# PARAMETER ESTIMATION OF 2D POLYNOMIAL PHASE SIGNALS: AN APPLICATION TO MOVING TARGET IMAGING WITH SAR

S. Barbarossa, P. Di Lorenzo, P. Vecchiarelli, A. Silvi, and A. Bruner

Sapienza University of Rome, DIET, Via Eudossiana 18, 00184 Rome, Italy. E-mail: dilorenzo, sergio@infocom.uniroma1.it, vecchiarelli@diet.uniroma1.it, a.silvi82@gmail.com, alessandro.bruner@alice.it

# ABSTRACT

Polynomial-Phase Signals (PPS) appear in a variety of applications and several algorithms are available to estimate their parameters in the presence of noise. Among the available tools, the Product High order Ambiguity Function (PHAF) has the merit of performing well in the presence of a superposition of PPS's. In this work, we generalize the PHAF to handle two-dimensional PPS's. Then we show an example of application motivating such an extension: the high resolution imaging of moving targets from synthetic aperture radars (SAR). Using the 2D-PHAF, we will propose an algorithm that compensates *jointly* for the range cell migration *and* the phase modulation induced by the relative radar-target motion in order to produce a focused image of the moving target.

*Index Terms*— Polynomial-Phase Signals, Synthetic Aperture Radar, Moving Target Imaging

# 1. INTRODUCTION

Polynomial-Phase Signals (PPS) are encountered in many applications [1]. In some cases, they represent an exact modeling of signals having a polynomial phase modulation, e.g. chirp signals in radars. In other cases, they constitute a good approximation of signals whose phase modulation is known to be a smooth function of time within a given observation interval. According to Weierstrass theorem, a continuous function defined on a closed interval can be uniformly approximated as closely as desired by a polynomial function. Hence, a continuous-phase signal observed in a finite interval can be well approximated by a PPS. In this second case, the selection of the polynomial degree affetcs the goodness of the approximation. Examples of continuous-phase signals are encountered in communications, as a way to generate signals with good spectral efficiency, as well in radars, where the phase modulation induced on the radar echo is impressed by the relative radar-target motion, which is certainly a continuous function of time. Not surprisingly, many algorithms have been proposed to estimate the parameters of PPS's embedded in noise, see e.g. [1-3]. A case of particular interest occurs when the observed signal is given by the superposition of multiple PPS's. In this case, of course, the superposition is no longer a PPS, but it is still important to track the instantaneous phases of the single components. This model arises, for example, in radar processing where the received signal is given by the superposition of echoes coming from different scatterers located at the same distance. The Product High-order Ambiguity Function (PHAF) was specifically introduced in [2] to estimate the parameters of the single components.

The PHAF was then proposed as a tool to produce a focused image of the ground in SAR in [4]. The analysis based on the PHAF can of course be extended to moving target imaging. However, when high resolution radar images are required, it is necessary to compensate both phase modulation and range cell migration (RCM) induced by the radar-target relative motion. Some approaches work directly on the SAR image and base the focusing mechanism on the adaptive sharpening of the moving target image, as in [5,7]. These methods do not attempt to track the phase modulation induced by the motion. Better performance can be achieved if the phase modulation is properly tracked and compensated. The PHAF-based method proposed in [2] assumed that the RCM was negligible. A method incorporating the compensation of RCM was proposed in [8]. However, the compensation used in [8] assumed that the slant range of the target was known. In this work, we extend the PHAF to the twodimensional (2D) case and we propose new methods to estimate the parameters of the 2D instantaneous phase, in the general multicomponent case. Then, we apply the new representation to the analysis of single antenna SAR signals and show how to use the 2D-PHAF to estimate and compensate the motion-induced phase modulation and range migration jointly.

