PROBLEM FORMULATION

We consider a phenomenon being observed by a set of N sensors denoted as $\mathcal{N} = \{1, 2, \ldots, N\}$. Each sensor's observation is $X_i = \theta + n_i, \forall i = 1, \ldots, N$, where θ is deterministic but unknown and n_i is zero mean Gaussian distributed noise. The observation noise is not independent across sensors due to which $\mathbf{X} = [X_1, X_2, \ldots, X_N]$ is dependent. Let Σ denote the covariance matrix of the noise, which is also the covariance matrix of $\mathbf{X}$.

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_1\sigma_2\rho_{1,2} & \cdots & \sigma_1\sigma_N\rho_{1,N} \\ \sigma_2\sigma_1\rho_{2,1} & \sigma_2^2 & \cdots & \sigma_2\sigma_N\rho_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_N\sigma_1\rho_{N,1} & \sigma_N\sigma_2\rho_{N,2} & \cdots & \sigma_N^2 \end{pmatrix}$$

where $\rho_{i,j} \in (-1, 1), \forall i \neq j$, is the correlation coefficient between n_i and n_j which can be determined from the distance between the two sensors [12]. Each sensor estimates θ using maximum likelihood estimation (MLE) based on its own measurements and those from collaborating sensors in the same coalition S, i.e., $\hat{\theta} = \max_\theta f_{\mathbf{X}_S}(\mathbf{x}_S; \theta)$, where $f_{\mathbf{X}_S}$ is the joint pdf of $\mathbf{X}_S = [X_1, \ldots, X_{|S|}], X_i \in S$. There are certain restrictions on the sensors in a coalition: (1) Each sensor can only join one coalition. (2) Once in a coalition, a sensor can request other sensors in the same coalition for their observations, and it has to transmit its observation upon request from other sensors.

However, each sensor's energy is finite and a communication cost is incurred when it transmits. Let z be the number of requests initiated by each sensor in the network within a certain time period T. Then, for a sensor in coalition S, it is requested to transmit $z(|S| - 1)$ times in T, where $|S|$ is the cardinality of coalition S. And its energy consumption per unit time is $E_S = z(|S| - 1)r/T$, where r is the energy consumption per single transmission per sensor. In order to guarantee sensor's lifetime, in designing the system, we put an energy consumption constraint, $E_S \leq \alpha$, on each sensor.

According to Cramer-Rao Lower Bound (CRLB), var $\left(\hat{\theta}(\mathbf{X})\right) \geq I(\theta)^{-1}$ where $I(\theta) = -E(\partial^2 \log(f(\mathbf{x}; \theta))/\partial\theta^2)$, is the FI corresponding to random vector $\mathbf{X}$. We use FI as the criterion of sensor estimation performance, since for the Gaussian case, CRLB is attainable using MLE. For each sensor in the coalition S, the performance it can achieve in terms of FI is

$$I_S(\theta) = -E(\frac{\partial^2 \log(f_{\mathbf{X}_S}(\mathbf{x}_S; \theta))}{\partial\theta^2}) = \mathbf{1}^T \Sigma_S^{-1} \mathbf{1} \tag{3}$$

where Σ_S is the covariance matrix corresponding to $\mathbf{X}_S$.

Proposition 1. *$I_S(\theta)$ is a nondecreasing function of the cardinality of S.*

Proof. We need to show that $I_S(\theta) \geq I_{S'}(\theta)$, for $S^{'} \subseteq S$.

$$\begin{aligned} I_S(\theta) &= -E\left[\frac{\partial^2}{\partial\theta^2} \log f_{\mathbf{X}_S}(\mathbf{x}_S; \theta)\right] \\ &= -E\left[\frac{\partial^2}{\partial\theta^2} \log f_{\mathbf{X}_{S'}}(\mathbf{x}_{S'}; \theta)\right] + \\ &\quad E_{S'}\left[-E_{S\setminus S'|S'}[\frac{\partial^2}{\partial\theta^2} \log f(\mathbf{x}_{S\setminus S'}|\mathbf{x}_{S'}; \theta)]\right] \\ &= I_{S'}(\theta) + E_{S'}\left[I_{S\setminus S'|S'}(\theta)\right] \\ &\geq I_{S'}(\theta) \end{aligned} \tag{4}$$

where $S \setminus S^{'} = \{i \in S | i \notin S^{'}\}$. The last inequality is because of the non-negativity of conditional FI. □

