ADMOT: COMPRESSIVE SENSING TECHNIQUES FOR CHANNEL MONITORING IN MULTIPLE ACCESS NETWORKS

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ABSTRACT

This paper studies the overhead for channel gain monitoring in wireless networks with time division multiple access. We first investigate the scenario in which a receiver needs to track the channel gains with respect to multiple transmitters. Suppose that there are n transmitters, and no more than k channels suffer significant variations since the last round. We prove that "$\Theta(k \log(n/k))$ time slots" is the *minimum* overhead needed to catch up with the k varied channels. We propose a *novel* channel-gain monitoring scheme named ADMOT. ADMOT leverages recent advances in compressive sensing in signal processing and interference processing in wireless communication, to enable the receiver to estimate all n channels in a *reliable* and *computationally efficient* manner within $\mathcal{O}(k \log(n/k))$ time slots. To our best knowledge, all previous channel-tracking schemes require $\Theta(n)$ time slots regardless of k.
Keywords: Wireless Network, Channel Gain Estimation, Compressive Sensing.

1. INTRODUCTION

The knowledge of channel gains is often needed in the design of high performance communication schemes [1–6]. In practice, the channel gains vary with time. Tracking and estimating channel gains of wireless channels is therefore fundamentally important [7–13]. An issue of interest is how to reduce the overhead of channel-gain estimation. On the one hand, if between two rounds of channel-gain estimation, the channels have varied significantly, then communication reliability will be jeopardized [4, 5, 10]. On the other hand, if the frequency of channel-gain estimation is high, the overhead will also be high [1, 7, 14]. Our approach is predicated on reducing the overhead in each round, while maintaining high accuracy.

We consider the case in which a receiver needs to estimate the channel gains from n transmitters [1, 2], and we assume the wireless channel conditions are more static. To achieve reliable bit-error-rate (BER), the frequency of estimation should be high enough [1]. Then it is likely that only a few of the n channels have suffered appreciable changes since the last estimation. We make use of the techniques of compressive sensing and interference signal processing to reduce the time needed to perform the estimation in each round. We propose the following question: Suppose that in the current round, there are at most $k \leq n$ channels suffering from appreciable channel gain variations. Given a target reliability for channel-gain estimation, can we reduce the overhead for probing all the channels? We prove that the minimum number time slots needed for estimation is $\Theta(k \log((n+1)/k))$. Then we propose a scheme named ADMOT, which utilizes the compressive sensing technique to probe a large-scale of channels simultaneously. We also show that ADMOT uses $\mathcal{O}(k \log(n/k))$ time slots for the probing. (Note that in each time slot, every transmitter transmits one symbol. Thus, one time slot is also one *symbol duration*.)

1.1. Illustrating Example

For illustration, let us consider the uplink of a cellular network with one receiver R and three transmitting nodes S_1, S_2, and S_3. We assume that this cellular network makes use of TDMA channel access. The three channels (S_1, R), (S_2, R) and (S_3, R) need to be estimated before data transmission. We assume all the initial channel gains of the three channels be 1, and suppose one of the channel gains changes to x in the current time. The goal of monitoring is to identify the updated channel and the value of x. A simple scheme is to schedule training data transmissions in different time slots, as shown in Figure 1. In time slots 1, 2, and 3, sender S_i, $i = 1, 2, 3$, sends training data 1 to node R, respectively, so that R can estimate the channel gain of (S_i, R). Thus, altogether three time slots are needed. However, us-

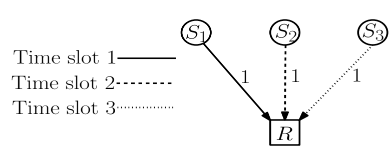

Figure 1: The monitoring scheme based on scheduling. The "solid-line", "dashed-line" and "dotted-line" are for the transmission of time slots 1, 2 and 3, respectively.

