SHORT-DATA-RECORD FILTERING OF PN-MASKED DATA

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ABSTRACT

Pseudo-noise (PN) masking is regarded as an effective means to combat data eavesdropping (for example in military-grade communications or positioning systems). At the same time, PN-masked data transmissions are considered vulnerable to interference/jamming due to lack of practical interference suppression solutions. In this work, (i) we derive an efficient minimum-mean-square-error (MMSE) optimal linear receiver of PN-masked data and (ii) develop an auxiliary-vector (AV) MMSE adaptive filter estimator with state-of-the-art small-sample-support estimation performance. Simulation studies included in this paper illustrate the effectiveness of the theoretical developments.

***Index Terms*—** Adaptive filtering, auxiliary-vector filtering, global-positioning systems, interference suppression, pseudo-noise masking, secure communications.

1. INTRODUCTION

Pseudo-noise (PN) masking is the data-hiding technique by which information symbols are masked (modulated) by a noise-like –yet deterministic– sequence, generated by a finite-state machine with a period that far exceeds the transmission data rate [1],[2]. The mask can be "stripped-off" and the data can be retrieved by a perfectly synchronized replica of the PN-generator by means of mask-matched-filtering. PN-masking applications can be found in the global navigation satellite systems (GNSS) [3], long-coded (aperiodic) spread-spectrum communication links [4], or even watermarking procedures for security tracing [5].

At the same time, there has been a long-standing debate on the vulnerability of PN-masked data to interference/jamming, stemming, arguably, from suggestions that linear minimum-mean-square-error (MMSE) interference suppression is either infeasible (no symbol-period-invariant MMSE filter exists) [6],[7] or "has a computational complexity per symbol interval proportional to the third power of the processing gain" [8]. On this basis, PN-masking is commonly thought as being non-amenable to MMSE filtering and a conventional mask-matched-filter receiver is employed instead.

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In this paper, our contribution is twofold. First, arguably contrary to common belief, we establish that efficient, low-complexity, true interference-suppressive MMSE reception of PN-masked data is, in fact, possible and requires only one covariance matrix inversion for the detection of the whole PN-masked symbol stream. Second, we derive for the first time an auxiliary-vector (AV) filter estimation algorithm for the protection of PN-masked data against highly non-stationary jamming via short-data-record adaptation.

2. SIGNAL MODEL AND NOTATION

We consider a PN-masked symbol stream transmitted over a quasi-static multipath fading channel in the presence of complex zero-mean colored interference (possibly malevolent). For simplicity in the presentation and without loss of generality, the information symbols and mask are to be both binary antipodal. We denote by $\mathbf{m}$ the modulating PN-sequence of some large length N and partition $\mathbf{m}$ into K segments $\mathbf{m}_1, \mathbf{m}_2, \cdots, \mathbf{m}_K$ of length $L = N/K$ each, to be used as mask to K corresponding information bits. Then, for the kth information bit $b_k \in \{\pm 1\}$ the transmitted signal vector is

$$\mathbf{s}_k = \sqrt{p_u} b_k \mathbf{m}_k \in \mathbb{R}^{L\times 1}, \;\; k = 1, \cdots, K, \tag{1}$$

where $\mathbf{m}_k \in \{\pm\frac{1}{\sqrt{L}}\}^L$ denotes the kth normalized mask segment and $p_u > 0$ is the total transmitted signal energy per information bit. If the channel exhibits q resolvable paths within the time scale of mask symbols, then the kth received signal has the following $(L+q-1)\times 1$ vector representation

$$\mathbf{r}_k = \sqrt{p_u} b_k \mathbf{H}\mathbf{m}_k + \mathbf{i}_k + \sqrt{p_j}\mathbf{j} + \mathbf{n} \in \mathbb{C}^{(L+q-1)\times 1} \tag{2}$$

