MULTI-STREAM SUM RATE MAXIMIZATION FOR MIMO AF RELAY NETWORKS

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ABSTRACT

We consider a multiple-antenna amplify-and-forward (AF) two-hop interference network with multiple links and multiple relays. Transmit precoders, receive decoders and relay AF matrices are optimized with the purpose to maximize the system sum rate. By pointing out that the existing models all lead to single data stream transmission for each user, we propose a novel multiple stream model. We maximize the Total Signal to Total Interference plus Noise Ratio (TSTINR), with the requirement of orthogonal columns of precoders and total relay transmit power constraint. An efficient algorithm is proposed to solve the corresponding problem. Simulations show that the system sum rate significantly benefits from multiple data streams in medium to high SNR scenarios.

Index Terms— MIMO relay network, total signal to total interference plus noise ratio, multiple stream, alternating iteration

1. INTRODUCTION

Transmission between users are often aided by relays, as they improve both the capacity and the reliability of the network [1]. Among various relay transmit schemes, Amplify-and-Forward (AF) protocol is standardized in [2] as layer 1 relaying, and thus is popular due to its simplicity and low complexity. In our paper we consider the multiple link multiple relay network with AF protocol.

In [3] the authors show that multiple data streams, corresponding to multiple Degrees of Freedom (DoFs), help to increase the capacity of single-hop network in high Signal-to-Noise-Ratio (SNR) scenarios. With Interference Alignment (IA) technique [4], we are able to eliminate the interference and achieve the required DoFs. In MIMO networks, IA technique as well as the IA algorithms have been deeply investigated [5–8]. There are also some related works for two-hop networks. Aiming to achieve the maximum DoFs of the $2 \times 2 \times 2$ MIMO relay network, [9] and [10] study similar technique of aligned interference neutralization to explore the optimal transmission scheme for single antenna and multiple antennas cases, respectively. [11] investigates the ergodic capacity of a class of fading 2-user 2-hop networks using the interference neutralization technique. In [12] the maximum achievable DoFs for different kinds of MIMO interference channels and MIMO multiple hop networks are listed and concluded. For the general MIMO relay network, the maximum DoFs are only analyzed with restriction to the number of relays. Recent works of [13] and [14] are based on the general MIMO AF relay networks with multiple links and multiple relays, which provide algorithms to jointly optimize users' precoders, decoders and the relay AF matrices with provided number of data streams, to maximize the system sum rate. In [13] the Total Leakage Interference plus Noise (TLIN) minimization and the Weighted Mean Square Error Minimization (WMMSE) models are investigated. [14] proposes the Total Signal to Total Interference plus Noise Ratio (TSTINR) maximization model. All these models have per user and total relay transmit power constraints, and they are extended to those with individual user and individual relay power constraints. The WMMSE model performs the best in low SNR scenario, while the TSTINR model outperforms the other two models in medium to high SNR scenarios [14]. Interestingly, we observe from simulations that the precoding matrices obtained by the three models are always rank one matrices, regardless of the predefined number of data streams. This means they all lead to single data stream per user.

To overcome the disadvantage of prior works that only single stream per user can be transmitted, we propose a multiple stream TSTINR model for general MIMO relay AF network in this paper. First we introduce the system model in Section 2. In Section 3, we propose our multiple stream TSTINR model as well as the corresponding algorithm. Simulation results are shown in Section 4.

Notation: Lowercase and uppercase boldface represent vectors and matrices, respectively. $\mathbb{C}$ represents the complex domain. $\mathrm{tr}(\cdot)$, $\|\cdot\|_F$ and $(\cdot)^H$ denote the trace, the Frobenius norm and the conjugate transpose, respectively. $\mathrm{Re}(\cdot)$ means the real part. $\mathbf{I}_d$ represents the $d \times d$ identity matrix. $\succeq 0$ means positive semi-definite. $\mathcal{K}$ and $\mathcal{R}$ represent the set of the user indices $\{1, 2, \dots, K\}$ and relay indices $\{1, 2, \dots, R\}$, respectively. $\{\mathbf{U}_{-k}\}$ represents the set $\{\mathbf{U}_q, q \in \mathcal{K} - \{k\}\}$. $\mathrm{vec}(\mathbf{A})$ turns the columns of $\mathbf{A}$ into a long column vector. Let $\otimes$ be the Kronecker product. $\mathbb{E}(\cdot)$ denotes the statistical expectation. $\nu^d_{\min}(\mathbf{A})$ is composed of the eigenvectors of $\mathbf{A}$ corresponding to its d smallest eigenvalues.

