A ROBUST MISO TRAINING SEQUENCE DESIGN

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ABSTRACT

In this paper, the problem of robust training sequence design for multiple-input single-output (MISO) channel estimation is investigated. The mean-squared error (MSE) of the channel estimates is considered as a performance criterion to design an optimized training sequence which is a function of channel covariance matrix. In practice, the channel covariance matrix is not perfectly known at the transmitter side. Our goal is to take such imperfection into account and propose a robust design following the worst-case philosophy which results in finding the optimal training sequences for the least favorable channel covariance matrix within a deterministic uncertainty set. In this work, we address the formulated minimax design problem under different assumptions of the uncertainty set, and we show that for a unitarily-invariant uncertainty set, the optimally robust training sequence shares its eigenvectors with the channel covariance matrix. Furthermore, we give analytical closed-form solutions for robust training sequences if the spectral norm or nuclear norm are considered as constraints to bound the existing uncertainty.

Index Terms— Robust training sequences, worst-case robustness, unitarily-invariant uncertainty set, imperfect covariance, MIMO channel estimation.

1. INTRODUCTION

A crucial step throughout the design of multiple array communication systems is the acquisition of channel state information (CSI). It has been shown in [1] that the full advantage of multiple-input multiple-output (MIMO) communication systems (e.g., throughput, capacity, etc.) is attained where a perfect knowledge of CSI is provided at the transmitter. There are many studies in which the authors tried to overcome the challenge of having imperfect CSI at the transmitter by proposing robust schemes or optimal designs, e.g., [2–8]. One approach to obtain channel coefficients is sending known *training sequences*, then, using an estimator at the receiver, the channel coefficients can be estimated given the observed received signal. Now, the design challenge is to optimize training sequences with respect to minimizing a distortion measure, e.g. the *mean squared error* (MSE) used in this paper, under a training power budget constraint. The MSE as a performance criterion is, itself, a function of the channel covariance matrix. The authors in [9–12] have derived optimal training sequences assuming certain statistical structures to model the covariance matrix of correlated MIMO channels. Now, the question is that how the MSE performance will change if the *true* channel covariance matrix is not necessarily equivalent to the *nominal* one (which is known *a priori*). Note that this is often the case, since, in practice, the channel covariance matrix itself has to be estimated and possibly also quantized and fed back to the transmitter, leading to an erroneous channel covariance matrix. In this paper, our goal is to take existing uncertainty in the channel covariance matrix into account, and to provide training sequences which are robust against such imperfections following the worst-case robust philosophy. We formulate a minimax robust training design problem by which we optimize the worst-case performance.

The technique of robust superimposed training sequences for MIMO channels has been brought up in [13] where robust sequences are obtained numerically using an iterative algorithm for the case of uncorrelated noise in the system. In [14], the authors have generalized this method in order to find robust multiplexed training sequences for the colored noise scenario. In both studies [13] and [14], the authors have considered the Frobenius norm constraint to model the uncertainty. In this paper, concentrating on multiple-input single-output (MISO) channels and uncorrelated noise, we widen the robust analysis for the training sequence design problem by considering different uncertainty models, and provide both structural results as well as closed form solutions, for specific special cases.

In robust designs where the uncertain parameter vector/matrix represents amplitude values, it often makes sense to use an Euclidean/Frobenius norm to express the uncertainty region, related to the power of the error. However, in the current application the uncertainty lies in the covariance matrix, where it sometimes can be more reasonable to express the uncertainty as a linear function of the eigenvalue deviations. One justification could be that this deviation directly relates to uncertainties in quadratic forms involving the uncertain covariance matrix. Therefore, we provide some insights into robustness first considering a general convex assumption, then, we focus on the uncertainty regions described in terms of the spectral norm and the nuclear norm of the error in the channel covariance matrix.