where $\mathbf{H} \in \mathbb{C}^{(L+q-1)\times L}$ is the multipath fading channel matrix taken to be invariant over the transmission of the K information bits, $\mathbf{i}_k$ identifies comprehensively the multipath-induced inter-symbol interference (ISI), $\mathbf{j}$ is a complex zero-mean colored jamming signal with covariance matrix $\mathbf{R}_j$, and $\mathbf{n}$ is complex zero-mean additive white Gaussian noise (AWGN) with covariance matrix $\mathbf{R}_n = \sigma^2\mathbf{I}_{L+q-1}$ for some $\sigma^2 > 0$ and $\mathbf{I}_{L+q-1}$ the size-$(L+q-1)$ identity matrix. With no loss of theoretical generality, we treat $\mathbf{R}_j$ as a positive-semidefinite matrix of rank $D \leq L+q-1$ and unitary trace, so that the overall power of the jamming signal equals $p_j > 0$.

3. MMSE RECEPTION OF PN-MASKED SIGNALS

The ideal MMSE filter for the recovery of the kth information bit is

$$\mathbf{w}_{k,\text{MMSE}} = \sqrt{p_u}\mathbf{R}_k^{-1}\mathbf{H}\mathbf{m}_k \tag{3}$$

where $\mathbf{R}_k$ is the input autocorrelation matrix of $\mathbf{r}_k$ defined as $\mathbf{R}_k \triangleq E\left\{\mathbf{r}_k\mathbf{r}_k^H\right\}$ with H being the conjugate-transpose operator and $E\{\cdot\}$ denoting statistical expectation taken with respect to b_k, $\mathbf{i}_k$, $\mathbf{j}$, $\mathbf{n}$ in (2), but not $\mathbf{m}_k$ which is treated as fixed deterministic. If we denote the autocorrelation matrix of the compound disturbance to signal $\mathbf{s}_k$ (ISI, jammer, and AWGN) $\mathbf{R}_{d,k} \triangleq E\left\{\left(\mathbf{i}_k + \sqrt{p_j}\mathbf{j} + \mathbf{n}\right)\left(\mathbf{i}_k + \sqrt{p_j}\mathbf{j} + \mathbf{n}\right)^H\right\}$, then if the number of resolvable paths is much less than the length of the bit mask (i.e., $q \ll L$) we can safely ignore the effects of ISI [9] and approximate $\mathbf{R}_{d,k} \approx \mathbf{R}_d \triangleq E\{\left(\sqrt{p_j}\mathbf{j} + \mathbf{n}\right)\left(\sqrt{p_j}\mathbf{j} + \mathbf{n}\right)^H\}$, so that the input autocorrelation matrix becomes

$$\mathbf{R}_k = p_u\mathbf{H}\mathbf{m}_k\mathbf{m}_k^H\mathbf{H}^H + \mathbf{R}_d. \tag{4}$$

Certainly, $\mathbf{R}_k$ in (4) changes on an information bit interval basis due to the mask and the MMSE filter in (3) appears at first sight to require an $(L+q-1)\times(L+q-1)$ matrix inversion per bit recovery as presented in [8]. In the following, we establish that this is not, in fact, necessary and a single matrix inversion – that of $\mathbf{R}_d$ – is sufficient for MMSE filtering of the whole masked bit stream. We begin with the comment that under post-filtering bit detection $\hat{b}_k = sign\left(\text{Re}\left\{\mathbf{w}_{k,\text{MMSE}}^H\mathbf{r}_k\right\}\right)$ ($sign(\cdot)$ is the sign operator and $\text{Re}\{\cdot\}$ extracts the real part of a complex number), any scaled version $s\mathbf{w}_{k,\text{MMSE}}$, $s > 0$, of $\mathbf{w}_{k,\text{MMSE}}$ in (3) is an equivalent filter. Then, using the Matrix Inversion Lemma (also known as Woodbury's Identity [10]) on $\mathbf{R}_k^{-1}$, we obtain

$$\mathbf{R}_k^{-1} = \mathbf{R}_d^{-1} - \frac{p_u\mathbf{R}_d^{-1}\mathbf{H}\mathbf{m}_k\mathbf{m}_k^H\mathbf{H}^H\mathbf{R}_d^{-1}}{1 + p_u\mathbf{m}_k^H\mathbf{H}^H\mathbf{R}_d^{-1}\mathbf{H}\mathbf{m}_k}. \tag{5}$$

Substituting (5) to (3), we show that

$$\mathbf{w}_{k,\text{MMSE}} \equiv \mathbf{R}_d^{-1}\mathbf{H}\mathbf{m}_k. \tag{6}$$

Thus, interference-suppressive MMSE filtering is, in fact, feasible for PN-masked data with a single matrix inversion for the whole data sequence $k = 1, 2, \cdots, K$, and one multiplication per data symbol of the form $(\mathbf{R}_d^{-1}\mathbf{H})\mathbf{m}_k$.

