OPTIMAL POWER ALLOCATION FOR AN ENERGY HARVESTING ESTIMATION SYSTEM

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ABSTRACT

Optimal transmit power allocation strategies are proposed for an energy harvesting estimation system, where energy can be harvested from the environment and buffered in a battery for future use. With the aim of minimizing the mean squared error at the receiver, two types of side information (SI) available to the transmitter are considered: causal SI (energy harvested in the past) and non-causal SI (energy harvested in the past, present and future). For the case where non-causal SI is available and battery storage is unlimited, it is shown that the optimal power allocation can be attained by a simple water-filling-like procedure, where the water level follows a non-decreasing staircase function. Dynamic programming is used to optimize the allocation policy when causal SI is available. The issue of unknown transmit power at the receiver is also addressed.

Index Terms— Energy harvesting, estimation, dynamic programming, convex optimization.

1. INTRODUCTION

Recent development in hardware design has empowered many wireless networks to support themselves by harvesting energy from nature through various sources such as solar cells, vibration absorption devices, among others, and store excess energy for future use. Unlike traditional battery-powered systems, where transmission is often subject to a constant power constraint, the energy available to an energy harvesting system typically fluctuates in time and is often modeled as a random process. This incurs additional difficulty in the system design, and in particular, how to allocate powers across time for improved system performance.

There has been recent works on estimation and communication systems with energy harvesters as the power source. Optimal communication and estimation strategies in a remote estimation problem with energy harvesting sensors are studied in [1]. In [2], optimal energy management schemes for energy harvesting communication systems operating in fading channels are developed. Throughput optimization via power allocation for a communication system with energy harvesting constraints is studied in [3]. Invariably, a key ingredient among those works is the use of dynamic programming [4] for power allocation when transmitter does not have non-causal information about energy available in the future.

In this work, we consider a remote estimation system with the transmitter powered by energy harvesting devices. Both the signals and the additive noises are assumed to be independent and identically distributed (i.i.d.) Gaussian sequences. The objective is to minimize the mean squared error (MSE) at the receiver (estimator), averaged over the entire sequence. The system is illustrated in Fig. 1. Notice that under a constant power constraint, uncoded transmission is known to be an optimal strategy [5]. While it is not clear whether this is still true with energy harvesting constraints, we apply the same uncoded transmission scheme due to its simplicity and minimum delay compared with a coded approach [6].

The central question we try to answer in the present work is how to determine transmit power across time under various assumptions as to what is known at the transmitter and/or receiver: whether the transmitter has non-causal side information (SI) of future harvested energy, and whether the receiver knows the transmit power *a priori*. The clairvoyant case, namely the transmitter has non-causal knowledge of future harvested energy and the receiver knows the exact transmit power, serves as a benchmark for performance comparison. We will then replace these idealized assumptions with more realistic ones and provide solutions to power allocation under each scenario. Performance evaluation will be conducted to identify conditions under which the optimal performance in terms of minimum mean square error (MMSE) with realistic assumptions is close to that of the clairvoyant case, therefore providing guidance on system design.

The rest of the paper is organized as follows. Section 2 describes the system model. In Section 3, we consider the optimal power allocation policy when the transmitter has non-causal SI. The case where causal SI is available to the trans-

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mitter is studied in Section 4. Section 5 includes performance comparison through numerical simulation and Section 6 concludes the paper.

2. SYSTEM MODEL

While the transmission of input sequence occurs 'instantaneously' using uncoded transmission (i.e., amplify and forward), we assume that the energy harvesting occurs in blocks with each block consisting of N input variables. Transmitter power is also allocated in a block-wise fashion: within each block the transmitter uses a given average transmit power, and power allocation occurs across different blocks. Let the entire sequence consist of K blocks, indexed using $j = 1, 2, \cdots, K$. The sequence within each block is indexed using $i = 1, 2, \cdots, N$. Thus the random sequence to be estimated can be denoted by S_i^j. Let S_i^j be i.i.d. Gaussian with mean zero and variance σ_S^2. Uncoded transmission [5] is used, hence the transmitted signal corresponding to S_i^j is

$$X_i^j = \sqrt{\frac{P^j}{\sigma_S^2}} S_i^j$$

where P^j is the average power constraint in block j. The transmitted signal goes through an AWGN channel characterized by

$$Y_i^j = X_i^j + Z_i^j$$

where Z_i^j is an i.i.d. Gaussian noise sequence with mean zero and variance σ_Z^2 and is independent of the transmitted signal. The receiver output $\hat{S}_i^j$ denotes the estimate of the transmitted signal S_i^j.