We start from the generic assumption of convex unitarily-invariant uncertainty sets [8] where we show that the robust design problem is convex and can be addressed efficiently, then we prove that diagonal structure is the optimal one, i.e., the optimally robust training sequence matrix is diagonalized by the eigenvectors of the nominal channel covariance matrix. This result reduces the complexity by allowing us to simplify a complex matrix-variable minimax problem to a real vector-variable power allocation problem. We proceed our analysis by deriving closed-form solutions to the minimax power allocation problem under different norm constraints used to determine the uncertainty sets, e.g., spectral norm and nuclear norm.

2. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a MISO communication system with n_T antennas at the transmitter side and single antenna at the receiver side. The received signal is presented as

$$\mathbf{y} = \mathbf{P}\mathbf{h} + \mathbf{n}, \tag{1}$$

where $\mathbf{P} \in \mathcal{C}^{B\times n_T}$ is the training matrix which comprises training sequences as its column at each channel use. B represents the length of the training sequence, or in other words, the number of channel uses, and $B \geq n_T$. $\mathbf{y} \in \mathcal{C}^{B\times 1}$ is the received signal and $\mathbf{n} \in \mathcal{C}^{B\times 1}$ is a white Gaussian noise vector with $\mathbb{E}\{\mathbf{n}\mathbf{n}^H\} = \sigma_n^2\mathbf{I}_B$ where $\mathbb{E}\{\cdot\}$ and $(\cdot)^H$ denote the expectation operator and Hermitian of a vector (or matrix), respectively. Moreover, $\mathbf{h} \in \mathcal{C}^{n_T\times 1}$ is the MISO channel with $\mathbb{E}\{\mathbf{h}\mathbf{h}^H\} = \mathbf{R} \in \mathcal{C}^{n_T\times n_T}$. Both the MISO channel $\mathbf{h}$ and the noise $\mathbf{n}$ are distributed according to circularly symmetric complex Gaussian distribution, i.e., $\mathbf{h} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R})$ and $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \sigma_n^2\mathbf{I}_B)$, respectively. We assume that the MISO channel covariance matrix is known at the receiver side. Then using the MMSE estimator, the estimate of the instantaneous channel coefficients vector $\hat{\mathbf{h}}_{mmse}$ is given by [15, Chapter 10]

$$\hat{\mathbf{h}}_{mmse} = [\mathbf{R}^{-1} + \frac{1}{\sigma_n^2}\mathbf{P}^H\mathbf{P}]^{-1}\frac{1}{\sigma_n^2}\mathbf{P}^H\mathbf{y}. \tag{2}$$

In order to optimize the training sequences under power constraint, a common method used in existing works such as [9–12] is to minimize the MSE with respect to the total power budget constraint $\mathcal{P}_T$

$$\begin{aligned} &\min_{\mathbf{P}} \ \mathrm{Tr}\left\{\left[\mathbf{R}^{-1} + \frac{1}{\sigma_n^2}\mathbf{P}^H\mathbf{P}\right]^{-1}\right\} \\ &\text{s.t.} \ \ \mathrm{Tr}\{\mathbf{P}^H\mathbf{P}\} \leq \mathcal{P}_T. \end{aligned} \tag{3}$$

This objective is a function of the channel covariance matrix $\mathbf{R}$ which itself should be estimated and/or quantized and fed back to the transmitter, and as a result, subject to errors. Following the worst-case robust optimization philosophy, we take this error into account formulating the robust design as a minimax optimization problem. Indeed, we find the training sequences which minimize the MSE for the least favorable covariance matrix within a deterministic uncertainty set. This set is defined as a neighborhood of the nominal channel covariance matrix $\hat{\mathbf{R}}$ which is bounded by a convex constraint on the Hermitian mismatch matrix $\mathbf{E}$. In other words, we assume that the true covariance matrix $\mathbf{R}$ is modelled as $\mathbf{R} = \hat{\mathbf{R}} + \mathbf{E}$ where $\mathbf{E} \in \mathcal{E}$ and $\mathcal{E}$ is a convex unitarily-invariant uncertainty set, i.e., if $\mathbf{E} \in \mathcal{E}$, then $\mathbf{U}\mathbf{E}\mathbf{U}^H \in \mathcal{E}$ for any arbitrary unitary matrix $\mathbf{U}$. This category of uncertainty sets is general enough to include most common sets used in the robust study literature, e.g., the Schatten p-norms which cover spectral norm, nuclear norm and also Frobenius norm for certain values of p. Now, let us define $\mathbf{W} = \frac{1}{\sigma_n^2}\mathbf{P}^H\mathbf{P} \in \mathcal{W}$ where $\mathcal{W} \triangleq \{\mathbf{W} \mid \mathbf{W} \succeq 0, \mathrm{Tr}\{\mathbf{W}\} \leq \frac{\mathcal{P}_T}{\sigma_n^2}\}$. Now, the robust design problem can be formulated as