4. SHORT-DATA-RECORD ADAPTIVE MMSE RECEPTION OF PN-MASKED SIGNALS

We consider now $\mathbf{R}_d$ being estimated over a finite number of received signal snapshots, say M, recorded during a silent period of the information-bearing signal component $\mathbf{s}_k$. If, for example, the jamming signal is non-stationary and its second-order statistics remain constant only for a short coherence period of, say, T information bit intervals, then we must keep $M < T$ and the need for effective MMSE filter estimation with small sample support becomes pronounced.

Conventionally, $\mathbf{R}_d$ is sample-average estimated [11],[12] over the M silent (signal-absent) snapshots, say $\mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_M$, by

$$\hat{\mathbf{R}}_d(M) = \frac{1}{M}\sum_{m=1}^{M}\mathbf{y}_m\mathbf{y}_m^H. \tag{7}$$

If $\mathbf{c}_i$, $i = 1, 2, \cdots, T-M$, are the transmission masks used within the coherence period of $\hat{\mathbf{R}}_d(M)$ and $L+q-1 \leq M \leq T-1$, then $[\hat{\mathbf{R}}_N(M)]^{-1}$ exists with probability 1 and

$$\hat{\mathbf{w}}_{i,\text{SMI}}(M) = [\hat{\mathbf{R}}_d(M)]^{-1}\mathbf{H}\mathbf{c}_i \tag{8}$$

is the well-known, widely used, unbiased signal-absent sample-matrix-inversion (SMI) estimator on the MMSE filter[1]. On the other hand, if the interferer changes second-order statistics fast enough to have $T < L+q$ forcing the receiver adaptation sample size to $M < L+q-1$, then $\hat{\mathbf{R}}_d(M)$ in (7) is singular and $\hat{\mathbf{w}}_{i,\text{SMI}}(M)$ in (8) cannot be defined. For this case, a simple and popular approach is the diagonally loaded signal-absent SMI filter estimator $\hat{\mathbf{w}}_{i,\text{DL-SMI}}(\beta, M) = [\hat{\mathbf{D}}_d(\beta, M)]^{-1}\mathbf{H}\mathbf{c}_i$, where $\hat{\mathbf{D}}_d(\beta, M) \triangleq \beta\mathbf{I}_{L+d-1} + \hat{\mathbf{R}}_d(M)$ and β is the diagonal loading factor to be appropriately (heuristically) chosen [13]. It is worth noting at this point that supervised recursive-least-squares (RLS) and least-mean-square (LMS) filter estimators do not apply herein due to the nature of the input autocorrelation matrix that changes on an information bit interval basis [14].

In this paper, for the first time in the literature, we derive a modified auxiliary-vector (AV) filtering algorithm [15]-[22] for effective adaptive PN-masked signal protection under small sample support. For each mask segment $\mathbf{c}_i$, our algorithm produces a sequence of filter estimators that converges to $\hat{\mathbf{w}}_{i,\text{SMI}}(M)$ when the latter exists. Most importantly, early elements in the sequence are shown to offer state-of-the-art mean-square filter estimation error (and post-filtering SINR). In the rest of the section, we derive analytically the AV filter estimator sequence for any generated mask segment $\mathbf{c}_i$.