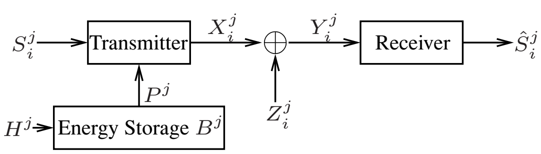

Fig. 1. System model

The blockwise energy harvesting and the transmission scheme can be modeled by the following parameters:

- P^j denotes the average amount of energy to be expended in each slot of block j. To ease our notation, we have implicitly assumed that the energy is normalized by the block length, thus P^j represents both the average power at block j as well as the energy expended at block j.
- H^j denotes the average amount of energy harvested in each slot of block j. We assume that energy is replenished at the end of each block, i.e., H^j is not available until the beginning of block $j+1$.
- B^j indicates the energy storage level at the beginning of block j. It varies linearly as long as the storage limit B_{max} is not exceeded such that

$$B^{j+1} = \min\{B^j - P^j + H^j, B_{\text{max}}\}.$$

The energy harvesting constraint is imposed on each block in the sense that the expended power can not exceed that available at the current block, i.e.,

$$P^j \leq B^j$$

for any $j = 1, \cdots, K$. The block length N is assumed to be sufficiently large such that the average power constraint is satisfied.

We aim to minimize the MSE averaged over the entire sequence

$$D = \frac{1}{K}\sum_{j=1}^{K}\frac{1}{N}\sum_{i=1}^{N}\left(\hat{S}_i^j - S_i^j\right)^2. \tag{1}$$

Apparently, D depends on how transmit power is allocated across blocks and the estimator used at the receiver (i.e., whether the receiver knows the transmit power).

3. NON-CAUSAL SI

Non-causal SI is said to be available if the transmitter has prior knowledge of the harvested power $[H^0 H^1 \cdots H^{K-1}]$ before transmission begins. To develop more insight on the structure of the optimal energy allocation scheme, we assume in this section that the battery storage is unlimited, i.e., $B_{\text{max}} \to \infty$.

3.1. Transmit Power Known to Receiver

If the receiver knows the transmit power, it can construct the optimal MMSE estimator

$$\hat{S}_i^j = \frac{\sqrt{P^j}\sigma_S Y_i^j}{\sigma_Z^2 + P^j}$$

and the corresponding MSE is

$$\mathcal{E}[D] = \frac{1}{K}\sum_{j=1}^{K}\frac{\sigma_S^2\sigma_Z^2}{\sigma_Z^2 + P^j} \triangleq \frac{1}{K}\sum_{j=1}^{K} h_1(P^j). \tag{2}$$

Then the optimal power allocation can be obtained by solving the following optimization problem

$$\begin{aligned} \text{minimize} \quad & \mathcal{E}[D] \\ \text{subject to} \quad & P^j \geq 0 \\ & \sum_{k=1}^{j} P^k - \sum_{k=0}^{j-1} H^k \leq 0 \end{aligned}$$

for $j = 1, \cdots, K$. It is easy to verify that the above problem is convex and satisfies the Slater's condition [7]. Hence, the Lagrange duality method can be used to obtain the global optimum. The Lagrangian associated with this problem is

$$\mathcal{L} = \frac{1}{K}\sum_{j=1}^{K}\frac{\sigma_S^2\sigma_Z^2}{\sigma_Z^2 + P^j} - \sum_{j=1}^{K}\mu_j P^j + \sum_{j=1}^{K}\lambda_j\left(\sum_{k=1}^{j} P^k - \sum_{k=0}^{j-1} H^k\right).$$