$$\min_{\mathbf{W}\in\mathcal{W}} \max_{\mathbf{E}\in\mathcal{E}} \ \mathrm{Tr}\left\{\left[(\hat{\mathbf{R}} + \mathbf{E})^{-1} + \mathbf{W}\right]^{-1}\right\}. \tag{4}$$

3. ROBUST TRAINING SEQUENCES FOR THE MISO CHANNEL

In this section, we study and analyze problem (4) whose solution gives the robust training sequences for the MISO channel. This problem is convex since the objective function, due to the convexity of $\mathrm{Tr}(\cdot)^{-1}$ in its argument, is a convex function in $\mathbf{W}$ and maximization preserve convexity. Then, the outer minimization problem becomes convex under the convex constraint $\mathbf{W} \in \mathcal{W}$, and can be solved numerically for example using the extended barrier method described in [16]. Note that recalling the assumption $B \geq n_T$, the robust training sequences $\mathbf{P}^\star$ can be extracted out from $\mathbf{W}^\star$, e.g., using Cholesky factorization. In the following, we focus on the convex unitarily-invariant set and investigate the optimal structure and optimal power allocation for problem (4).

3.1. Optimal structure: Diagonalization

Now, consider the eigenvalue decomposition (EVD) of $\hat{\mathbf{R}}$, $\mathbf{E}$ and $\mathbf{W}$ as $\mathbf{U}_\mathbf{R}\mathbf{\Lambda}_\mathbf{R}\mathbf{U}_\mathbf{R}^H$, $\mathbf{U}_\mathbf{E}\mathbf{\Lambda}_\mathbf{E}\mathbf{U}_\mathbf{E}^H$ and $\mathbf{U}_\mathbf{W}\mathbf{\Lambda}_\mathbf{W}\mathbf{U}_\mathbf{W}^H$, respectively, then we have the following theorem, whose proof, due to the space limitation, is omitted. The detailed proof is given in [17].

Theorem 1. *For MISO channel* $\mathbf{h}$*, the solution to the minimax robust design problem* (4) $\mathbf{W}^\star$ *is given as* $\mathbf{W}^\star = \mathbf{U}_{\mathbf{W}^\star}^H\mathbf{\Lambda}_{\mathbf{W}^\star}\mathbf{U}_{\mathbf{W}^\star}$ *where* $\mathbf{U}_{\mathbf{W}^\star} = \mathbf{U}_\mathbf{R}$ *and* $\mathbf{\Lambda}_{\mathbf{W}^\star}$ *is the solution to the following minimax power allocation problem*

$$\min_{\mathbf{\Lambda}_\mathbf{W}\in\mathcal{W}} \max_{\mathbf{\Lambda}_\mathbf{E}\in\mathcal{E}} \ \mathrm{Tr}\left\{\left[(\mathbf{\Lambda}_\mathbf{R} + \mathbf{\Lambda}_\mathbf{E})^{-1} + \mathbf{\Lambda}_\mathbf{W}\right]^{-1}\right\}. \tag{5}$$