Denote by $\mathbf{P}_i^{\perp}$ the orthogonal projector onto the nullspace of the signal vector direction (channel-processed mask segment) $\mathbf{v}_i = \mathbf{H}\mathbf{c}_i$, that is

$$\mathbf{P}_i^{\perp} \triangleq \mathbf{I}_{L+d-1} - \|\mathbf{v}_i\|^{-2}\mathbf{v}_i\mathbf{v}_i^H. \tag{9}$$

Assume data record size $1 \leq M \leq T-1$ being available at the receiver for the computation of $\hat{\mathbf{R}}_d(M)$. We begin

[1] The SMI estimator is unbiased for multivariate elliptically contoured input distributions, i.e. $E\{\hat{\mathbf{w}}_{i,\text{SMI}}(M)\} = \mathbf{w}_{i,\text{MMSE}} = \mathbf{R}_d^{-1}\mathbf{H}\mathbf{c}_i$ [11].

the algorithmic developments by initializing our estimator sequence to the conventional (scaled) matched filter

$$\hat{\mathbf{w}}_{i,0} = \mathbf{w}_{i,\mathrm{MF}} = \|\mathbf{v}_i\|^{-2}\mathbf{v}_i. \tag{10}$$

Thereafter, following the theory developed in [15], each new filter estimate in the sequence is produced by incorporating to the previous filter estimate a weighted, orthogonal to $\mathbf{v}_i$ auxiliary-vector component. Specifically, the inductive step of the generated sequence is defined as

$$\hat{\mathbf{w}}_{i,n} = \hat{\mathbf{w}}_{i,n-1} - \mu_{i,n}\mathbf{g}_{i,n}, \ \ n = 1, 2, \cdots, \tag{11}$$

where $\mathbf{g}_{i,n} \in \mathbb{C}^L$, $\mu_{i,n} \in \mathbb{C}$, and $\mathbf{g}_{i,n}^H\mathbf{v}_i = 0$. Considering for a moment the nth auxiliary vector $\mathbf{g}_{i,n}$ to be arbitrarily fixed, we will choose $\mu_{i,n}$ to minimize the (estimated) variance of the output of $\hat{\mathbf{w}}_{i,n}$

$$\mu_{i,n} = \arg\min_{\mu\in\mathbb{C}} \hat{\mathbf{w}}_{i,n}^H\hat{\mathbf{R}}_d(M)\hat{\mathbf{w}}_{i,n}, \tag{12}$$

which has solution

$$\mu_{i,n} = \frac{\mathbf{g}_{i,n}^H\hat{\mathbf{R}}_d(M)\hat{\mathbf{w}}_{i,n-1}}{\mathbf{g}_{i,n}^H\hat{\mathbf{R}}_d(M)\mathbf{g}_{i,n}}. \tag{13}$$

Next, we turn to the auxiliary vector $\mathbf{g}_{i,n}$ and choose the vector that maximizes the magnitude of the (estimated) cross-correlation between the disturbance filtered by $\hat{\mathbf{w}}_{i,n-1}$ and by $\mathbf{g}_{i,n}$, while remaining orthogonal to the signal direction of interest $\mathbf{v}_i$. Mathematically stated,

$$\begin{aligned} \mathbf{g}_{i,n} &= \arg\max_{\mathbf{g}\in\mathbb{C}^L} |\hat{\mathbf{w}}_{i,n-1}^H\hat{\mathbf{R}}_d(M)\mathbf{g}|, \\ &\text{subject to} \ \ \mathbf{g}^H\mathbf{v}_i = 0. \end{aligned} \tag{14}$$

The solution to (14), derived through conventional Lagrange multipliers, is given by

$$\mathbf{g}_{i,n} = \mathbf{P}_i^{\perp}\hat{\mathbf{R}}_d(M)\hat{\mathbf{w}}_{i,n-1}. \tag{15}$$

Following the initialization of the filter sequence and the fact that $\mathbf{g}_{i,n}$ lies in the nullspace of $\mathbf{v}_i$, we observe that

$$\hat{\mathbf{w}}_{i,n}^H\mathbf{v}_i = 1 \ \ \forall n = 0, 1, 2, \cdots. \tag{16}$$

For clarity in presentation, the algorithm developed above is summarized in Fig. 1.

Convergence analysis is summarized in the form of the Lemmas and Theorems below. Proofs are omitted due to lack of space.