Using the Karush-Kuhn-Tucker (KKT) conditions

$$\begin{aligned} P^j &\geq 0 \\ \mu_j &\geq 0 \\ \mu_j P^j &= 0 \end{aligned}$$

$$\begin{aligned}\sum_{k=1}^{j} P^k - \sum_{k=0}^{j-1} H^k &\leq 0 \\ \lambda_i &\geq 0 \qquad (3)\\ \lambda_i \left(\sum_{k=1}^{j} P^k - \sum_{k=0}^{j-1} H^k \right) &= 0 \qquad (4)\\ \frac{\partial \mathcal{L}}{\partial P^j} = \frac{-\sigma_S^2 \sigma_Z^2}{(\sigma_Z^2 + P^j)^2} - \mu_j + \sum_{k=j}^{K} \lambda_k &= 0\end{aligned}$$

we obtain the optimal solution as

$$P^j = \max\left(\frac{\sigma_S \sigma_Z}{\sqrt{\sum_{k=j}^{K} \lambda_k}} - \sigma_Z^2, 0 \right). \qquad (5)$$

Equation (5) can be interpreted as a *staircase water-filling* procedure with a flat bottom of height σ_Z^2 [3]. Denote the water level at block j to be

$$\nu_j = \frac{\sigma_S \sigma_Z}{\sqrt{\sum_{k=j}^{K} \lambda_k}}. \qquad (6)$$

Comparing (5) with the solution to the standard water-filling problem (Example 5.2 in [7]), we see that the optimal water level is not a constant, but changes over the blocks. We define the following for convenience:

- If the water level changes after block t, i.e., $\nu_t \neq \nu_{t+1}$, then block t is defined to be a *transition block*. Block K is also a transition block, as we define $\nu_{K+1} = \infty$. Let $\mathcal{S} = \{t_1, t_2, \cdots, t_{|S|}\}$ be a sequence that contains all the indices of the transition blocks such that $t_{|S|} = K$.
- If $t_i, t_{i+1} \in \mathcal{S}$, then the blocks indexed by $t_i + 1, \cdots, t_{i+1}$ are called the ith *transition interval*.

The optimal power allocation satisfies the following properties:

- The water level is non-decreasing over the blocks. This can be easily seen from (3) and (6). In fact, the water level is a staircase-like function.
- The battery storage is empty at the end of a transition block, i.e., $B^{j+1} = 0$ if $j \in \mathcal{S}$. This can be observed from the definition of the water level (6) and the complimentary slackness condition (4).
- The optimal power allocation can be obtained by conventional water-filling for each of the transition intervals subject to the sum power constraint of the total energy harvested within this transition interval. This follows naturally from the previous property that energy harvested in any transition interval is depleted.

A backward-search procedure (Algorithm 2 in [3]) can be implemented to find the optimal transition blocks that solve the problem.

3.2. Transmit Power Not Known to Receiver

If the transmit power is not readily available, the receiver uses an estimate of the transmit power to construct a MMSE estimator. Choose the following $\hat{P}^j$ to be the estimate of the transmit power in block j and $\tilde{D}$ to be the corresponding MSE obtained using $\hat{P}^j$ as the transmit power:

$$\begin{aligned}\hat{P}^j &= \max\left(\frac{1}{N}\sum_{i=1}^{N} (Y_i^j)^2 - \sigma_Z^2, 0 \right) \qquad (7)\\ \tilde{D} &= \frac{1}{K}\sum_{j=1}^{K} \frac{\left(\hat{P}^j \sigma_Z^2 + \left(\sigma_Z^2 + \hat{P}^j - \sqrt{\hat{P}^j P^j} \right)^2 \right) \sigma_S^2}{(\sigma_Z^2 + \hat{P}^j)^2} \\ &\triangleq \frac{\sigma_S^2}{K}\sum_{j=1}^{K} f(\hat{P}^j).\end{aligned}$$

Then we form the optimization problem with $\mathcal{E}_{\tilde{P}^j}[\tilde{D}]$ as the objective function

$$\begin{aligned}\text{minimize} \quad & \mathcal{E}_{\tilde{P}^j}[\tilde{D}] \qquad (8)\\ \text{subject to} \quad & P^j \geq 0 \\ & \sum_{k=1}^{j} P^k - \sum_{k=0}^{j-1} H^k \leq 0.\end{aligned}$$