In Theorem 1, it is proved that the optimally robust solution is obtained when the variables $\mathbf{W}$ and $\mathbf{E}$ are diagonalized by the eigenvectors of the nominal covariance matrix, i.e., $\mathbf{U}_{\mathbf{W}^\star} = \mathbf{U}_{\mathbf{E}^\star} = \mathbf{U}_{\mathbf{R}}$ implying that the diagonal structure is the optimal structure so that the optimal direction is the eigen-direction of the nominal covariance matrix $\hat{\mathbf{R}}$. Following the same argument mentioned earlier in this section, we conclude that this power allocation problem also is convex and can be addressed numerically for any convex unitarily-invariant set. In the next subsection, we take a deeper analytic look into the problem (5) by considering different vector norm constraints to describe the unitarily-invariant uncertainty set.

3.2. Optimal power allocation for different uncertainty models

In this subsection, we provide closed-form solutions for the power allocation problem (5) considering two specific uncertainty models expressed using the *spectral norm* and *nuclear norm*, respectively. These analytical solutions give insights regarding the dependency of robust training sequences on the parameters of the MISO system as well as the aspects of uncertainty sets. Let $\lambda_{\mathbf{A}_i}$ for $i = 1, ..., n$ and $\lambda_{max}(\mathbf{A})$ represent each eigenvalue and the largest eigenvalue of the $n \times n$ matrix $\mathbf{A}$, respectively, also diag($\mathbf{a}$) is a diagonal matrix which has the vector $\mathbf{a}$ as its main diagonal elements.

3.2.1. Spectral norm

We assume that $\mathbf{\Lambda_E} \in \mathcal{E}_s$ where $\mathcal{E}_s \triangleq \{\mathbf{\Lambda_E} \mid \|\mathbf{\Lambda_E}\|_2 \triangleq \lambda_{max}(\mathbf{\Lambda_E}) \leq \epsilon\}$.

Theorem 2. *Let the uncertainty set $\mathcal{E} = \mathcal{E}_s$. Then, the solution to the power allocation problem* (5) *is given by*

$$\mathbf{\Lambda}_{\mathbf{W}^\star} = diag(\lambda_{\mathbf{W}_i^\star}) = diag(max\{k - \frac{1}{\lambda_{\mathbf{R}_i} + \epsilon}, 0\}) \quad (6)$$

for $i = 1, 2, ..., n_T$, where $k = \frac{\mathcal{P}_T + \sigma_n^2 \mathrm{Tr}\{(\mathbf{\Lambda_R} + \epsilon \mathbf{I}_{n_T})^{-1}\}}{\sigma_n^2 n_T}$.

Proof: The proof is based on, first, using the matrix inversion lemma and exploiting the fact that $\mathbf{\Lambda_E} \preceq \epsilon \mathbf{I}_{n_T}$ to turn the inner maximization problem into a minimization problem for which $\mathbf{E} = \epsilon \mathbf{I}_{n_T}$ is the solution, second, inserting this solution back to the objective function, the outer minimization can be, equivalently, rewritten as the following minimization problem

$$\begin{aligned} \min_{\lambda_{\mathbf{W}_i}} \quad & \sum_{i=1}^{n_T} \frac{\lambda_{\mathbf{R}_i} + \epsilon}{\lambda_{\mathbf{W}_i}(\lambda_{\mathbf{R}_i} + \epsilon) + 1} \\ \text{s.t.} \quad & \sum_{i=1}^{n_T} \lambda_{\mathbf{W}_i} \leq \frac{\mathcal{P}_T}{\sigma_n^2}, \lambda_{\mathbf{W}_i} \geq 0, \end{aligned}$$

which can be solved by applying Karush-Kuhn-Tucker (KKT) conditions since we have a convex problem (the objective function is convex with respect to $\lambda_{\mathbf{W}_i}$). Then, the robust power allocation for $i = 1, ..., n_T$ is given by

$$\lambda_{\mathbf{W}_i^\star} = \max\left\{\frac{1}{n_T}\left[\frac{\mathcal{P}_T}{\sigma_n^2} + \sum_{i=1}^{n_T}(\frac{1}{\lambda_{\mathbf{R}_i} + \epsilon})\right] - \frac{1}{\lambda_{\mathbf{R}_i} + \epsilon}, 0\right\}. \quad (7)$$