ing the algebraic approach to exploit the nature of wireless medium, **two** time slots are enough. As shown in Figure 2, in time slot 1 S_1 and S_2 and S_3 all send training data 1 to node R. These three signals "collide" in the air, but the collided signals turn out to be useful for our estimation. Let $y[1]$ denote the signal received by R in the first time slot. We have $y[1] = 3 + (x - 1)$. In time slot 2 S_1, S_2 and S_3 send training data 1, 2 and 3, respectively. Thus, the received signal is $y[2] = 6 + i(x-1)$ if (S_i, R) is the updated channel. At the end of the second time slot, R computes $[y(1), y(2)] - [3, 6] = (x - 1)[1, i]$. Since $[1, 1]$ and $[1, 2]$ and $[1, 3]$ are mutually linearly independent, R can uniquely decode i and x.

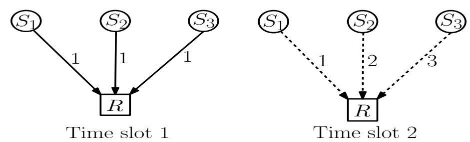

Figure 2: A better monitoring scheme. The first and second sub-figure show the transmissions in time slots 1 and 2, respectively.

1.2. Related Work

The works [9–13] designed probing data and estimation algorithm for estimating channel gains, but interference has not been shown to be an advantage. The benefits of network coding has been shown in [4–6, 15]. We remark that [4] has shown the advantage of interference, and later the work [5] proposed an amplify-and-forward relaying strategy for easy implementation. Our paper here demonstrates that not shunning away from interference is also advantageous in channel-gain monitoring. ADMOT proposed in this paper uses recent advances of compressive sensing developed for sparse signal recovering [16, 17]. Compressive sensing has been used to recover the channel's delay-Doppler sparsity [18], channel's sparse multipath structure [19], sparse-user detection [20, 21] and channel's sparse response [22]. When applying the above schemes to estimate all the n channels from the transmitters, the overhead is at least $\Theta(n)$. In contrast, ADMOT achieves optimal overhead $O(k \log(n/k))$.

2. PROBLEM SETTING

2.1. Notation Conventions and Preliminaries

For a vector $V \in \mathbb{R}^n$, define $\|V\|_1 = \sum_{i=1}^{n} |V(i)|$ and $\|V\|_2 = \sqrt{\sum_{i=1}^{n} |V(i)|^2}$, where $V(i)$ is the ith entry of V. Define the "distance" between V and *k-sparsity* by:

$$d_k(V) = ||V - V^k||_1, \qquad (1)$$

where V^k is V with all but the largest k components set to 0. V is be *k-sparse* if and only if $d_k(V) = 0$.

For vector $H \in \mathbb{C}^n$, $\|H\|_2^2 = \|Re(H)\|_2^2 + \|Im(H)\|_2^2$, where vector $Re(H)$ be the real part of X and $Im(H)$ be the imaginary part of X. Let $\mathcal{N}(\mu, \sigma^2)$ denote the normal distribution with mean μ and variance σ^2. Throughout the paper, the logarithm function $\log(.)$ is computed over base 2, *i.e.*, $\log(.) = \log_2(.)$.

Let M be a matrix in $\mathbb{R}^{m \times n}$ with $m \ll n$, and each column has unit ℓ_2-norm. M satisfies *restricted isometry property*(RIP) of order k if for all k-sparse vector $X \in \mathbb{R}^n$, $(1-\delta_k)||X||_2^2 \leq ||MX||_2^2 \leq (1+\delta_k)||X||_2^2$ holds for some $\delta_k > 0$ [23].

Let $X \in \mathbb{R}^n$ be the data vector and $Y = MX + Z$ be the noisy measurement, where $Z \in \mathbb{R}^m$ is the noise with $||Z||_2 \leq \sigma$. Let ConvexOPT(M, Y, σ) denote the solution to the following problem:

$$\min ||X||_1 \quad \text{subject to} \quad ||MX - Y||_2 \leq \sigma. \qquad (2)$$

If M satisfies RIP for $\delta_{2k} < \sqrt{2} - 1$, then

Theorem 1 *[23]*

$$||X - X^*||_2 \leq C_1 d_k(X)/\sqrt{k} + C_2\sigma/\sqrt{m}, \qquad (3)$$

for constants C_1 and C_2, where X^* =ConvexOPT(M, Y, σ)
.