### 2. 2D MULTILAG HIGH ORDER AMBIGUITY FUNCTION FOR 2D POLYNOMIAL PHASE SIGNALS

A discrete-time 2D Polynomial Phase Signal (2D-PPS) of degree M is defined as a constant amplitude signal having an instantaneous phase given by a 2D polynomial of order M:

$$s[u,v] = Ae^{j\Phi[u,v]} = Ae^{j2\pi\sum_{m=0}^{M}\sum_{l=0}^{m}a_{m-l,l}u^{m-l}v^{l}}, \quad (1)$$

with u = 0, ..., N - 1 and v = 0, ..., P - 1. This formulation makes explicit the phase terms having a common degree: All the terms corresponding to an index m have degree m. We define the 2D multilag High order Instantaneous Moment (2D ml-HIM) of order  $M, s_M[u, v; \tau_{M-1}, \theta_{M-1}]$ , through the following iterative rule:

$$s_{1}[u, v] = s[u, v];$$

$$s_{2}[u, v; \tau_{1}, \theta_{1}] = s_{1}^{*}[u, v] \cdot s_{1}[u + \tau_{1}, v + \theta_{1}];$$

$$\cdots$$

$$s_{M}[u, v; \tau_{M-1}, \theta_{M-1}] = s_{M-1}^{*}[u, v; \tau_{M-2}, \theta_{M-2}]$$

$$\cdot s_{M-1}[u + \tau_{M-1}, v + \theta_{M-1}; \tau_{M-2}, \theta_{M-2}],$$
(2)

where  $\boldsymbol{\tau}_k := [\tau_1, \dots, \tau_k]$  and  $\boldsymbol{\theta}_k := [\theta_1, \dots, \theta_k]$  are the vectors of delays used up to step k. Given the 2D ml-HIM, the 2D multilag High Order Ambiguity Function (2D ml-HAF) is defined as the 2D Fourier Transform of the 2D ml-HIM:

$$S_M(f,\nu;\boldsymbol{\tau}_{M-1},\boldsymbol{\theta}_{M-1}) =$$

This work has been supported by SIMTISYS Project, Nr. 263268, within the FP7-SPACE-2010-1 program.

$$=\sum_{u=0}^{N-1-T_{M-1}}\sum_{v=0}^{N-1-\Delta_{M-1}}s_{M}[u,v;\boldsymbol{\tau}_{M-1},\boldsymbol{\theta}_{M-1}]e^{-j2\pi(fu+\nu v)}$$
(3)

where  $T_{M-1} = \sum_{k=1}^{M-1} \tau_k$  and  $\Delta_{M-1} = \sum_{k=1}^{M-1} \theta_k$ . The motivation underlying the previous definitions is quite simple: If the signal is a single component 2D PPS, every product in (2) reduces the phase polynomial by one. Hence, if the signal has a phase of degree M, its 2D ml-HIM of order M has a linear phase. As a consequence, the 2D ml-HAF has a peak whose position is related to the phase coefficients of order M. Hence, estimating the peaks of the 2D ml-HAF allows to recover the phase coefficients. Some examples are useful to grasp the main properties of the ml-HAF.

M = 2

Applying (2) and (3), the 2D ml-HIM of a PPS of order M=2 is

$$s_2[u, v; \tau_1, \theta_1] = A^2 e^{j\Phi_2[u, v]}$$

with

$$\Phi_2[u,v] = \phi_0 + 2\pi [(2\tau_1 a_{2,0} + \theta_1 a_{1,1})u + (\tau_1 a_{1,1} + 2\theta_1 a_{0,2})v],$$
(4)

where  $\phi_0$  is a constant term. Hence, the 2D ml-HAF has a peak in the point of coordinates

$$f(\tau_1, \theta_1) = 2\tau_1 a_{2,0} + \theta_1 a_{1,1}$$
(5)

$$\nu(\tau_1, \theta_1) = \tau_1 a_{1,1} + 2\theta_1 a_{0,2}.$$
 (6)

The two frequencies have been expressed explicitly in terms of the delays  $\tau_1$  and  $\theta_1$ . For a given pair of delays, we have two observations  $(f(\tau_1, \theta_1)), \nu(\tau_1, \theta_1)$  and three unknowns:  $a_{2,0}, a_{1,1}$ , and  $a_{0,2}$ ). However, if we compute the ml-HAF using multiple sets of lags, we get multiple observations related to the the same set of phase coefficients. Denoting the *l*-th set of lags as  $\tau_1^{(l)}, \theta_1^{(l)}$ , if we use *L* sets of lags, we can write the following system of equations:

$$\begin{pmatrix} f(\tau_{1}^{(1)}, \theta_{1}^{(1)}) \\ \nu(\tau_{1}^{(1)}, \theta_{1}^{(1)}) \\ \vdots \\ f(\tau_{1}^{(L)}, \theta_{1}^{(L)}) \\ \nu(\tau_{1}^{(L)}, \theta_{1}^{(L)}) \end{pmatrix} = \begin{pmatrix} 2\tau_{1}^{(1)} & \theta_{1}^{(1)} & 0 \\ 0 & \tau_{1}^{(1)} & 2\theta_{1}^{(1)} \\ \vdots & \vdots & \vdots \\ 2\tau_{1}^{(L)} & \theta_{1}^{(L)} & 0 \\ 0 & \tau_{1}^{(L)} & 2\theta_{1}^{(L)} \end{pmatrix} \begin{pmatrix} a_{2,0} \\ a_{1,1} \\ a_{0,2} \end{pmatrix}$$

$$(7)$$

or, in compact form,

$$= \Theta a,$$
 (8)

where  $f \in \mathbb{R}^{2L \times 1}$  contains the measured frequencies,  $a \in \mathbb{R}^{3 \times 1}$  is the set of phase coefficients pertaining to the second degree, and  $\Theta \in \mathbb{R}^{2L \times 3}$ . If the the number of different sets of lags is  $L \geq 2$  and the sets of lags are chosen so that the columns of  $\Theta$  are linearly independent, the previous system can be inverted to find the phase coefficient vector a as

f

$$\hat{\boldsymbol{a}} = (\boldsymbol{\Theta}^T \boldsymbol{\Theta})^{-1} \boldsymbol{\Theta}^T \boldsymbol{f}.$$
(9)

Once the second order coefficients have been estimated, the second order phase term can be removed by the received signal by the multiplication

$$s_c[u,v] = s[u,v] e^{-j2\pi(\hat{a}_{2,0}u^2 + \hat{a}_{1,1}uv + \hat{a}_{0,2}v^2)}.$$
 (10)

If the estimated coefficients are correct, the compensated signal  $s_c[u, v]$  is a PPS of order one, whose remaining phase coefficients

can be estimated using a Fourier Transform<sup>1</sup>.

M = 3

The 2D ml-HIM of order 3 of a PPS of degree M = 3 is

$$s_3[u,v;\boldsymbol{\tau}_2,\boldsymbol{\theta}_2] = A^2 e^{j\Phi_3[u,v]},$$

with  $\Phi_3[u, v] = \phi_0 +$ 

$$+2\pi[(6\tau_1\tau_2a_{3,0}+2\tau_2\theta_1a_{2,1}+2\tau_1\theta_2a_{2,1}+2a_{1,2}\theta_1\theta_2)u$$

+
$$(6\theta_1\theta_2a_{0,3} + 2\tau_1\tau_2a_{2,1} + 2\tau_2\theta_1a_{1,2} + 2a_{1,2}\tau_1\theta_2)v].$$
 (11)

where  $\phi_0$  is a constant term. The relation between the estimated frequencies and the polynomial coefficients is as in (8), where the matrix  $\mathbf{\Theta} \in \mathbb{R}^{2L \times 4}$  is now

$$\boldsymbol{\Theta} = \begin{pmatrix} 6\tau_1^{(1)}\tau_2^{(1)} & 2\rho_{12}^{(1)} & 2\theta_1^{(1)}\theta_2^{(1)} & 0\\ 0 & 2\tau_1^{(1)}\tau_2^{(1)} & 2\rho_{12}^{(1)} & 6\theta_1^{(1)}\theta_2^{(1)}\\ \vdots & \vdots & \vdots & \vdots\\ 6\tau_1^{(L)}\tau_2^{(L)} & 2\rho_{12}^{(L)} & 2\theta_1^{(L)}\theta_2^{(L)} & 0\\ 0 & 2\tau_1^{(L)}\tau_2^{(L)} & 2\rho_{12}^{(L)} & 6\theta_1^{(L)}\theta_2^{(L)} \end{pmatrix}$$
(12)

with  $\rho_{12}^{(i)} := \tau_2^{(i)} \theta_1^{(i)} + \tau_1^{(i)} \theta_2^{(i)}$ , whereas the phase coefficient vector is  $\boldsymbol{a} := (a_{3,0}, a_{2,1}, a_{1,2}, a_{0,3})^T$ . Again, the phase coefficients of order 3 can be recovered provided that a number  $L \ge 2$  of sets of lags are used and the lags are chosen so that the matrix  $\boldsymbol{\Theta}$  is full-column rank. Once the third order phase coefficients have been estimated, the overall third order phase term can be removed from the received signal. If the estimation is correct, the resulting signal is a second order PPS, whose phase coefficients can be estimated using the second order ml-HAF, and so on.