2. SYSTEM MODEL

Consider a two-hop interference channel with K user pairs and R relays as in Fig. 1. Transmitter k, Receiver k and Relay r are equipped with M_k, N_k and L_r antennas, respectively, for any $k \in \mathcal{K}$, $r \in \mathcal{R}$. And User k wishes to transmit d_k parallel data streams. $\mathbf{s}_k \in \mathbb{C}^{d_k \times 1}$ denotes the transmit signal vector of User k, where $\mathbb{E}(\mathbf{s}_k\mathbf{s}_k^H) = \mathbf{I}_{d_k}$. Due to the poor channel conditions between user pairs, there is no direct links among users. Therefore, low-complex relays aid to communicate and the AF transmit protocol is used. We assume perfect channel state information is available at a central controller.

Transmission process includes two time slots. First, all sources transmit signals to all relays. For all $r \in \mathcal{R}$, Relay r receives $\mathbf{x}_r = \sum_{k \in \mathcal{K}} \mathbf{G}_{rk}\mathbf{U}_k\mathbf{s}_k + \mathbf{n}_r$, where $\mathbf{U}_k \in \mathbb{C}^{M_k \times d_k}$ is the precoding matrix of User k, $\mathbf{G}_{rk} \in \mathbb{C}^{L_r \times M_k}$ is the channel coefficient

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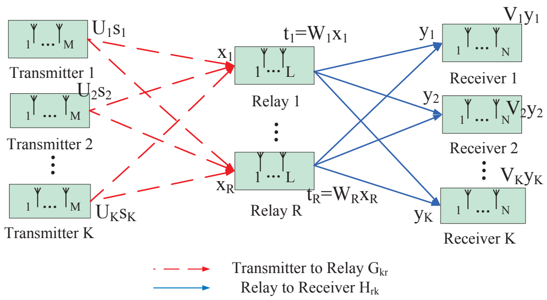

Fig. 1 MIMO relay AF network

between Transmitter k and Relay r, and $\mathbf{n}_r$ with zero mean and variance matrix $\sigma_1^2\mathbf{I}_{L_r}$ is the noise at Relay r. Then by the AF protocol all relays broadcast to all receivers $\mathbf{t}_r = \mathbf{W}_r\mathbf{x}_r$, for all $r \in \mathcal{R}$, where $\mathbf{W}_r \in \mathbb{C}^{L_r\times L_r}$ is the beamforming matrix of Relay r.

Receiver k observes $\mathbf{y}_k = \sum_{r\in\mathcal{R}}\mathbf{H}_{kr}\mathbf{t}_r + \mathbf{z}_k$, for all $k \in \mathcal{K}$, where $\mathbf{H}_{kr} \in \mathbb{C}^{N_k\times L_r}$ is the channel coefficient between Relay r and Receiver k, and $\mathbf{z}_k$ with zero mean and variance matrix $\sigma_2^2\mathbf{I}_{N_k}$ is the addictive noise vector at Receiver k. Multiplying the decoding matrix $\mathbf{V}_k \in \mathbb{C}^{N_k\times d_k}$, Receiver k obtains:

$$\begin{aligned}\tilde{\mathbf{y}}_k &= \underbrace{\mathbf{V}_k^H\mathbf{T}_{kk}\mathbf{s}_k}_{\text{desired signal}} + \underbrace{\sum_{q\in\mathcal{K},q\neq k}\mathbf{V}_k^H\mathbf{T}_{kq}\mathbf{s}_q}_{\text{interference}} \\ &\quad \underbrace{+\sum_{r\in\mathcal{R}}\mathbf{V}_k^H\mathbf{H}_{kr}\mathbf{W}_r\mathbf{n}_r + \mathbf{V}_k^H\mathbf{z}_k}_{\text{noise}}, \qquad (1)\end{aligned}$$

which contains three terms: the desired signal, the interference from other users and the noise including relay enhanced noise and the local noise. The effective channel from Transmitter k to Receiver q is given by $\mathbf{T}_{kq} = \sum_{r\in\mathcal{R}}\mathbf{H}_{kr}\mathbf{W}_r\mathbf{G}_{rq}\mathbf{U}_q$. Suppose all the transmit signals and noise in the system are independent of each other. The transmit power at each user and the total relay transmit power are, respectively:

$$P_k^T = \mathbb{E}(\|\mathbf{U}_k\mathbf{s}_k\|_F^2) = \text{tr}(\mathbf{U}_k^H\mathbf{U}_k), k \in \mathcal{K},$$

$$P^R = \sum_{r\in\mathcal{R}}\mathbb{E}(\|\mathbf{t}_r\|_F^2) = \sum_{r\in\mathcal{R}}(\sum_{k\in\mathcal{K}}\|\mathbf{W}_r\mathbf{G}_{rk}\mathbf{U}_k\|_F^2 + \sigma_1^2\|\mathbf{W}_r\|_F^2).$$