2.2. Communication Model

We consider the uplink of a multiple access channel with one receiver R. Let $\mathcal{S} = \{S_1, S_2, ..., S_n\}$ be the set of transmitting nodes. We assume all transmissions are slotted and synchronized. In each time slot, every transmitter transmits one symbol. For the sake of exposition, we assume that channels are narrowbanded and channel gains are flat. We assume the channel conditions don't change dramatically across time. As shown in Figure 3, the data

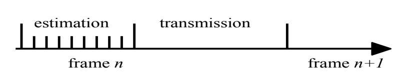

Figure 3: Systematical implementation of ADMOT.

stream is divided into frames. Consider time slot s in the estimation period of frame j, each $S_i \in \mathcal{S}$ transmits symbol $X_i[j, s] \in \mathbb{C}$. Then the received signal at R is $Y[j, s] = \sum_{i=1}^{n} H_i[j, s]X_i[j, s] + Z[j, s]$, where $H_i[j, s] \in \mathbb{C}$ is the channel gain of (S_i, R) and $Z[j, s] \in \mathbb{C}$ is the noise. Note that both $Re(Z[j, s])$ and $Im(Z[j, s])$ are *identically and independently distributed* (i.i.d.)$\sim \mathcal{N}(0, 1)$ across all frames and time slots[1]. The **state** of R is defined to be a vector $H[j, s] \in \mathbb{C}^n$, whose i'th component is $H_i[j, s]$. We assume channel gain stays unchanged within a frame transmission. In the following, we use $H[j] = H[j, 0]$ to denote the state at the j'th frame.

[1]For the simplicity, we normalize the noise variance to be 1.

2.3. State Estimation in Dynamic Network

Let $\hat{H}[j-1] \in \mathbb{C}^n$ be the estimation of $H[j-1]$ at R. Our objective is to estimate $H[j]$ by $\hat{H}[j-1]$ and the signals received in the estimation period of the j'th frame. For $\epsilon > 0$ and non-negative integer $k \leq n$, the difference $H[j] - \hat{H}[j-1]$ is said to be (k,ϵ)-sparse if and only if $d_k(Re(H[j] - \hat{H}[j-1])) \leq \epsilon$ and $d_k(Im(H[j] - \hat{H}[j-1])) \leq \epsilon$. Channel (S_i, R) is suffering α-variation if and only if $|H_i[j] - \hat{H}_i[j-1]| \geq \alpha$. Thus, for "$(k,\epsilon)$-sparse" state variation $H[j] - \hat{H}[j-1]$, there are at most k channels suffering from ϵ/k-variation.

3. ADMOT STATE ESTIMATION

3.1. Construction of ADMOT

The training data of ADMOT is denoted by matrix Φ with dimensions $m \times n$. Here, n is the number of transmitters in the network and m is the number of the time slots. Each component $\Phi(s,i)$ is generated independently from $\{-1, 1\}$ with equal probability, for all s and all i. The i-th column of matrix Φ is assumed to be known *a priori* to transmitter S_i in the network, for all $i \in \{1, 2, \ldots, n\}$. Knowledge of Φ can be broadcast by R in the network setting-up stage. [2]

The training data of each $S_i \in \mathcal{S}$ (*i.e.*, the i'th column of Φ) is in fact the "algebraic fingerprint" of channel (S_i, R). These fingerprints are "highly independent" such that the varying channels would expose their fingerprints even under interference. Let m be the system parameter denoting the number of time slots used by ADMOT. We construct ADMOT as follows.