Remark 1. *A grand coalition forms when $r = 0$. This implies that all the sensors in the network will collaborate with each other. Proposition 1 implies that FI will not decrease by including more sensors in a coalition. Thus, if there is no communication cost, all the sensors will collaborate for a better estimation performance.*

It is clear from Proposition 1 and the definition of $E_S = z(|S| - 1)r/T$ that, as $|S|$ increases, both $I_S(\theta)$ and E_S increase. There is a tradeoff between the estimation performance and energy consumption. Each sensor selfishly aims at maximizing its estimation performance, i.e., FI contained in coalition S to which it belongs, subject to an energy constraint. Let S be a coalition in partition $\mathcal{S}$ and let $\mathcal{P}$ be the set of all possible partitions. The problem is

$$\max_{\mathcal{S} \in \mathcal{P}} I_{S(i \in S)}(\theta) \;\text{ subject to } E_S \leq \alpha, \; \forall i \in \mathcal{N} \tag{5}$$

where $I_{S(i\in S)}(\theta)$ represents the FI of sensor i when it is in coalition S. Eq. (5) describes a multi-objective optimization problem in which coupling exists among sensors. If each sensor solves its optimization problem iteratively by itself, the overall system algorithm may not converge. An exhaustive approach in which we search over all possible partitions will render us a computational complexity of $N!/(\beta! M! M)$, where $\beta = (\alpha T/zr) + 1, M = N/\beta$ (assuming $\beta, M \in \mathbb{Z}$), even when the observations are independent. When N is large and dependent observations come into play, the problem become intractable. So, we use a game theoretical approach. The difficulty of this problem partially comes from dependence among observations which will be explained in detail in the next section.

3. COLLABORATIVE DISTRIBUTED ESTIMATION

3.1. Diversity gain & Redundancy loss

To analyze the effect of inter-sensor dependence on the FI in coalition S, we express the joint pdf of random variables in coalition S in terms of the marginal pdf and copula density function c_s, as in (1).

By copula theory, when $\log c_S(\cdot;\theta,\mathbf{\Phi})$ is twice differentiable with respect to θ, $I_S(\theta)$, the Fisher information contained in $\mathbf{X}_S$ can be written as

$$\begin{aligned} I_S(\theta) &= -E\left[\frac{\partial^2 \log\{\left(\prod_{i=1}^{|S|} f_i(x_i;\theta)\right) c_S(\cdot;\theta,\mathbf{\Phi})\}}{\partial\theta^2}\right] \\ &= \sum_{i\in S} I_i(\theta) - E\left[\frac{\partial^2 \log c_S(\cdot;\theta,\mathbf{\Phi})}{\partial\theta^2}\right] \\ &= \sum_{i\in S} I_i(\theta) + I_{c_S}(\theta) \end{aligned} \quad (6)$$

Thus, FI of a random vector can be written as the summation of FI corresponding to each random variable and I_{c_S}, the generalized Fisher information (GFI) of the copula density c_S. We name I_{c_S} the generalized FI because it may not satisfy the non-negativity of FI. Proposition 2 provides us some insights into the properties of the GFI of bivariate Gaussian pdf. A bivariate Gaussian pdf can be written as a product of Gaussian marginals and a Gaussian copula.

Proposition 2. *$[X,Y]^T$ is a bivariate Gaussian distributed vector, $[X,Y]^T \sim N(\boldsymbol{\mu},\Sigma_{XY})$, where $\boldsymbol{\mu} = [\mu_X(\theta),\mu_Y(\theta)]^T$*

$$\Sigma_{XY} = \begin{pmatrix} \sigma_X^2 & \sigma_X\sigma_Y\rho_{XY} \\ \sigma_Y\sigma_X\rho_{YX} & \sigma_Y^2 \end{pmatrix}$$

and θ is the parameter to be estimated (Without loss of generality, let $\left|\frac{\sigma_X}{\sigma_Y}\frac{\mu_Y'(\theta)}{\mu_X'(\theta)}\right| \le 1$, where the derivative is taken with respect to θ) then we have: (1) $I_{c_{XY}}$, the GFI of copula c_{XY}, is a convex function of ρ_{XY} and $\min_{\rho_{XY}} I_{c_{XY}} = -\frac{\mu_Y'^2(\theta)}{\sigma_Y^2}$ is reached at $\rho_{XY} = \frac{\sigma_X}{\sigma_Y}\frac{\mu_Y'(\theta)}{\mu_X'(\theta)}$; (2) $I_{c_{XY}} \le 0$ for ρ_{XY} between 0 and $\rho_B(\boldsymbol{\mu},\Sigma_{XY})$, where $\rho_B(\boldsymbol{\mu},\Sigma_{XY}) = \frac{2\mu_X'(\theta)\mu_Y'(\theta)\sigma_X\sigma_Y}{\mu_X'^2(\theta)\sigma_Y^2+\mu_Y'^2(\theta)\sigma_X^2}$.