The desired signal power, the leakage interference and the noise power at Receiver k are, respectively:

$$\begin{aligned}P_k^S &= \mathbb{E}(\|\mathbf{V}_k^H\mathbf{T}_{kk}\mathbf{s}_k\|_F^2) \\ &= \|\mathbf{V}_k^H\sum_{r\in\mathcal{R}}\mathbf{H}_{kr}\mathbf{W}_r\mathbf{G}_{rk}\mathbf{U}_k\|_F^2, \qquad (2)\end{aligned}$$

$$\begin{aligned}P_k^I &= \mathbb{E}(\|\sum_{q\in\mathcal{K},q\neq k}\mathbf{V}_k^H\mathbf{T}_{kq}\mathbf{s}_q\|_F^2) \\ &= \sum_{q\in\mathcal{K},q\neq k}\|\mathbf{V}_k^H\sum_{r\in\mathcal{R}}\mathbf{H}_{kr}\mathbf{W}_r\mathbf{G}_{rq}\mathbf{U}_q\|_F^2, \qquad (3)\end{aligned}$$

$$\begin{aligned}P_k^N &= \mathbb{E}(\|\sum_{r\in\mathcal{R}}\mathbf{V}_k^H\mathbf{H}_{kr}\mathbf{W}_r\mathbf{n}_r + \mathbf{V}_k^H\mathbf{z}_k\|_F^2) \\ &= \sigma_1^2\sum_{r\in\mathcal{R}}\|\mathbf{V}_k^H\mathbf{H}_{kr}\mathbf{W}_r\|_F^2 + \sigma_2^2\|\mathbf{V}_k\|_F^2. \qquad (4)\end{aligned}$$

Before introducing the detailed optimization model, we predefine some symbols for the sake of expression simplicity: $\bar{\mathbf{G}}_{rk} = \mathbf{G}_{rk}\mathbf{U}_k$, $\bar{\mathbf{H}}_{kr} = \mathbf{H}_{kr}\mathbf{W}_r$, $\bar{\mathbf{V}}_{kr} = \mathbf{V}_k^H\mathbf{H}_{kr}$, $\bar{\mathbf{W}}_{rk} = \mathbf{W}_r\mathbf{G}_{rk}$, $k \in \mathcal{K}, r \in \mathcal{R}$.

3. MULTIPLE STREAM MODEL

In this section, we propose a novel multiple data stream model, based on the single stream TSTINR model in [14]. Sufficient motivation for the construction of the new model is also provided.

3.1. Analysis of single stream models

Although we expect $\text{rank}(\mathbf{U}_k) = d_k$, the single stream TSTINR, TLIN and WMMSE models in [14] and [13] all lead to rank one $\mathbf{U}_k$, for any $k \in \mathcal{K}$, regardless of d_k, as observed in simulations.

For the single stream TSTINR model in [14] and the TLIN model in [13], the subproblems to solve precoder $\mathbf{U}_k$ share the same structure as the following problem:

$$\begin{aligned}\min_{\mathbf{X}\in\mathbb{C}^{M_k\times d_k}} \quad & \text{tr}(\mathbf{X}^H\mathbf{A}_1\mathbf{X}) \\ \text{s.t.} \quad & \|\mathbf{X}\|_F^2 = p_0^T, \\ & \text{tr}(\mathbf{X}^H\mathbf{A}_2\mathbf{X}) = a. \qquad (5)\end{aligned}$$

And the following theorem provides theoretical evidence for the phenomena of the two models, where the detailed proof is shown in [14]:

Theorem 1 *The programming problem (5) always has a rank one optimal solution, regardless of* d_k.

Shown in Theorem 1, the optimizations always have rank one precoders in TSTINR and TLIN as solutions. Simulations verify this behavior in all cases. The same phenomenon is observed for the WMMSE model of [13], but the conclusion of Theorem 1 cannot be extended to WMMSE, due to the extra linear term in the objective function of the precoder subproblem. With the existing three models only a single data stream for each user pair can be achieved. A new model should be proposed to support multiple data streams.