Lemma 1 *For any mask segment $i = 1, 2, \cdots, T - M$, the sequence of auxiliary-vector weights $\{\mu_{i,n}\}$, $n = 1, 2, \cdots$, defined by* (13) *is real-valued, positive, and bounded as follows*

$$0 < \frac{1}{\lambda_1} \leq \mu_{i,n} \leq \frac{1}{\lambda_P}, \quad n = 1, 2, \cdots, \tag{17}$$

where λ_1 and λ_P are the maximum and minimum, respectively, non-zero eigenvalues of $\hat{\mathbf{R}}_d(M)$. □

Input: $\mathbf{H}$, $\mathbf{c}_1$, $\mathbf{c}_2$, $\ldots$, $\mathbf{c}_{T-M}$, $\hat{\mathbf{R}}_d(M)$.
for $i = 1, 2, \cdots, T - M$ do
  Initialization:
  $\mathbf{v}_i := \mathbf{H}\mathbf{c}_i$;
  $\hat{\mathbf{w}}_{i,0} := \|\mathbf{v}_i\|^{-2}\mathbf{v}_i$;
  $\mathbf{P}_i^{\perp} := \mathbf{I} - \|\mathbf{v}_i\|^{-2}\mathbf{v}_i\mathbf{v}_i^H$.
  Iterative computation:
  for $n = 1, 2, \cdots$ do
    $\mathbf{g}_{i,n} := \mathbf{P}_i^{\perp}\hat{\mathbf{R}}_d(M)\hat{\mathbf{w}}_{i,n-1}$
    if $\mathbf{g}_{i,n} = \mathbf{0}$, then EXIT
    $\mu_{i,n} := \mathbf{g}_{i,n}^H\hat{\mathbf{R}}_d(M)\hat{\mathbf{w}}_{i,n-1}/\mathbf{g}_{i,n}^H\hat{\mathbf{R}}_d(M)\mathbf{g}_{i,n}$
    $\hat{\mathbf{w}}_{i,n} := \hat{\mathbf{w}}_{i,n-1} - \mu_{i,n}\mathbf{g}_{i,n}$
  end for
end for
Output:
Filter estimate sequences $\{\hat{\mathbf{w}}_{1,n}\}$, $\{\hat{\mathbf{w}}_{2,n}\}$, $\cdots$, $\{\hat{\mathbf{w}}_{T-M,n}\}$, $n = 1, 2, \cdots$.

Fig. 1: Proposed auxiliary-vector (AV) algorithm for the generation of MMSE filter estimate sequences $\{\hat{\mathbf{w}}_{1,n}\}$, $\{\hat{\mathbf{w}}_{2,n}\}$, $\cdots$, $\{\hat{\mathbf{w}}_{T-M,n}\}$, $n = 1, 2, \cdots$.

Lemma 2 *For any mask segment $i = 1, 2, \cdots, T - M$, successively generated auxiliary vectors, defined as in* (15)*, are orthogonal; that is,*

$$\mathbf{g}_{i,j}^H\mathbf{g}_{i,j+1} = 0, \quad j = 1, 2, \cdots. \qquad \square \tag{18}$$

Theorem 1 *For any mask segment $i = 1, 2, \cdots, T - M$, the sequence of auxiliary vectors $\{\mathbf{g}_{i,n}\}$, $n = 1, 2, \cdots$, converges to the zero vector as n tends to infinity,*

$$\lim_{n\to\infty} \mathbf{g}_{i,n} = \mathbf{0}. \qquad \square \tag{19}$$

Theorem 2 *For any mask segment $i = 1, 2, \cdots, T - M$ and sample support M large enough for $\hat{\mathbf{R}}_d(M)$ to be invertible ($M \geq L + q - 1$), the sequence of auxiliary-vector filters delivered by* (11) *converges as follows,*

$$\lim_{n\to\infty} \hat{\mathbf{w}}_{i,n} = \frac{[\hat{\mathbf{R}}_d(M)]^{-1}\mathbf{v}_i}{\mathbf{v}_i^H[\hat{\mathbf{R}}_d(M)]^{-1}\mathbf{v}_i}. \qquad \square \tag{20}$$

By Theorem 2, we conclude that our filter sequence converges to the signal-absent SMI filter estimate in (8). At the same time, as the sample support $M \to \infty$ the convergence point of the sequence tends to (3) and the algorithm offers the means to derive the MMSE filter without any form of explicit covariance matrix inversion, decomposition, or diagonalization. Theorem 3 establishes convergence when the sample support is not enough for $\hat{\mathbf{R}}_d(M)$ to be invertible. The proof, again omitted due to lack of space, shall be credited to [17].