Proof.

$$\begin{aligned} I_{c_{XY}} &= \frac{-1}{\sigma_X^2\sigma_Y^2(1-\rho_{XY}^2)}\{2\rho_{XY}\mu_X'(\theta)\mu_Y'(\theta)\sigma_X\sigma_Y \\ &\quad -\rho_{XY}^2(\mu_X'^2(\theta)\sigma_Y^2+\mu_Y'^2(\theta)\sigma_X^2)\} \end{aligned} \quad (7)$$

It can be shown that $\frac{\partial^2 I_{c_{XY}}}{\partial\rho_{XY}^2} \ge 0, \forall\rho_{XY} \in (-1,1)$. By setting $\frac{\partial I_{c_{XY}}}{\partial\rho_{XY}} = 0$, we get $\rho^* = \frac{\sigma_X}{\sigma_Y}\frac{\mu_Y'(\theta)}{\mu_X'(\theta)}$ and $I_{c_{XY}}^* = -\frac{\mu_Y'^2(\theta)}{\sigma_Y^2}$ when $\left|\frac{\sigma_X}{\sigma_Y}\frac{\mu_Y'(\theta)}{\mu_X'(\theta)}\right| \le 1$.

By setting (7) equal to zero, we get two solutions: $\rho_1 = 0$ and $\rho_2 = \frac{2\mu_X'(\theta)\mu_Y'(\theta)\sigma_X\sigma_Y}{\mu_X'^2(\theta)\sigma_Y^2+\mu_Y(\theta)'^2\sigma_X^2}$. Combined with the convexity of the function, it can be concluded that $I_{c_{XY}} \le 0$ when $\rho_{XY} \in [\min\{0,\rho_B(\boldsymbol{\mu},\Sigma_{XY})\},\max\{0,\rho_B(\boldsymbol{\mu},\Sigma_{XY})\}]$. Also note that when $\rho_{XY} = 0$, $I_{c_{XY}} = 0$, meaning that FI is solely the summation of individual FIs of X and Y; when $\rho_{XY} = \frac{\sigma_X}{\sigma_Y}\frac{\mu_Y'(\theta)}{\mu_X'(\theta)}$, $I_{c_{XY}}$ is just the smaller individual FI of the two with a minus sign. □

Remark 2. *For our problem formulation, a sensor i prefers to cooperate with sensor j with a positive $I_{c_{X_iX_j}}$ than sensor k with a negative $I_{c_{X_iX_k}}$, when sensor i and j have identical individual performance. This is because to sensor i, sensor j is more "valuable" than sensor k in the sense that some of sensor k's information is redundant for sensor i.*

Definition 1. *If $I_{c_{XY}} < 0$, we define -$I_{c_{XY}}$ to be pairwise redundancy loss denoted as $I_{rl_{XY}}$, otherwise we define $I_{c_{XY}}$ to be pairwise diversity gain denoted as $I_{dg_{XY}}$.*

The definitions of diversity gain and redundancy loss allow for a better characterization of the different roles pairwise inter-sensor dependence may play. General properties of the GFI of multivariate Gaussian copula may be analyzed using *vines* which is a graphical method of constructing multivariate copulas proposed by Kurowicka and Cooke [13]. The joint pdf of N random variables expressed in terms of a D-vine decomposition is given by:

$$\begin{aligned} &\prod_{i=1}^{N} f(x_i) \prod_{j=1}^{N-1}\prod_{k=1}^{N-j} c_{j,j+k|j+1,\ldots,j+k-1} \\ &(F(x_j|j+1,\ldots,j+k-1), F(x_{j+k}|j+1,\ldots,j+k-1)) \end{aligned} \quad (8)$$