3.2. Analysis of user transmit power allocation

Here we wish to maximize the system sum rate

$$R_{\text{sum}} = \frac{1}{2}\sum_{k\in\mathcal{K}}\log_2\det(\mathbf{I}_{N_k} + \mathbf{F}_k^{-1}\mathbf{T}_{kk}\mathbf{T}_{kk}^H) \qquad (6)$$

with $\mathbf{F}_k = \sum_{q\neq k,q\in\mathcal{K}}\mathbf{T}_{kq}\mathbf{T}_{kq}^H + \sigma_1^2\sum_{r\in\mathcal{R}}\bar{\mathbf{H}}_{kr}\bar{\mathbf{H}}_{kr}^H + \sigma_2^2\mathbf{I}_{N_k}$. The coefficient $\frac{1}{2}$ is due to the two time slots transmission process. The direct optimization of the system sum rate is complicated. So we apply TSTINR maximization approach to approximate the sum rate maximization, where

$$\text{TSTINR} = \frac{P^S}{P^I + P^N} = \frac{\sum_{k\in\mathcal{K}}P_k^S}{\sum_{k\in\mathcal{K}}(P_k^I + P_k^N)}.$$

It is proved that $\log_2(1 + \text{TSTINR})$ is a lower bound of R_{sum} [14]. Similar to the TSTINR model in [14], we require decoders $\mathbf{V}_k, k \in \mathcal{K}$ to be orthogonal as the bases of the d_k-dimensional solution subspaces, because TSTINR remains invariant with $\mathbf{V}_k$ replaced by $\mathbf{V}_k\mathbf{Q}$, where $\mathbf{Q}$ is any $d_k \times d_k$ unitary matrix. Also, we add the total relay transmit power constraint. To achieve the required number of parallel data streams, we should have independent columns of precoder $\mathbf{U}_k$ for all $k \in \mathcal{K}$. Without loss of generality we require the columns of $\mathbf{U}_k$ to be orthogonal. Since there is a transmit power constraint for each user, we have power allocation among d_k parallel data streams for User k. First we state our multiple stream TSTINR model as follows:

$$\begin{aligned}\max_{\substack{\{\mathbf{U}\},\{\mathbf{V}\},\\ \{\mathbf{W}\},\{\mathbf{\Phi}\}}} \quad & \text{TSTINR} = \frac{\sum_{k\in\mathcal{K}}P_k^S}{\sum_{k\in\mathcal{K}}(P_k^I + P_k^N)} \\ \text{s.t.} \quad & \mathbf{U}_k^H\mathbf{U}_k = \mathbf{\Phi}_k, \mathbf{V}_k^H\mathbf{V}_k = \mathbf{I}_{d_k}, k \in \mathcal{K}, \\ & \sum_{r\in\mathcal{R}}\left(\sum_{k\in\mathcal{K}}\|\mathbf{W}_r\mathbf{G}_{rk}\mathbf{U}_k\|_F^2 + \sigma_1^2\|\mathbf{W}_r\|_F^2\right) \leq p_{\max}^R, \\ & \text{tr}(\mathbf{\Phi}_k) \leq p_0^T, \mathbf{\Phi}_k \text{ is diagonal}, \mathbf{\Phi}_k \succeq 0, k \in \mathcal{K}. \qquad (7)\end{aligned}$$

Here $\mathbf{\Phi}_k$ is a $d_k \times d_k$ matrix, which contains the data stream power allocation of User k. And p_0^T and $p_{\max}^R$ represent the maximum per user and total relay transmit power, respectively. The feasible set of a precoder $\mathbf{U}_k$ is restricted to have orthogonal columns. This avoids the phenomenon observed in the models in [14] and [13] that all columns of the rank one precoders $\mathbf{U}_k$ are nonzero but linearly dependent. Different from (5), here the rank one case of $\mathbf{U}_k$ only happens when there is one positive diagonal element of $\mathbf{\Phi}_k$, which result in all columns of $\mathbf{U}_k$ but one are all zeros. Hence, we focus on the analysis of the subproblem to solve $\mathbf{U}_k$ as well as $\mathbf{\Phi}_k$.

The reformulation of the objective function of (7) is the same as that for the TSTINR model in [14], which is shown in Section 3.3. Then given $k \in \mathcal{K}$, fixing all variables other than $\mathbf{U}_k$ and $\mathbf{\Phi}_k$, the subproblem has the following form:

$$\begin{aligned} \min_{\mathbf{X}\in\mathbb{C}^{M_k\times d_k},\mathbf{\Phi}_k\in\mathbb{C}^{d_k\times d_k}} \quad & \mathrm{tr}(\mathbf{X}^H\mathbf{B}_1\mathbf{X}) \\ \text{s.t.} \quad & \mathbf{X}^H\mathbf{X} = \mathbf{\Phi}_k, \\ & \mathrm{tr}(\mathbf{X}^H\mathbf{B}_2\mathbf{X}) \le b, \\ & \mathrm{tr}(\mathbf{\Phi}_k) \le p_0^T, \mathbf{\Phi}_k \text{ is diagonal}, \mathbf{\Phi}_k \succeq 0, \end{aligned} \quad (8)$$

where $\mathbf{X}$ represents $\mathbf{U}_k$, and $\mathbf{B}_1$, $\mathbf{B}_2$ and b are all parameters.