■

3.2.2. Nuclear norm

We assume that $\mathbf{\Lambda_E} \in \mathcal{E}_n$ where $\mathcal{E}_n \triangleq \{\mathbf{\Lambda_E} \mid \|\mathbf{\Lambda_E}\|_* \triangleq \mathrm{Tr}\{(\mathbf{\Lambda_E}^H \mathbf{\Lambda_E})^{\frac{1}{2}}\} \leq \epsilon\}$.

Theorem 3. *Let the uncertainty set $\mathcal{E} = \mathcal{E}_n$. Then, for $\epsilon \geq \epsilon' \triangleq \lambda_{max}(\hat{\mathbf{R}}) - \sum_{i=1}^{n_T} \lambda_{\mathbf{R}_i}$, the solution to the power allocation problem* (5) *is given as*

$$\mathbf{\Lambda}_{\mathbf{W}^\star} = \frac{\mathcal{P}_T}{\sigma_n^2 n_T} \mathbf{I}_{n_T}. \quad (8)$$

Proof: Consider first solving (5) under the alternative constraint $\mathcal{E}_t \triangleq \{\mathbf{\Lambda_E} \mid \mathrm{Tr}\{\mathbf{\Lambda_E}\} \leq \epsilon\}$. It can be shown that the solution is given by

$$(\mathbf{\Lambda}_{\mathbf{W}^\star}, \mathbf{\Lambda}_{\mathbf{E}^\star}) = \left(\frac{\mathcal{P}_T}{\sigma_n^2 n_T}\mathbf{I}_{n_T}, \frac{1}{n_T}\left(\epsilon + \sum_{i=1}^{n_T} \lambda_{\mathbf{R}_i}\right)\mathbf{I}_{n_T} - \mathbf{\Lambda_R}\right).$$

For detailed proof, refer to [17]. Note that if $\forall \lambda_{\mathbf{E}_i} \geq 0, i = 1, ..., n_T$, the two uncertainty sets $\mathcal{E}_n$ and $\mathcal{E}_t$ are equivalent. It can be easily shown that if $\epsilon \geq \epsilon' \triangleq \lambda_{max}(\hat{\mathbf{R}}) - \sum_{i=1}^{n_T} \lambda_{\mathbf{R}_i}$, then $\lambda_{\mathbf{E}_i} \geq 0$. Therefore, it is concluded that for $\epsilon \geq \epsilon'$, we have found the solution to the nuclear norm problem and obtained the optimal power allocation which is equivalent to (8). ■

4. NUMERICAL RESULTS

In this section, we evaluate the performance of our proposed robust training sequence design. We consider a 4×1 MISO channel where the nominal covariance matrix $\hat{\mathbf{R}}$ is generated according to the exponential model [18] with the correlation coefficient ρ.

In Fig.1, we have plotted the cumulative distribution function (CDF) of the worst-case MSE for the proposed robust design and non-robust scheme. We have solved (5) under the Frobenius norm constraint, i.e., $\mathbf{E} \in \mathcal{E}_f$ where $\mathcal{E}_f \triangleq \{\mathbf{E} \mid \|\mathbf{E}\|_F \triangleq (\mathrm{Tr}\{\mathbf{E}^H\mathbf{E}\})^{\frac{1}{2}} \leq \epsilon\}$ to obtain the worst-case MSE values for robust design. We have chosen the Frobenius norm constraint here due to its differentiability which allows us to use the extended barrier method. For the non-robust design, the outer minimization in (4) is solved by only considering the nominal covariance matrix $\hat{\mathbf{R}}$, and then the resulting solution is inserted back to the objective where we solve the