- **ADMOT**$(\hat{H}[j-1], \mathcal{S}, R, m)$.
- Variables Initialization: Vector $\hat{H}[j] \in \mathbb{C}^n$ is the estimation of $H[j]$, which is initialized to be zero vector. Vector $Y \in \mathbb{C}^m$ is initialized to be zero vector.
- Step A: For $s = 1, 2, ..., m$, in the s'th time slot:
  - For any $S_i \in \mathcal{S}$, S_i sends $\Phi(s,i)$.
  - Node R sets $Y(s)$ (*i.e.*, the s'th component of Y) to be the received sample in the time slot. Thus, $Y(s) = \sum_{i=1}^{n} \Phi(s,i) H_i[j] + Z[j,s]$, where $Z[j,s]$ is the noise in the time slot (see Section 2.2 for details).
- Step B: Node R computes $D \in \mathbb{C}^m$ as $D = Y - \Phi\hat{H}[j-1]$. Thus, $D = \Phi(H[j] - \hat{H}[j-1]) + Z[j]$, where $Z[j] \in \mathbb{C}^m$ is the noise vector that is comprised of $\{Z[j,s] : s = 1, 2, ..., m\}$.
- Step C: Node R runs ConvexOPT($\Phi, Re(D), \sqrt{2m}$) and ConvexOPT($\Phi, Im(D), \sqrt{2m}$). Let the solutions be denoted by $Re(\Delta^*) \in \mathbb{R}^n$ and $Im(\Delta^*) \in \mathbb{R}^n$, respectively.
- Step D: Node R estimates $H[j]$ by $\hat{H}[j] = \hat{H}[j-1] + \Delta^*$.
- Step E: **End**

To avoid the accumulative errors, we can apply ADMOT consecutively for k frames, and then re-initialize the states.

[2] To avoid the overhead of broadcasting Φ, we can generate Φ by practical pseudorandom generators [24]. Since ADMOT can be simulated within polynomial time, pseudo randomness suffices.

3.2. Estimation Performance of ADMOT

Theorem 2 *If $m \geq C_0 k \log(n/k)$ for a constant C_0, and $H[j] - \hat{H}[j-1]$ is $(k, \delta\sqrt{k})$-sparse for some $\delta > 0$, then the estimation error of ADMOT satisfies $||\hat{H}[j] - H[j]||_2 \leq \sqrt{2}C_1\delta + 2C_2$ with a probability $1 - \mathcal{O}(e^{-0.15m})$, for some constants C_1 and C_2.*

We need the following lemma for proof of Theorem 2.

Lemma 3 *If $Z \in \mathbb{R}^m$ has i.i.d. $\mathcal{N}(0,1)$ entries, then $||Z||_2 \leq \sqrt{2m}$ with probability at least $1 - e^{-0.15m}$.*

Proof of Lemma 3 For any $i \neq j$, the probability density function of $X = Z(i)^2 + Z(j)^2$ is $f_X(x) = e^{-x/2}/2$ for $x \geq 0$ [25]. Thus, $E(e^{X/4}) = \int_0^{+\infty} e^{(-x/4)}/2dx = 2$. Without loss of generality we assume m is *even*. Then we have:

$$Pr(||Z||_2^2 > 2m) = Pr\Big(\sum_{i=1}^{d} Z(i)^2/4 > m/2\Big)$$

$$= Pr\Big(e^{\sum_{i=1}^{m} Z(i)^2/4} > e^{m/2}\Big) \leq \frac{E\Big(e^{\sum_{i=1}^{m} Z(i)^2/4}\Big)}{e^{m/2}} \quad (4)$$

$$= \frac{\prod_{j=1}^{m/2} E\Big(e^{\frac{Z(2j-1)^2}{4} + \frac{Z(2j)^2}{4}}\Big)}{e^{m/2}} \leq \frac{2^{m/2}}{e^{m/2}} \leq e^{-0.15m}, \quad (5)$$

where the inequality in (4) is by the Markov Inequality, and (5) is by the independence between the random variables. It completes the proof of Lemma 3. □