#### 3. 2D PRODUCT HIGH ORDER AMBIGUITY FUNCTION

The previous analysis shows that the ml-HAF is a valid tool to process single component PPS's. However, if the signal is composed by the superposition of multiple PPS's having different phase modulations, the approach based on the ml-HAF is no longer valid, as the multiplications present in the computation of the ml-HIM create cross products that can alter the estimation of the phase coefficients substantially. A robust approach for analyzing 2D multi-component PPS's can be achieved by generalizing the Product High order Ambiguity Function (PHAF), introduced in [2] for one-dimensional signals, to the two-dimensional case.

The computation of the ml-HIM of multicomponent PPS's gives rise to auto-terms (products of each component with itself) and cross-terms (products between different components). The main idea underlying the 1D PHAF is to enhance the peaks of the autoterms with respect to the cross terms by multiplying the ml-HAF obtained with different sets of lags, after proper rescaling in the frequency domain. The scope of the scaling operation is to align the peaks of the auto-terms. This idea can be extended to the 2D case, provided that the lags are properly chosen. An example of application to 2D PPS's may be helpful to clarify the proposed approach.

Let us consider the superposition of K 2D PPS's of order M:

$$s[u,v] = \sum_{k=1}^{K} A_k e^{j2\pi \sum_{m=0}^{M} \sum_{l=0}^{m} a_{m-l,l}^{(k)} u^{m-l} v^l}.$$
 (13)

<sup>&</sup>lt;sup>1</sup>In most applications, the zero order coefficient is not important or it can be incorporated in the amplitude A, which becomes complex.

Our goal is now to estimate the phase coefficients  $a_{m,l}^{(k)}$  for each signal component. From (7), we notice that the estimated frequency, for an arbitrary choice of the lags, is not proportional to a single phase coefficients, so that scaling is not always possible. However, if we choose the two sets of lags as follows:  $(\tau_1^{(1)} = \tau_1, \theta_1^{(1)} = 0)$  and  $(\tau_1^{(1)} = 0, \theta_1^{(1)} = \theta_1)$ , we can generalize the definition of the PHAF to the 2D case as follows. For the sake of clarity, we illustrate the approach for the cases M = 2 and 3.

M = 2

In this case, we define two PHAF's, corresponding to two choices of the lags, as follows

$$P_2(f,\nu;\boldsymbol{\tau}_1,\mathbf{0}) = \prod_{l=1}^L S_2\left(\frac{\tau_1^{(l)}f}{\tau_1^{(1)}},\frac{\tau_1^{(l)}\nu}{\tau_1^{(1)}};\tau_1^{(l)},0\right)$$
(14)

and

$$P_2(f,\nu;\mathbf{0},\boldsymbol{\theta}_1) = \prod_{l=1}^{L} S_2\left(\frac{\theta_1^{(l)}f}{\theta_1^{(1)}}, \frac{\theta_1^{(l)}\nu}{\theta_1^{(1)}}; 0, \theta_1^{(l)}\right)$$
(15)