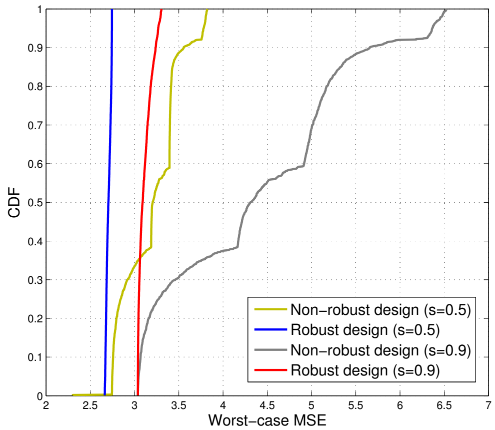

Fig. 1. CDF comparison of the robust and non-robust training sequence design for arbitrary correlated MISO channel.

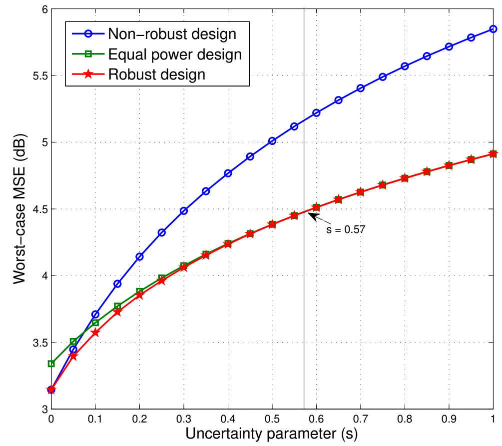

Fig. 2. Worst-case MSE comparison of three different training sequence designs for different sizes of the nuclear norm uncertainty set, where $s = \epsilon/\|\hat{\mathbf{R}}\|_*$.

inner maximization to find the worst covariance matrix. A relative uncertainty parameter $s \in [0, 1]$ used in Fig.1 is defined as $s \triangleq \epsilon/\|\hat{\mathbf{R}}\|_F$. The curves in Fig.1 are plotted using 1000 Monte Carlo simulations where the magnitude of the correlation coefficient $|\rho|$ is drawn according to the uniform distribution in $(0, 1)$. We also assume that SNR $\triangleq \frac{\mathcal{P}_T}{\sigma_n^2}$ is set to 5 dB. As can be seen from this figure, the worst-case MSE value variations are much larger for the non-robust design as compared to the robust design. This observation implies that we have met our design purpose in the sense of finding training sequences which provide robust performance against perturbed covariance matrix. Also, robust deign outperforms non-robust one in terms of the worst-case performance since the probability of having the worst-case MSE smaller than a certain value is higher for the curves denoting the robust design.

In Fig.2, we compare three different training design schemes under the nuclear norm constraint where the plotted results are based on a single channel realization. Here, $\rho = 0.3e^{-0.83j\pi}$. As it would be expected, the robust approach outperforms the non-robust one and for $\epsilon \geq \epsilon'$ (defined in Theorem 3), the robust one approaches the equal power allocation scheme which can be understood according to Theorem 3.

5. CONCLUSION

A robust scheme has been investigated for the problem of optimal training sequence design for correlated MISO channel estimation. Existing uncertainty in the channel covariance matrix has been taken into account using deterministic uncertainty modelling, in general, where our goal was to optimize the training sequences under total power budget constraint. We have formulated this design problem as a minimax optimization problem by following the worst-case robust philosophy. Then, focusing on the convex unitarily-invariant uncertainty sets, we have proved that the diagonalization is the optimal strategy which provides robust training sequences and simplifies the problem to a power allocation problem. We have provided closed-form solutions as optimal power allocations under different uncertainty models which have been defined according to different norms. Numerical examples have been drawn to evaluate the performance of the proposed strategy and to show that our design goal, i.e., providing robustness against channel covariance mismatch, has been satisfied.