Since it is difficult to evaluate $\mathcal{E}_{\tilde{P}^j}[\tilde{D}]$ directly, we consider its approximation which is asymptotically accurate as N becomes large. For simplicity, we also replace $\hat{P}^j$ in (7) using the following estimate

$$\tilde{P}^j = \frac{1}{N}\sum_{i=1}^{N} (Y_i^j)^2 - \sigma_Z^2.$$

Define Δ such that

$$\tilde{P}^j = P^j + \Delta.$$

It is straightforward to verify that

$$\begin{aligned}\mathcal{E}[\Delta] &= 0 \\ \mathcal{E}[\Delta^2] &= \frac{2}{N}(\sigma_Z^2 + P^j)^2.\end{aligned}$$

Then it follows from Tailor approximation that

$$\begin{aligned}\mathcal{E}_{\tilde{P}^j}[\tilde{D}] &\doteq \frac{\sigma_S^2}{K}\sum_{j=1}^{K} \mathcal{E}\left[f(P^j) + f'(P^j)\Delta + \frac{f''(P^j)}{2}\Delta^2 \right] \\ &= \frac{\sigma_S^2}{K}\sum_{j=1}^{K} \left(f(P^j) + f'(P^j)\mathcal{E}[\Delta] + \frac{f''(P^j)}{2}\mathcal{E}[\Delta^2] \right) \\ &= \frac{\sigma_S^2}{K}\sum_{j=1}^{K} \left(\frac{\sigma_Z^2}{\sigma_Z^2 + P^j} + \frac{(P^j)^2 + 14P^j\sigma_Z^2 + \sigma_Z^4}{2NP^j(\sigma_Z^2 + P^j)} \right) \qquad (9)\\ &\triangleq \frac{1}{K}\sum_{j=1}^{K} h_2(P^j). \qquad (10)\end{aligned}$$

where $f'(\cdot)$ and $f''(\cdot)$ denote the first and second derivative of $f(\cdot)$. It is straightforward to verify that $h_2(\cdot)$ is convex.

Substituting the objective function by (9), we obtain from the KKT conditions that

$$\sum_{k=j}^{K} \lambda_k = \frac{\sigma_S^2 \sigma_Z^2}{K} \frac{(2N + 13)(P^j)^2 + 2P^j\sigma_Z^2 + \sigma_Z^4}{2N(P^j)^2(\sigma_Z^2 + P^j)^2} \triangleq g(P^j).$$

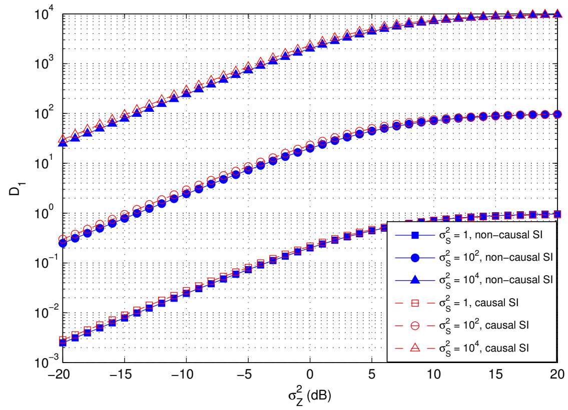

Fig. 2. D_1 vs σ_Z^2 with, $K = 4$ and H^j i.i.d. uniformly distributed on $\{1, 2, \cdots, 9\}$ for $j = 0, 1, 2, 3$

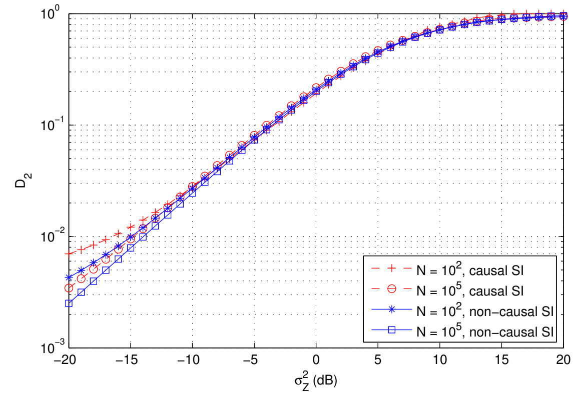

Fig. 3. D_2 vs σ_Z^2 with $\sigma_S^2 = 1$, $K = 4$ and H^j i.i.d. uniformly distributed on $\{1, 2, \cdots, 9\}$ for $j = 0, 1, 2, 3$

It is easily seen that the derivative of both $g(\cdot)$ and its inverse $g^{-1}(\cdot)$ are negative, which means P^j is decreasing in $\sum_{k=j}^{K} \lambda_k$. Comparing with (5) where P^j is also a decreasing function of $\sum_{k=j}^{K} \lambda_k$, we observe that the optimal power allocation can, again, be obtained through staircase water-filling, but with a flat bottom of height 0.