Now we are ready to prove Theorem 2. **Proof of Theorem 2**. When $m \geq C_0 k \log(n/k)$, with overwhelming probability (*i.e.*, $1 - \mathcal{O}(2^{-n})$), the matrix $\Phi/\sqrt{m}$ satisfies RIP with $\delta_{2k} < \sqrt{2} - 1$ [23]. From Lemma 3, we have $Pr(||Z||_2 \geq \sqrt{2m}) > 1 - e^{-0.15m}$. Now we assume both events happen, which is true with probability at least $1 - \mathcal{O}(2^{-n}) - e^{-0.15m}$. By Theorem 1, $Re(\Delta*)$ satisfies

$$\begin{aligned} & ||Re(\Delta^*) - Re(H[j] - \hat{H}[j-1])||_2 \\ \leq \quad & C_1 d_k(Re(\hat{H}[[j-1] - H[j]))/\sqrt{k} + \sqrt{2}C_2. \end{aligned}$$

Since $H[j] - \hat{H}[j-1]$ is $(k, \delta\sqrt{k})$-sparse, we have $||Re(\hat{H}[j] - H[j])||_2 \leq C_1\delta + \sqrt{2}C_2$ by setting $\hat{H}[j] = \hat{H}[j-1] + \Delta^*$. Similarly we have $||Im(\hat{H}[j] - H[j])||_2 \leq C_1\delta + \sqrt{2}C_2$. In the end, we have $||\hat{H}[j] - H[j]|| \leq \sqrt{2}C_1\delta + 2C_2$. It completes the proof of Theorem 2. □

In fact, $\mathcal{O}(k\log(n/k))$ is the best we can achieve for channel state estimation. It is proved Theorem 4.

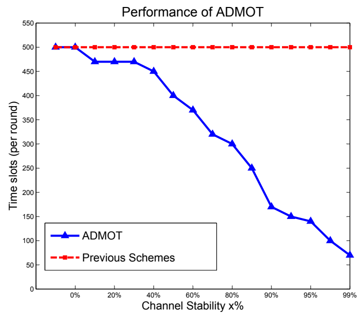

Figure 4: Comparison between ADMOT and previous monitoring schemes

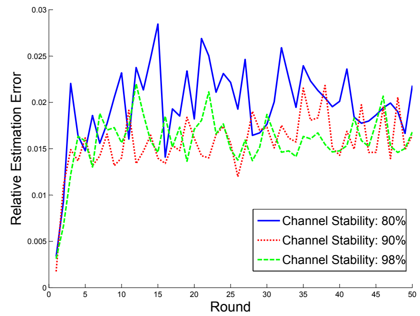

Figure 5: Relative estimating error for each stage of running ADMOT

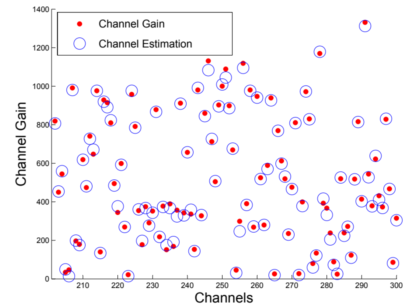

Figure 6: Detailed drawing of channel gains and the corresponding estimations

Theorem 4 *For any $k \leq n$, when $H[j] - \hat{H}[j-1]$ is $(k, \delta\sqrt{k})$-sparse, any monitoring scheme achieving estimation error $||\hat{H}[j] - H[j]||_2 \leq \mathcal{O}(\delta)$ requires at least $\Theta(k \log ((n+1)/(k)))$ time slots.*

Proof of Theorem 4: We first consider a sub-problem: Assuming $Re(H[j] - \hat{H}[j-1])$ is $(k, \sqrt{k}C)$-variation and $Im(H[j] - \hat{H}[j-1])$ is a all-zero vector, what is the minimum slots to find $Re(\hat{H}[j])$ such that $||Re(\hat{H}[j]) - Re(H[j])||_2^2 = \mathcal{O}(1)$?