It is straightforward to check that all the ml-HAF's appearing in the definition of  $P_2(f, \nu; \tau_1, \mathbf{0})$  exhibit peaks of the auto-terms lying always in the point of coordinates  $(2\tau_1^{(1)}a_{2,0}^{(k)}, \tau_1^{(1)}a_{1,1}^{(k)})$ , irrespective of the lag set. Hence the product enhances the auto-terms. Similarly, the all the ml-HAF's appearing in the definition of  $P_2(f,\nu;\mathbf{0},\boldsymbol{\theta}_1)$  exhibit peaks of the auto-terms in the points of coordinates  $(\theta_1^{(1)}a_{1,1}^{(k)}, 2\theta_{1,2}^{(1)}a_{0,2}^{(k)})$ . Hence, again, the products enhance the auto-terms. The second order phase coefficients of each component can then be estimated as follows:

$$\hat{a}_{2,0}^{(k)} = \arg\max_{f} \left\{ |P_2(f,\nu;\boldsymbol{\tau}_1,\mathbf{0})| \right\} \frac{1}{2\tau_1^{(1)}}$$
(16)

$$\hat{a}_{1,1}^{(k)} = \arg\max_{\nu} \left\{ |P_2(f,\nu;\boldsymbol{\tau}_1, \mathbf{0})| \right\} \frac{1}{\tau_1^{(1)}}$$
(17)

$$\hat{a}_{0,2}^{(k)} = \arg\max_{\nu} \left\{ |P_2(f,\nu;\mathbf{0},\boldsymbol{\theta}_1)| \right\} \frac{1}{2\theta_1^{(1)}}$$
(18)

Actually,  $a_{1,1}$  can also be recovered from  $P_2(f,\nu;\mathbf{0},\boldsymbol{\theta}_1)$ , so that two estimates of this coefficient are available and can be combined to reduce the estimation error variance.

M = 3

Extending the previous approach to multicomponent cubic phase signals, multiple 2D PHAF's can be defined as follows

$$P_{3}(f,\nu;\boldsymbol{\tau}_{2},\mathbf{0}) = \prod_{l=1}^{L} S_{3}\left(\frac{\tau_{1}^{(l)}\tau_{2}^{(l)}f}{\tau_{1}^{(1)}\tau_{2}^{(1)}}, \frac{\tau_{1}^{(l)}\tau_{2}^{(l)}\nu}{\tau_{1}^{(1)}\tau_{2}^{(1)}}; \boldsymbol{\tau}_{2}, \mathbf{0}\right)$$
(19)

$$P_{3}(f,\nu;\mathbf{0},\boldsymbol{\theta}_{2}) = \prod_{l=1}^{L} S_{3} \left( \frac{\theta_{1}^{(l)}\theta_{2}^{(l)}f}{\theta_{1}^{(1)}\theta_{2}^{(1)}}, \frac{\theta_{1}^{(l)}\theta_{2}^{(l)}\nu}{\theta_{1}^{(1)}\theta_{2}^{(1)}}; \mathbf{0}, \boldsymbol{\theta}_{2} \right)$$
(20)

The third order phase coefficients of each component can then be estimated as follows:

$$\hat{a}_{3,0}^{(k)} = \arg\max_{f} \left\{ |P_3(f,\nu;\boldsymbol{\tau}_2,\mathbf{0})| \right\} \frac{1}{6\tau_1^{(1)}\tau_2^{(1)}}$$
(21)

$$\hat{a}_{2,1}^{(k)} = \arg\max_{\nu} \left\{ |P_3(f,\nu;\boldsymbol{\tau}_2,\mathbf{0})| \right\} \frac{1}{2\tau_1^{(1)}\tau_2^{(1)}}$$
(22)

$$\hat{a}_{1,2}^{(k)} = \arg\max_{\nu} \left\{ |P_3(f,\nu;\mathbf{0},\boldsymbol{\theta}_2)| \right\} \frac{1}{2\theta_1^{(1)}\theta_2^{(1)}}$$
(23)



Fig. 1. HAF of a signal composed of three PPS's of order 3.



Fig. 2. PHAF of the same signal analyzed in Fig. 1.

$$S_{0,3}^{(k)} = \arg\max_{\nu} \left\{ |P_3(f,\nu;\mathbf{0},\boldsymbol{\theta}_2)| \right\} \frac{1}{6\theta_1^{(1)}\theta_2^{(1)}}$$
(24)

A numerical example is useful to assess the ability of the PHAF to enhance the auto terms. To this purpose, in Fig. 1 we report the third order HAF of a signal composed of three superimposed PPS's of degree M = 3. We can clearly see that the cross terms are comparable with the auto terms. In Fig. 2 we report the PHAF of the same signal analyzed in Fig. 1. We can clearly see how the PHAF is effective in reducing the cross terms with respect to the auto terms.