Theorem 3 *For any mask segment $i = 1, 2, \cdots, T - M$, consider M small enough ($M < L + q - 1$) such that*

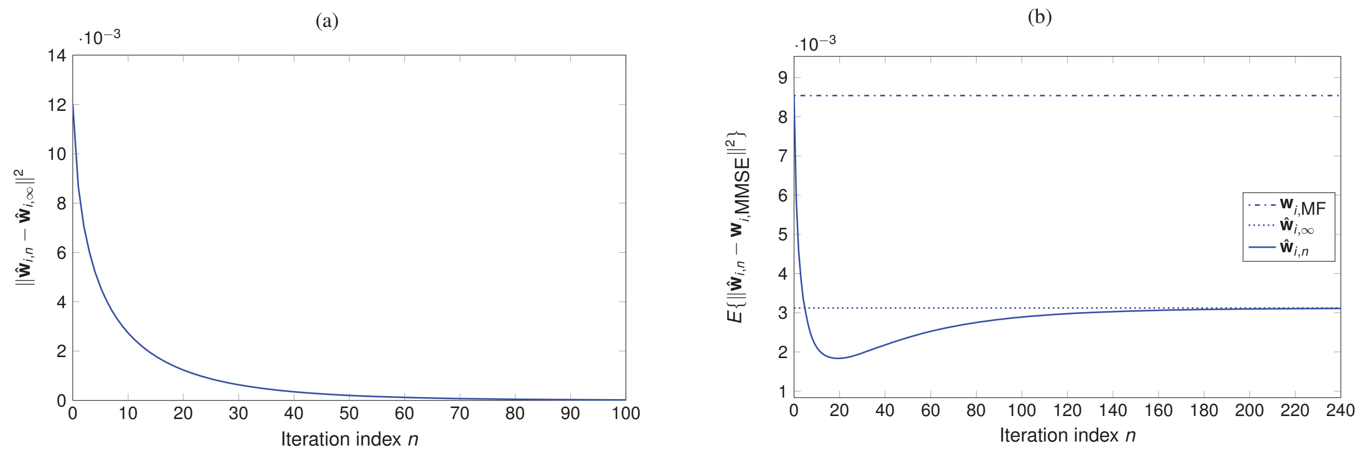

Fig. 2: Convergence of AV sequence $\{\hat{\mathbf{w}}_{i,n}\}$, $n = 1, 2, \cdots$, when $\hat{\mathbf{R}}_d(M)$ is invertible, captured in terms of (a) the squared euclidean norm and (b) the mean-square estimation error calculated over 10 000 independent realizations.

$\hat{\mathbf{R}}_d(M)$ is singular with eigenvector decomposition representation $\hat{\mathbf{U}}\hat{\mathbf{\Lambda}}\hat{\mathbf{U}}^H$. Let $\mathbf{P}_{\hat{\mathbf{U}}}^{\perp}$ be the orthogonal projector onto the nullspace of $\hat{\mathbf{R}}_d(M)$ such that $\mathbf{P}_{\hat{\mathbf{U}}}^{\perp}\hat{\mathbf{R}}_d(M) = \mathbf{0}_{(L+q-1)\times(L+q-1)}$. Then,

$$\lim_{n\to\infty} \hat{\mathbf{w}}_{i,n} = \frac{\mathbf{P}_{\hat{\mathbf{U}}}^{\perp}\mathbf{v}_i}{\mathbf{v}_i^H\mathbf{P}_{\hat{\mathbf{U}}}^{\perp}\mathbf{v}_i}. \qquad \square \quad (21)$$

Therefore, by Theorem 3, when the sample support size is less than the length of the (multipath extended) mask segment, the AV filter sequence converges to the projection of the matched filter $\mathbf{v}_i$ onto the subspace orthogonal to the collected data.