4. CAUSAL SI

In this section we assume H^j follows the first-order stationary Markov model over j and consider the case of causal SI available at the transmitter. Therefore, the transmitter only knows $[H^0 H^1 \cdots H^{j-1}]$ at the beginning of block j. For finite and arbitrary B_{max}, we construct the following problem to find the optimal power allocation

$$\begin{aligned} \text{minimize} \quad & \frac{1}{K}\sum_{j=1}^{K}\frac{1}{N}\sum_{i=1}^{N}\mathcal{E}[(\hat{S}_i^j - S_i^j)^2] \triangleq \frac{1}{K}\sum_{j=1}^{K} h(P^j) \\ \text{subject to} \quad & 0 \le P^j \le B^j \\ & B^j = \min\{B^{j-1} + H^{j-1} - P^{j-1}, B_{\text{max}}\} \end{aligned}$$

for $j = 1, \cdots, K$, where $h(\cdot) = h_1(\cdot)$ in (2) if the transmit power is known to the receiver; otherwise $h(\cdot) = h_2(\cdot)$ in (10).

In general, this problem cannot be solved by independently minimizing $h(P^j)$ due to energy harvesting constraints. Instead, the optimal power allocation policy can be obtained by recursively computing $J_K, \cdots, J_1$ based on Bellman's equation, where

$$\begin{aligned} J_K(H, B) &= \min_{0 \le P \le B} h(P) = h(B) \\ J_j(H, B) &= \min_{0 \le P \le B} \left(h(P) + \bar{J}_{j+1}(H, B - P)\right) \end{aligned}$$

for $j = 1, \cdots, K - 1$ and

$$\bar{J}_{j+1}(H, x) = \mathcal{E}_{\tilde{H}}\left[J_{j+1}\left(\tilde{H}, \min\left(B_{\text{max}}, x + \tilde{H}\right)\right) | H\right]. \quad (11)$$

In (11), $\bar{J}_{j+1}$ indicates the expected MSE accumulated from block $[j+1, \cdots, K]$. As expected, the solution ensures that all power is depleted at block K whereas the power allocated to any intermediate block is determined by the past harvested energy and power allocations.

5. NUMERICAL RESULTS

Denote D_1 and D_2 to be the distortions (defined in (1)) achieved with and without knowledge of the transmit power respectively. In Fig. 2, we fix $K = 4$, $\sigma_S^2 \in \{10, 10^2, 10^4\}$ and vary σ_Z^2 from -20dB to 20dB to examine the corresponding D_1. The harvested energy H^j is assumed to be i.i.d. uniformly distributed on $\{1, 2, \cdots, 9\}$, for $j = 0, 1, 2, 3$. It can be observed that the distortion with both causal SI and non-causal SI increases with increasing noise variance. The distortion with causal SI always dominates that with non-causal SI though the difference between the two is very small under this setup.

Fig. 3 illustrates the performance when the knowledge of transmit power is not available to the receiver. The settings of K and H^j are the same as in Fig. 2. The signal variance σ_S^2 is fixed to be 1. It can be seen that D_2 increases with increasing σ_Z^2. Here we compare two different N values because the number of time slots in a block has a significant impact on the accuracy of transmit power estimation and therefore on the overall performance. As expected, smaller N results in larger distortion.

6. CONCLUSION

This paper studied the problem of minimizing the mean squared error of an energy harvesting estimation system via power allocation over a finite number of blocks. Optimal power allocation strategies are developed for cases where either non-causal SI or causal SI is available at the transmitter, with or without transmit power knowledge at the receiver.

7. REFERENCES

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