# 4. APPLICATION TO MOVING TARGET SAR IMAGING

In this section, we apply the 2D-PHAF to the imaging of moving targets observed by a SAR. The 2D-PHAF in this case is used to compensate the phase modulation *and* the range cell migration induced by the radar-target motion *jointly*. The target is supposed to be a rigid body moving at constant velocity during the observation interval. Furthermore, we also suppose that the target is constituted by a set of dominant scatterers. We consider a spaceborne SAR system flying at an altitute z = H, moving at constant velocity v along the y-axis, and illuminating the scene with an off-nadir angle  $\alpha$  and zero squint angle. The target is supposed to move over the plane of



Fig. 3. SAR image of a moving target using Chirp Scaling Algorithm matched to the fixed ground.

equation z = 0, with velocity vector having components  $(v_x, v_y)$ . For each position of the satellite, the radar-target distance is:

$$R(t_a) = \sqrt{\left(H\tan\alpha + v_x t_a\right)^2 + \left(v - v_y\right)^2 t_a^2 + H^2} \qquad (25)$$

where  $t_a = u \cdot PRT$  is the slow time (or azimuth time) and PRT is the Pulse Repetition Time. Since the shifts of the radar position are typically much smaller than the radar-target distance  $R_0$  at time 0, we can approximate  $R(t_a)$  with its Taylor series expansion:

$$R(t_{a}) \cong R_{0} + v_{x} \sin \alpha t_{a} + \frac{v_{x}^{2} \cos^{2} \alpha + (v - v_{y})^{2}}{2R_{0}} t_{a}^{2} + \frac{v_{x} \sin \alpha \left[v_{x}^{2} \cos^{2} \alpha + (v - v_{y})^{2}\right]}{2R_{0}^{2}} t_{a}^{3} + O(t_{a}^{4})$$

$$(26)$$

The transmitted signal is a chirp signal with sweep rate  $\mu$  and carrier frequency  $f_0$ . The antenna beam pattern, in the azimuth plane, is indicated as  $G(\phi)$ . Using the usual start-stop approximation, the echo received from a pointlike moving target at (fast) time  $t_r$  and (slow) time  $t_a$  is

$$s(t_a, t_r) = A_0 \operatorname{rect}_T \left( t_r - \frac{2R(t_a)}{c} \right) G(v(t_a - t_0)/R_0) e^{j\Psi(t_a, t_r)}$$
(27)

with

$$\Psi(t_a, t_r) = -\frac{4\pi}{\lambda} R(t_a) + \pi \mu \left[ t_r - \frac{2R(t_a)}{c} \right]^2 \qquad (28)$$

where c is the speed of light,  $\lambda = c/f_0$  is the radar wavelength,  $t_0$  is a reference time associated to the point-like target,  $A_0$  is the amplitude of the echo. This expression shows that the echo undergoes a phase modulation and a time-varying delay that depend on the relative radar-target motion, which is unknown. To produce a focused image of the target, it is necessary to compensate both effects. To this end, we apply a Fourier Transform along the time axis  $t_r$ . This converts the time-varying delay into a phase modulation in the transformed domain. Using the stationary phase principle [9], the signal in the range transformed-azimuth time domain has a phase:

$$\Psi(t_a, f_{t_r}) \cong -\frac{4\pi}{c} (f_0 + f_{t_r}) R(t_a) - \frac{\pi}{\mu} f_{t_r}^2.$$
(29)



**Fig. 4**. SAR image of the same moving target as in Fig.3 after motion compensation via 2D-PHAF.

Substituting  $R(t_a)$  in this expression with its Taylor series expansion (26) and neglecting the terms of order greater than 3, we obtain the following approximated expression

$$\Psi(t_a, f_{t_r}) \cong -\frac{4\pi}{\lambda} R_0 - \frac{4\pi}{c} b_{11} \left(f_0 + f_{t_r}\right) t_a - \frac{4\pi}{c} R_0 f_{t_r} + \frac{4\pi}{c} b_{21} \left(f_0 + f_{t_r}\right) t_a^2 - \frac{\pi}{\mu} f_{t_r}^2$$
(30)