Assume T time slots are used for estimating $Re(H[j])$. For any $s = 1, 2, ..., T$, and $i = 1, 2, ..., n$, in the t'th time slot let S_i send $A(s, i) \in \mathbb{C}$. Let $Y(s) \in \mathbb{C}$ be the received data of R in the s'th time slot, and $Y \in \mathbb{C}^T$ be a length-T vector whose s'th component is $Y(s)$. Thus $Y = AH[j]$. Note that assuming no noise only reduces the complexity of estimating $H[j]$. Since $\hat{H}[j-1]$ and A are known by R as *a priori*, the original problem is equivalent to estimating $\Delta = H[j] - \hat{H}[j-1]$ by $D = Y - A\hat{H}[j-1] = A(H[j] - \hat{H}[j-1])$. As $Im(H[j]) = Im(\hat{H}[j-1])$, $Y = A(Re(H[j]) - Re(\hat{H}[j-1]))$. Thus the problem is equal to estimating $Re(\Delta)$ by $Re(D)$ and $Im(D)$. A recent result [26] shows that provided $d_k(Re(\Delta)) \leq C\sqrt{k}$ for some constant C, it requires at least $\Theta(k \log((n+1)/k))$ linear samples (over $\mathbb{R}$) for reliably finding $\Delta^* \in \mathbb{R}^n$ such that $||Re(\Delta) - \Delta^*||_2^2 \leq \mathcal{O}(1)$. Thus we have $T \geq \Theta(k \log((n+1)/k))$. For the original problem, which considers noise and the variations of imaginary parts of channel gains, the complexity can only be higher. $\square$

4. PERFORMANCE EVALUATION

Let the $n = |\mathcal{S}| = 500$, average channel SNR= 20db. A channel preserves *stability* $x\%$ if with $< (1-x\%)$ probability the channel suffers significant variations. Let $H[r] = H[r-1] + \Delta[r]$, where $\Delta[r]$ is the variation. Each component of $\Delta[r] \in \mathbb{C}^n$, say $\Delta[r](i)$, is *independently* generated as: with a probability $x\%$ (or $1-x\%$), $Re(\Delta[r](i))$ and $Im(\Delta[r](i))$ are uniformly and independently chosen from $[-10, 10]$ (or $[-500, 500]$).

We proceed ADMOT$(\hat{H}[r-1], \mathcal{S}, R, m_r)$ for the r'th frame. Figure 4 shows the average time slots (per round) used by ADMOT. From the figure, we can see that ADMOT significantly reduces the overheads when x is large, *i.e.*, high channel stability is required. In the region where x is small, ADMOT also preserves reliable performance. We also provide the detailed simulations for the cases where channel preserves stabilities 80%, 90%, and 98%, respectively. The average time slots used per round is 320, 252, and 140, respectively. Figure 5 shows the relative estimation errors $||\hat{H}[r] - H[r]||_2 / ||H[r]||_2$ of ADMOT for 50 randomly selected frames. Note that we bound estimation error regardless the channel stability $x\%$. Thus, lower channel stability only corresponds to more overheads (as shown in Figure 4). For a detailed look, we also show the estimations at 50 randomly chosen frames for the case of 80% channel stability. Figure 6 draws (the absolute value of) channel gains and the corresponding estimations for the 200,201, ... , 300-th channels.

5. CONCLUSION

In the paper, we investigate the scenario where a receiver needs to track the channel gains of the channels with respect to n transmitters. We assume that in each round of channel gain estimation, no more than $k < n$ channels suffer significant variations since the last round. We prove that "$\Theta(k \log((n+1)/k))$ time slots" is the *minimum* number of time slots needed to catch up with k varying channels. We propose a novel scheme ADMOT to reduce the overhead for estimating a large number of channels simultaneously by leveraging the recent technique compressive sensing. We also analyze the theoretical performance of ADMOT and show that ADMOT can achieve the overhead lower bound.

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