Theorem 2 *The optimal $\mathbf{\Phi}_k$ of (8) is of rank one, i.e., there is only one positive element on the diagonal of $\mathbf{\Phi}_k$.*

The detailed proof is shown in [14]. Then at the optimal solution of (8) the complete transmit power should be assigned to one data stream. This means here the optimization of stream power allocation leads to $\mathrm{rank}(\mathbf{U}_k) = 1$ and thus only one data stream can be transmitted for each user.

3.3. New model and the algorithm framework

In our multiple stream model, we assume each user has fixed transmit power p_0^T, and require equal power allocation among d_k parallel data streams for User k. This choice accords with the optimal power allocation scheme to maximize the system sum rate in the high SNR scenario [15]. The corresponding optimization problem of the new model becomes:

$$\begin{aligned} \max_{\{\mathbf{U}\},\{\mathbf{V}\},\{\mathbf{W}\}} \quad & \mathrm{TSTINR} = \frac{\sum_{k\in\mathcal{K}} P_k^S}{\sum_{k\in\mathcal{K}}(P_k^I + P_k^N)} \\ \text{s.t.} \quad & \mathbf{U}_k^H\mathbf{U}_k = \frac{p_0^T}{d_k}\mathbf{I}_{d_k}, \mathbf{V}_k^H\mathbf{V}_k = \mathbf{I}_{d_k}, k \in \mathcal{K}, \\ & \sum_{r\in\mathcal{R}}\left(\sum_{k\in\mathcal{K}}\|\mathbf{W}_r\mathbf{G}_{rk}\mathbf{U}_k\|_F^2 + \sigma_1^2\|\mathbf{W}_r\|_F^2\right) \le p_{\max}^R. \end{aligned} \quad (9)$$

Similar to the technique in [14], we reformulate the objective function of (9) with parameter C, and combine the total desired signal power P^S and the total leakage interference plus noise $P^I + P^N$ as: $C(P^I + P^N) - P^S$. In each iteration C is updated as $C = \frac{P^S(\{\mathbf{U}\},\{\mathbf{V}\},\{\mathbf{W}\})}{P^I(\{\mathbf{U}\},\{\mathbf{V}\},\{\mathbf{W}\})+P^N(\{\mathbf{U}\},\{\mathbf{V}\},\{\mathbf{W}\})}$. We minimize the new objective function with the same constraint set and update C with the above strategy. Then the new problem shares the same stationary points as (9) [14].

As the reformulated problem is still nonconvex and quite complicated, we apply the alternating iteration method to solve it. First fixing $\mathbf{U}_k, k \in \mathcal{K}$ and $\mathbf{W}_r, r \in \mathcal{R}$, then all $\mathbf{V}_k, k \in \mathcal{K}$ are independent of each other. The subproblem for $\mathbf{V}_k$ becomes:

$$\min_{\mathbf{X}^H\mathbf{X}=\mathbf{I}_{d_k}} \mathrm{tr}(\mathbf{X}^H\mathbf{C}\mathbf{X}), \quad (10)$$

where $\mathbf{X}$ represents variable $\mathbf{V}_k$, and $\mathbf{C} = C\mathbf{F}_k - \mathbf{T}_{kk}\mathbf{T}_{kk}^H$. Noticing that $\mathbf{C}$ is Hermitian, we obtain the closed form solution of (10) as $\mathbf{X} = \nu_{\min}^{d_k}(\mathbf{C})$.

Next, we solve the subproblem for $\mathbf{W}_r$. Given a certain index $r \in \mathcal{R}$, we fix $\mathbf{U}_k, \mathbf{V}_k, k \in \mathcal{K}$ and $\{\mathbf{W}_{-r}\}$. Thus the optimization subproblem for $\mathbf{W}_r$ is:

$$\begin{aligned} \min_{\mathbf{X}\in\mathbb{C}^{L_r\times L_r}} \quad & \sum_{k\in\mathcal{K}} \mathrm{tr}\big[\mathbf{X}(\mathbf{P}_{rr}^k + \sigma_1^2\mathbf{I}_{L_r})\mathbf{X}^H\bar{\mathbf{V}}_{kr}^H\bar{\mathbf{V}}_{kr}\big] \\ & +2\mathrm{Re}\Big\{\sum_{k\in\mathcal{K}}\sum_{\substack{l\neq r,\\ l\in\mathcal{R}}}\mathrm{tr}(\mathbf{X}\mathbf{P}_{rl}^k\mathbf{W}_l^H\bar{\mathbf{V}}_{kl}^H\bar{\mathbf{V}}_{kr})\Big\} \\ \text{s.t.} \quad & \mathrm{tr}\big[\mathbf{X}(\sum_{k\in\mathcal{K}}\bar{\mathbf{G}}_{rk}\bar{\mathbf{G}}_{rk}^H + \sigma_1^2\mathbf{I}_{L_r})\mathbf{X}^H\big] \le \eta_1, \end{aligned} \quad (11)$$