with  $b_{11} = v_x \sin\alpha$  and  $b_{21} = (v_x^2 \cos^2 \alpha + (v - v_y)^2)/(2R_0)$ . Sampling the signal at  $t_a = uPRT$  and  $f_{t_r} = vF_s$ , where  $F_s$  is the sampling frequency, the phase in the transformed domain can be written as

$$\Phi[u,v] := \Psi(uPRT, vF_s) \cong 2\pi(a_{00} + a_{01}v + a_{10}u + a_{11}uv + a_{02}v^2 + a_{20}u^2 + a_{21}u^2v),$$
(31)

where  $a_{0,0} = -\frac{2}{\lambda}R_0$ ,  $a_{0,1} = -\frac{2}{c}R_0F_s$ ,  $a_{1,0} = -\frac{2}{c}b_{11}f_0PRT$ ,  $a_{1,1} = -\frac{2}{c}b_{11}F_sPRT$ ,  $a_{2,0} = -\frac{2}{c}b_{21}f_0PRT^2$ ,  $a_{0,2} = -\frac{1}{2\mu}F_s^2$   $a_{2,1} = -\frac{2}{c}b_{21}F_sPRT^2$ . To produce a focused image, we apply the 2D PHAF of order M = 3, described in the previous section to estimate and then compensate the phase behavior given in (31).

In the following, we compare the synthetic images obtained with chirp scaling (Fig. 3) and with the proposed 2D PHAF (Fig. 4). The system parameters are chosen as follows: the satellite velocity is 7500m/s; the off-nadir angle is  $20^{\circ}$ , the (nominal) azimuth and range resolutions are 2m; the target velocity is 20m/s and the target moves at a  $45^{\circ}$  angle with respect to the satellite trajectory; the number of dominant target scatterers is 11. We can clearly see the effectiveness of the 2D-PHAF-based approach in compensating both range migration and phase modulation and producing a well focused target image.

In summary, the extension of the PHAF to the 2D domain provides a new powerful tool to process 2D multi-component PPS's. The generalization to the 2D case offers additional degrees of freedom in selecting the lags to enhance the PHAF capabilities to track multiple components. Moving target imaging with SAR constitutes an interesting application of the proposed 2D-PHAF.

## 5. REFERENCES

- S. Peleg, B. Porat, "Estimation and classification of polynomial-phase signals," *IEEE Trans. on Information Theory*, pp. 422–430, Mar. 1991.
- [2] S. Barbarossa, A. Scaglione, G.B. Giannakis, "Product High-Order Ambiguity Function for Multicomponent Polynomial-Phase Signal Modeling," *IEEE Trans. on Signal Process.*, vol. 46, no. 3, pp. 691–708, Mar. 1998.
- [3] D. S. Pham, A. M. Zoubir, "Analysis of Multicomponent Polynomial Phase Signals," *IEEE Trans. on Signal Process.*, vol. 55, no. 1, pp. 56– 65, Jan. 2007.
- [4] S. Barbarossa, A. Scaglione, "Autofocusing of SAR Images Based on the Product High Order Ambiguity Function," *IEE Proceedings, Radar Sonar Navigation*, pp. 269–273, Oct. 1998.
- [5] J. R. Fienup, "Detecting Moving Targets in SAR Imagery by Focusing," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 37, no. 3, pp. 794–809, July 2001.
- [6] P.A.C. Marques, J.M.B. Dias, "Velocity Estimation of Fast Moving Tagets Using a Single SAR Sensor," *IEEE Trans. on Aerospace and Electronic Systems*, pp. 75–89, Jan. 2005.
- [7] F. Berizzi, G. Corsini, "Autofocusing of Inverse Synthetic Aperture Radar Images Using Contrast Optimization," *IEEE Trans. on Aerospace* and Electronic Systems, pp. 1185–1191, July 1996.
- [8] T. K. Sjogren, V. T. Vu, M. I. Pettersson, A. Gustavsson, L. M. H. Ulander, "Moving Target Relative Speed Estimation and Refocusing in Synthetic Aperture Radar Images," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 48, no. 3, pp. 2426–2436, July 2012.
- [9] A. Papoulis, Signal Analysis, McGraw-Hill, May 1977.