Thus, a multivariate copula is decomposed into the product of bivariate conditional copulas. Therefore, I_{c_S}, the corresponding GFI of the copula in any coalition S can by written as:

$$\begin{aligned} I_{c_S} &= \sum_{j=1}^{|S|-1}\sum_{k=1}^{|S|-j} I_{c_{j,j+k|j+1,\ldots,j+k-1}} \\ &= \sum_{j=1}^{|S|-1}\sum_{k=1}^{|S|-j} I_{dg_{j,j+k|j+1,\ldots,j+k-1}} \mathbb{1}_{[I_{c_{j,j+k|j+1,\ldots,j+k-1}}\ge 0]} \\ &\quad - \sum_{j=1}^{|S|-1}\sum_{k=1}^{|S|-j} I_{rl_{j,j+k|j+1,\ldots,j+k-1}} \mathbb{1}_{[I_{c_{j,j+k|j+1,\ldots,j+k-1}}<0]} \\ &= I_{dg_S} - I_{rl_S} \end{aligned} \quad (9)$$

I_{dg_S} represents the diversity gain in the coalition S and I_{rl_S} represents the amount of redundant information included in coalition S. By noting that I_{dg_S} and I_{rl_S} are nonnegative and nondecreasing function of $|S|$, we view I_{dg_S} together with $\sum_{i\in S} I_i(\theta)$ as the gain of forming S, and I_{rl_S} together with E_S as the cost.

3.2. Game Formulation and Properties

We propose a $(\mathcal{N}, v)$ coalitional game to model our collaborative estimation framework, where $\mathcal{N}$ is the set of players (the sensors) and v is the utility function of a coalition S. $v(S)$ is defined as an increasing function of the gain $\left[\sum_{i\in S} I_i(\theta) + I_{dg_S}\right]$ and a decreasing function of the cost I_{rl_S}, E_S:

$$v(S) = \left[\sum_{i\in S} I_i(\theta) + I_{dg_S}\right] - [I_{rl_S} + C(E_S)] \quad (10)$$

$C(E_S)$ reflects the energy cost of each sensor in the coalition S. There are certain properties that a well designed cost function

$C(E_S)$ should satisfy; here we use the logarithmic barrier penalty function given by [14]

$$C(E_S) = \begin{cases} -1/t \cdot \log(1 - \frac{E_S}{\alpha}) & \text{if } E_S < \alpha \\ +\infty & \text{otherwise} \end{cases} \quad (11)$$

where α is the constraint on E_S, and t is a control parameter.

Proposition 3. *In the proposed game, the utility is non-transferable. That is the utility for each of the sensor in coalition S is equal to the utility of the coalition, i.e., $v(S) = \phi_i(S), \forall i \in S$, where $\phi_i(S)$ denotes the utility of sensor i when it belongs to a coalition S.*

Now, we have a non-transferable $(\mathcal{N}, v)$ coalitional game and a distributed algorithm for forming coalitions among sensors will be derived.

3.3. Coalition formation algorithm

For autonomous coalition formation in wireless sensor networks, we propose a distributed algorithm based on two simple rules denoted as *merge* and *split* [11] that allow us to modify a partition $\mathcal{S}$ of the set $\mathcal{N}$.

Merge Rule: Merge any set of coalitions $\{S_1, \ldots, S_m\}$, where $\{\cup_{j=1}^m S_j\} \triangleright \{S_1, \ldots, S_m\}$, therefore, $\{S_1, \ldots, S_m\} \rightarrow \{\cup_{j=1}^m S_j\}$.

Split Rule: Split any coalition $\{\cup_{j=1}^m S_j\}$, where $\{S_1, \ldots, S_m\} \triangleright \{\cup_{j=1}^m S_j\}$, thus $\{\cup_{j=1}^m S_j\} \rightarrow \{S_1, \ldots, S_m\}$.

For the proposed collaborative estimation game $(\mathcal{N}, v)$, we construct a coalition formation algorithm based on merge-and-split rules, the result of which is a network partition composed of disjoint independent coalitions of sensors.

To collaboratively estimate the parameter of interest, after the sensors make their own observations, they seek to form coalitions through merge-and-split. Let the initial partition be $\mathcal{S} = \{S_1, \ldots, S_m\}$. Then,successive merge-and-split processes will go on until the iterations terminate.

repeat
$\mathcal{R}$ = Merge($\mathcal{S}$): coalitions in $\mathcal{S}$ merge according to the merge rule, until no further merge occurs
$\mathcal{S}$ = Split($\mathcal{R}$): coalitions in $\mathcal{R}$ split according to the split rule, until no further split occurs.
until No merge or split occurs

The stability of this resulting network structure can be investigated using the concept of a defection function $\mathbb{D}$ [9, 11]. A partition $\mathcal{T} = \{T_1, \ldots, T_m\}$ of $\mathcal{N}$ is $\mathbb{D}_{hp}$-stable, if no players in $\mathcal{T}$ are interested in leaving $\mathcal{T}$ through merge-and-split to form other partitions in $\mathcal{N}$. A partition $\mathcal{T}$ is $\mathbb{D}_c$-stable, if no players in $\mathcal{T}$ are interested in leaving $\mathcal{T}$ through any operation to form other collections in $\mathcal{N}$ [9].