where $\mathbf{P}_{rl}^k = C\sum_{q\neq k,q\in\mathcal{K}}\bar{\mathbf{G}}_{rq}\bar{\mathbf{G}}_{lq}^H - \bar{\mathbf{G}}_{rk}\bar{\mathbf{G}}_{lk}^H, k \in \mathcal{K}, r,l \in \mathcal{R}$ and $\eta_1 = p_{\max}^R - \sum_{l\neq r,l\in\mathcal{R}}\big(\sum_{k\in\mathcal{K}}\|\mathbf{W}_l\bar{\mathbf{G}}_{lk}\|_F^2 + \sigma_1^2\|\mathbf{W}_l\|_F^2\big)$.

The problem (11) is equivalent to a specific Quadratic Constrained Quadratic Programming (QCQP) with $\mathbf{x} = \mathrm{vec}(\mathbf{X})$:

$$\begin{aligned} \min_{\mathbf{x}\in\mathbb{C}^{L_r^2\times 1}} \quad & \bar{f}(\mathbf{x}) = \mathbf{x}^H\mathbf{D}_1\mathbf{x} + \mathbf{q}^H\mathbf{x} + \mathbf{x}^H\mathbf{q} \\ \text{s.t.} \quad & \mathbf{x}^H\mathbf{D}_2\mathbf{x} \le \eta_1. \end{aligned} \quad (12)$$

Here $\mathbf{D}_1 = \sum_{k\in\mathcal{K}}(\mathbf{P}_{rr}^k + C\sigma_1^2\mathbf{I}_{L_r})^T \otimes (\bar{\mathbf{V}}_{kr}^H\bar{\mathbf{V}}_{kr})$,
$\mathbf{D}_2 = (\sum_{k\in\mathcal{K}}\bar{\mathbf{G}}_{rk}\bar{\mathbf{G}}_{rk}^H + \sigma_1^2\mathbf{I}_{L_r})^T \otimes \mathbf{I}_L$ and
$\mathbf{q} = \mathrm{vec}(\sum_{k\in\mathcal{K}}\sum_{l\neq r,l\in\mathcal{R}}\bar{\mathbf{V}}_{kr}^H\bar{\mathbf{V}}_{kl}\mathbf{W}_l\mathbf{P}_{rl}^k)$.

As $\mathbf{D}_2$ is positive definite, with $\mathbf{D}_2 = \mathbf{Q}^H\mathbf{Q}, \mathbf{Q} \succ 0, \mathbf{p} = \mathbf{Q}\mathbf{x}$, $\bar{\mathbf{D}}_1 = \mathbf{Q}^{-1}\mathbf{D}_1\mathbf{Q}^{-1}$ and $\bar{\mathbf{q}} = \mathbf{Q}^{-1}\mathbf{q}$, (12) is equivalent to

$$\min_{\mathbf{p}^H\mathbf{p}\le\eta_1} \mathbf{p}^H\bar{\mathbf{D}}_1\mathbf{p} + \bar{\mathbf{q}}^H\mathbf{p} + \mathbf{p}^H\bar{\mathbf{q}}. \quad (13)$$

Problem (13) is a typical trust region (TR) subproblem in trust region optimization method. [16, Chapter 6.1.1] provides an efficient algorithm to achieve its optimal solution.

For the precoder $\mathbf{U}_k$, the corresponding subproblem becomes, while fixing $\mathbf{V}_q, q \in \mathcal{K}$, $\mathbf{W}_r, r \in \mathcal{R}$ and $\{\mathbf{U}_{-k}\}$:

$$\min_{\mathbf{X}\in\mathbb{C}^{M_k\times d_k}} \quad \mathrm{tr}(\mathbf{X}^H\mathbf{Q}_k\mathbf{X}) \quad (14a)$$

$$\text{s.t.} \quad \mathbf{X}^H\mathbf{X} = \frac{p_0^T}{d_k}\mathbf{I}_{d_k}, \quad (14b)$$

$$\mathrm{tr}(\mathbf{X}^H\mathbf{L}_k\mathbf{X}) \le \eta_2, \quad (14c)$$

where $\mathbf{X}$ represents the variable $\mathbf{U}_k$, and

$$\mathbf{Q}_k = \sum_{r\in\mathcal{R}}\sum_{l\in\mathcal{R}}\bar{\mathbf{W}}_{rk}^H\Big(C\sum_{q\neq k,q\in\mathcal{K}}\bar{\mathbf{V}}_{qr}\bar{\mathbf{V}}_{ql} - \bar{\mathbf{V}}_{kr}\bar{\mathbf{V}}_{kl}\Big)\bar{\mathbf{W}}_{lk},$$