In Fig. 2(a), we illustrate the convergence of the AV sequence $\{\hat{\mathbf{w}}_{i,n}\}$, $n = 1, 2, \cdots$, as derived in (20) for $\hat{\mathbf{R}}_d(M)$ being invertible. Denoting by $\hat{\mathbf{w}}_{i,\infty}$ the convergence point, we depict the convergence in terms of the squared euclidean norm $\|\hat{\mathbf{w}}_{i,n} - \hat{\mathbf{w}}_{i,\infty}\|^2$ as a function of the iteration step. System specifics per Section 2 notation are $L = 80$, $q = 1$, $\sigma^2 = 0$dB, $p_j/\sigma^2 = 10$dB (signal-to-noise ratio of the jamming signal), $p_u/\sigma^2 = 0$dB (signal-to-noise ratio of the user signal), sample support $M = 240$. In Fig. 2(b) we show the mean-square filter estimation error, $E\{\|\hat{\mathbf{w}}_{i,n} - \mathbf{w}_{i,\text{MMSE}}\|^2\}$, for the ith information bit interval as a function of the iteration step n, calculated over 10 000 independent filter estimator realizations. As reference lines, we include the mean-square filter estimation error of the signal-absent SMI estimator $\hat{\mathbf{w}}_{i,\infty}$ and the squared euclidean distance between the conventional matched filter and the ideal MMSE filter $\|\mathbf{w}_{i,\text{MF}} - \mathbf{w}_{i,\text{MMSE}}\|^2$. Fig. 2(b) demonstrates that early, non-asymptotic elements of the generated sequence of AV filter estimators outperform the signal-absent SMI estimator (when the latter exists, i.e. when $\hat{\mathbf{R}}_d(M)$ is invertible).

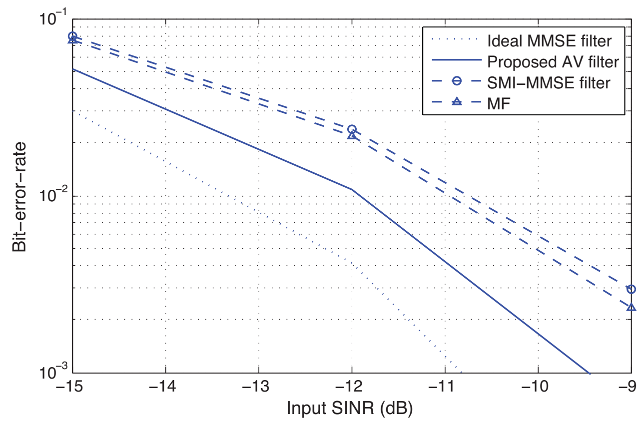

Fig. 3: Bit-error-rate for MF, SMI-MMSE, proposed AV, and ideal MMSE receivers of PN-masked signals (adaptation sample support $M = 70$).

5. SIMULATION STUDIES

In this section, we carry out a simulation study on the protection of PN-masked bitstreams against highly non-stationary jamming by means of the proposed technique. Each PN-masked information bit is filtered by the best, in MS filter estimation error, AV filter-estimate generated by the algorithm in Fig. 1 with sample support $M < T - 1$. In Fig. 3, we set the system parameters to sample support value $M = 70$, coherence interval length $T = 250$, mask segment length $L = 32$, number of mulitpaths $q = 1$, $\sigma^2 = 0$dB, $p_j/\sigma^2 = 20$dB. Then, we plot the bit-error-rate (BER) attained by the AV filter as a function of the input SINR calculated over 10 000 independent simulation runs. For reference purposes, we include in the plot the BER of the conventional matched filter in (10) and the SMI-MMSE filter estimate in (8). As a theoretical lower bound, we also add the curve of the ideal MMSE filter. The effectiveness –and superiority– of the proposed AV scheme can be easily seen.

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