$\mathbb{D}_{hp}$-stable can be thought of as a state of equilibrium where no coalitions have an incentive to pursue coalition formation through merge or split. It has been proved in [15] that a partition is $\mathbb{D}_{hp}$-stable if and only if it is the outcome of iterating the merge-and-split rules. Thus, for the proposed $(\mathcal{N}, v)$ collaborative distributed estimation game, the proposed merge-and-split algorithm converges to a $\mathbb{D}_{hp}$-stable partition.

It is known that if $\mathcal{T}$ is $\mathbb{D}_c$-stable, then $\mathcal{T}$ is the outcome of every iteration of the merge-and-split rules and it is a unique $\mathbb{D}_c$-stable partition [11]. Thus, if a $\mathbb{D}_c$-stable partition exists for our proposed game, then the $\mathbb{D}_{hp}$-stable partition that our algorithm converges to, is also the optimal $\mathbb{D}_c$-stable partition. Nonetheless, a $\mathbb{D}_c$-stable partition dose not always exits [15]. A $\mathbb{D}_c$-stable partition is not guaranteed for our collaborative game and its existence depends on the covariance matrix and the parameters of the cost function in (11).

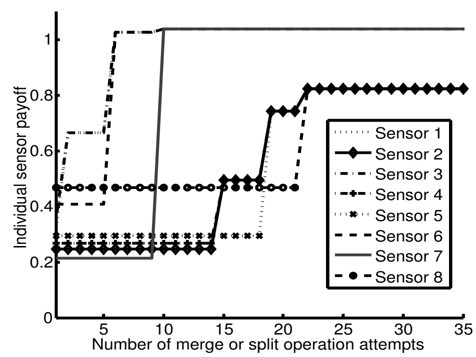

Fig. 1. Individual sensor payoffs keep increasing with each merge or split operation attempt until a $\mathbb{D}_{hp}$-stable partition is reached

4. SIMULATION RESULTS

We consider a wireless sensor network with eight sensors, i.e., $\mathcal{N} = \{1, \ldots, 8\}$. The covariance matrix of the eight random variables is such that X_1, X_3, X_6 are independent of each other, while X_2, X_4, X_5 are independent of each other.

We set α such that the largest coalition size is four. We set the initial partition to be $\mathcal{F} = \{\{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\}, \{7\}, \{8\}\}$. By applying the merge-and-split algorithm we proposed in the previous section, the final partition is $\mathcal{F} = \{\{1, 3, 6, 7\}, \{2, 4, 5, 8\}\}$, as shown in Figure 1.

By the utility function (10), sensors that tend to increase the diversity gain are more likely to be grouped together; on the other hand, sensors that tend to be redundant to each other are more likely to be placed in different coalitions. For X_1 and X_2, whose correlation coefficient is $\rho = 0.3$, assigning them in the same coalition will incur a redundancy loss term. The final partition avoided redundancy loss by putting $1, 3, 6$ in one coalition and $2, 4, 5$ in another. Note that it may not always be the case that independent observations are assigned to the same coalition, since it may happen that some inter-sensor dependence may add to diversity gain, under such circumstances dependent sensors are preferred than independent ones. For the overall system performance, the average payoff of the sensor network, which is defined as $1/N \sum_{S \in \mathcal{S}} |S| I_S(\theta)$, also increases as shown in Figure 2. Sensor's estimation performance improves by collaborating with others in the network, while communication efficiency constraint is satisfied. A more detailed study of these concepts will continue.

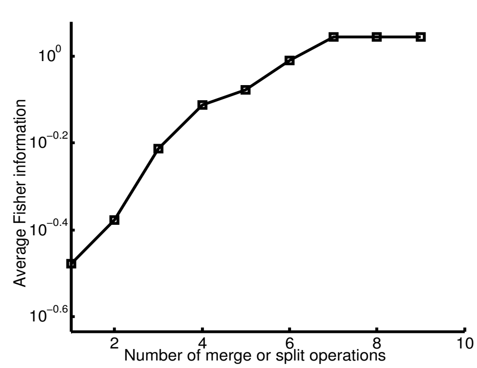

Fig. 2. Average FI increases with each merge or split operation

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