$$\mathbf{L}_k = \sum_{r\in\mathcal{R}}\bar{\mathbf{W}}_{rk}^H\bar{\mathbf{W}}_{rk},$$

$$\eta_2 = p_{\max}^R - \sum_{q\neq k,q\in\mathcal{K}}\sum_{r\in\mathcal{R}}\|\bar{\mathbf{W}}_{rq}\mathbf{U}_q\|_F^2 - \sigma_1^2\sum_{r\in\mathcal{R}}\|\mathbf{W}_r\|_F^2.$$

The optimality conditions for (14) are as follows. (The detailed proof for the optimality is shown in [14].)

There exists $\mu^* \ge 0$ as the Lagrange multiplier of (14c), such that:

OC1 $\mathbf{X}^*(\mu^*) = \sqrt{\frac{p_0^T}{d_k}}\nu_{\min}^{d_k}(\mathbf{Q}_k + \mu^*\mathbf{L}_k)$ is the optimal solution for:

$$\min_{\mathbf{X}^H\mathbf{X}=\frac{p_0^T}{d_k}\mathbf{I}_{d_k}} \mathrm{tr}[\mathbf{X}^H(\mathbf{Q}_k + \mu^*\mathbf{L}_k)\mathbf{X}]. \quad (15)$$

OC2 Complimentary condition holds: $\mu^*\{\mathrm{tr}[(\mathbf{X}^*)^H\mathbf{L}_k\mathbf{X}^*] - \eta_2\} = 0$.

OC3 $c(\mu)$ as the function of μ satisfies (14c):

$$c(\mu^*) = \mathrm{tr}\{[\mathbf{X}^*(\mu^*)]^H\mathbf{L}_k\mathbf{X}^*(\mu^*)\} \le \eta_2.$$

We want to have $\mu^* \ge 0$, to satisfy the optimality conditions OC2 and OC3. If $c(0) \le \eta_2$, then $\mu = 0$ is the optimal Lagrange multiplier. Otherwise μ^* should be strictly greater than 0. Thus we should always have $c(\mu^*) = \mathrm{tr}[(\mathbf{X}^*)^H\mathbf{L}_k\mathbf{X}^*] = \eta_2$ from the condition OC2. Also from the constraint (14c) we should have $c(\infty) \le \eta_2$ for a feasible problem. Then with $c(\mu)$ as a continuous function, there exists $\mu^* \in (0,\infty)$ such that $c(\mu^*) = \eta_2$. Thus we use Newton's root finding method [16] to search for μ^*. With the methods for all the three subproblems, the algorithm for (9) is presented as:

Algorithm for multiple stream TSTINR model

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1. Set initial value of $\mathbf{U}_k, k \in \mathcal{K}$ and $\mathbf{W}_r, r \in \mathcal{R}$. $C = 1$.
2. Update decoder $\mathbf{V}_k$ by solving (10), $k \in \mathcal{K}$.
3. Update relay beamforming matrix $\mathbf{W}_r$ by solving (11), $r \in \mathcal{R}$.
4. Update precoder $\mathbf{U}_k$ by solving (14), $k \in \mathcal{K}$.
5. Update C as $C := \frac{P^S}{P^I + P^N}$. Go back to Step 2. Iterate until convergence.

As we obtain the global optimal solution of each subproblem, in each iteration sufficient improvement of the objective function value TSTINR is guaranteed. Thus the objective function value of our algorithm is convergent. As we enforce the orthogonal constraints to the columns of each precoder, it is guaranteed that rank$(\mathbf{U}_k) = d_k, k \in \mathcal{K}$. Thus User k has d_k parallel data streams as expected.

4. SIMULATIONS

In this section, we evaluate the performance of our proposed multiple stream TSTINR model. Each element of $\mathbf{G}_{rk}$ and $\mathbf{H}_{kr}, k \in \mathcal{K}, r \in \mathcal{R}$ are generated as i.i.d complex Gaussian distribution with zero mean and unit variance. The noise variances are set as $\sigma_1^2 = \sigma_2^2 = \sigma^2 = 1$. The initial values of $\mathbf{U}_k, k \in \mathcal{K}$ and $\mathbf{W}_r, r \in \mathcal{R}$ are randomly generated, and scaled to be feasible. Initially, $C = 1$. For each plotted point, 100 random realization of different channel coefficients are generated to evaluate the average performance. Here we define SNR as SNR$= \frac{p_0^T}{\sigma^2} = \frac{p_0^R}{\sigma^2}$, and $p_{\max}^R = R \cdot p_0^R$. The system sum rate R_{sum} is applied as the measure of Quality of Service. We investigate three kinds of $2 \times 2 \times 2$ networks with different number of antennas, that is, $K = R = 2$. Here for the scheme $d_1 = d_2 = 1$ we choose the maximum system sum rate results between the single stream TSTINR model in [14] and the WMMSE model with power control in [13].