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6. REFERENCES

[1] E. Biglieri, R. Calderbank, A. Constantinides, A. Goldsmith, A. Paulraj, and H. V. Poor, *MIMO Wireless Communications*, New York: Cambridge University Press, 2007.

[2] E. Jorswieck and H. Boche, "Optimal transmission with imperfect channel state information at the transmit antenna array," *Wireless Personal Communications,*

Kluwer Academic Publishers, vol. 27, pp. 33–56, Oct. 2003.

[3] A. Pascual-Iserte, D.P. Palomar, A.I. Perez-Neira, and M.A. Lagunas, “A robust maximin approach for MIMO communications with imperfect channel state information based on convex optimization,” *IEEE Transactions on Signal Processing*, vol. 54, no. 1, pp. 346 – 360, Jan. 2006.

[4] J. Wang and D.P. Palomar, “Worst-case robust MIMO transmission with imperfect channel knowledge,” *IEEE Transactions on Signal Processing*, vol. 57, no. 8, pp. 3086 –3100, Aug. 2009.

[5] N. Vucic, H. Boche, and S. Shi, “Robust transceiver optimization in downlink multiuser MIMO systems,” *IEEE Transactions on Signal Processing*, vol. 57, no. 9, pp. 3576 –3587, Sept. 2009.

[6] J. Wang and D.P. Palomar, “Robust MMSE precoding in MIMO channels with pre-fixed receivers,” *IEEE Transactions on Signal Processing*, vol. 58, no. 11, pp. 5802 –5818, Nov. 2010.

[7] J. Wang and M. Bengtsson, “Joint optimization of the worst-case robust MMSE MIMO transceiver,” *IEEE Signal Processing Letters*, vol. 18, no. 5, pp. 295 –298, May 2011.

[8] J. Wang, M. Bengtsson, B. Ottersten, and D.P. Palomar, “Robust MIMO precoding for several classes of channel uncertainties,” *Accepted for publication in IEEE Transactions on Signal Processing*, 2013.

[9] J.H. Kotecha and A.M. Sayeed, “Transmit signal design for optimal estimation of correlated MIMO channels,” *IEEE Transactions on Signal Processing*, vol. 52, no. 2, pp. 546 – 557, Feb. 2004.

[10] M. Biguesh and A.B. Gershman, “Training-based MIMO channel estimation: a study of estimator tradeoffs and optimal training signals,” *IEEE Transactions on Signal Processing*, vol. 54, no. 3, pp. 884 – 893, Mar. 2006.

[11] Y. Liu, T.F. Wong, and W.W. Hager, “Training signal design for estimation of correlated MIMO channels with colored interference,” *IEEE Transactions on Signal Processing*, vol. 55, no. 4, pp. 1486 –1497, Apr. 2007.

[12] E. Björnson and B. Ottersten, “A framework for training-based estimation in arbitrarily correlated Rician MIMO channels with Rician disturbance,” *IEEE Transactions on Signal Processing*, vol. 58, no. 3, pp. 1807 –1820, Mar. 2010.

[13] C.T. Chiang and C.C. Fung, “Robust training sequence design for spatially correlated MIMO channel estimation,” *IEEE Transactions on Vehicular Technology*, vol. 60, no. 7, pp. 2882 –2894, Sept. 2011.

[14] N. Shariati and M. Bengtsson, “Robust training sequence design for spatially correlated MIMO channels and arbitrary colored disturbance,” in *IEEE Personal Indoor and Mobile Radio Communication (PIMRC)*, Sept. 2011.

[15] S. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*, Prentice Hall, Englewood Cliffs, NJ, 1993.

[16] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge University Press, 2004.

[17] N. Shariati, J. Wang, and M. Bengtsson, “Robust training sequence design for correlated MIMO channel estimation,” *Submitted to IEEE Transactions on Signal Processing*, 2013.

[18] S.L. Loyka, “Channel capacity of MIMO architecture using the exponential correlation matrix,” *IEEE Communications Letters*, vol. 5, no. 9, pp. 369 –371, Sept. 2001.