First we consider a network with 2 antennas for each user and 4 antennas for each relay. The number of data streams for User k, d_k, varies from 1 to 2 for both $k = 1, 2$. And for different choices of $d_k, k = 1, 2$ the average sum rate corresponding to different SNR values are shown in Fig. 2. As expected, in the low SNR scenario the single stream scheme with $d_1 = d_2 = 1$ outperforms other schemes; in medium to high SNR scenarios, the scheme $d_1 = d_2 = 2$ becomes dominant and the scheme $d_1 = d_2 = 1$ is worse than all others, in terms of sum rate.

In Fig. 3, the considered network has the same parameters as the previous one, except that each relay owns 2 antennas. Similar to Fig. 2, the average achieved sum rate results corresponding to different SNR values for different requirements of data streams are shown. The curves are quite different from the previous example. Here the schemes $d_1 = 1, d_2 = 2$ and $d_1 = 2, d_2 = 1$ outperform the other two schemes in medium to high SNR scenarios. And in general the scheme $d_1 = d_2 = 2$ performs very bad.

The performances shown in Fig. 2 and Fig. 3 accord with the recent theoretical result on DoF of MIMO relay networks. From the cut-set bound theory, the maximum DoFs of the first network is no greater than 2 for each user [17, Theorem 15.1]. And simulations verify the benefit to transmit 2 data streams for each user over other schemes in Fig. 2. However in the second example, there is no extra relay antenna to align interference besides transmitting the desired signal. Without symbol extension or time division, 2 DoFs for each user is not achievable. This accords with the performances in Fig. 3. In [9] the authors show that $\frac{3}{2}$ DoFs for each user are achievable in $2 \times 2 \times 2$ network with 2 antennas for each user and each relay. And the essential idea of the transmission scheme is to sacrifice one data stream for interference and make full use of all other streams. Correspondingly, in Fig. 3 the schemes $d_1 = 1, d_2 = 2$ and $d_1 = 2, d_2 = 1$ perform the best in medium to high SNR. This indicates substantial benefit of system sum rate from such schemes. In general, multiple stream schemes improve the system sum rate in medium to high SNR scenarios.

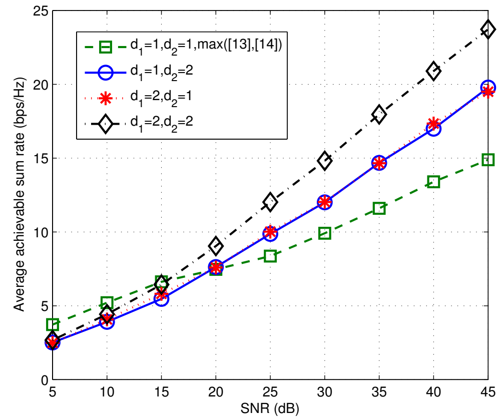

Fig. 2 Average achieved sum rate versus SNR, $M_k = N_k = 2, L_r = 4$

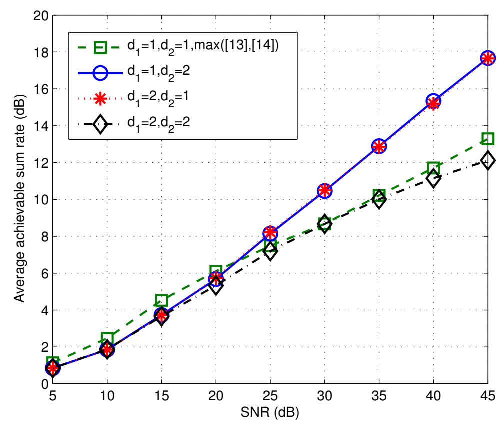

Fig. 3 Average achieved sum rate versus SNR, $M_k = N_k = L_r = 2$

5. CONCLUSION

In this paper, we considered the general $K \times R \times K$ MIMO AF relay network. First we analyzed and observed by simulations that the existed models all lead to rank one precoders, resulting in single stream transmission. Then with sufficient motivations, we set up a multiple stream TSTINR model with orthogonal precoder column requirement and total relay transmit power constraint. Simulation results show that with multiple stream transmission the system sum rate is significantly improved.

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