JOINT RESOURCE ALLOCATION AND RECEIVER MAP ESTIMATION IN UNDERLAY COGNITIVE RADIOS

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ABSTRACT

Conventional spectrum sensing schemes can detect active transmitters but not passive *receivers*, which have to be nevertheless protected from excessive interference whenever their bands are reused. In this paper, a resource allocation scheme for underlay cognitive radios is formulated, taking into account uncertainty of *both* propagation gains *and* locations of incumbent receivers. The performance of orthogonal access by secondary users is maximized under average interference constraints, using channel statistics and maps that pinpoint areas where primary receivers are likely to reside. These maps are tracked using a Bayesian approach, based on a 1-bit message sent by the primary system whenever a communication disruption occurs due to interference.

Index Terms— Cognitive radios, underlay access, resource allocation, receiver localization, Lagrange dual, Bayesian estimator.

1. INTRODUCTION

In the underlay spectrum access paradigm, cognitive radios (CRs) can reuse the frequency bands licensed to a primary user (PU) network, provided PU-to-PU communications are not overly disrupted [1]. However, assessing the interference that secondary users (SUs) will inflict to PUs is not an easy task. For instance, spectrum sensing schemes can generally detect and localize active PU *sources* [2–4], but not "passive" PU *receivers*, which may remain silent for the majority of the time. But even if the locations of these PUs were available, SU-to-PU channels can not be acquired accurately, since PUs have generally no reason to exchange synchronization and channel training signals with SUs.

The present paper advocates the notion of *receiver map* as a tool for unveiling areas where PU receivers are located, with the objective of limiting the average interference inflicted to those locations. These maps are tracked using a recursive Bayesian estimator, which is based on a 1-bit message sent by the primary transmitter whenever the instantaneous interference inflicted to *at least* one PU exceeds a tolerable level.

Based on these maps, and assuming that the distribution of SU-to-PU channels is available, a resource allocation (RA) scheme is formulated, where the SU design variables are adapted to the time-varying SU-to-SU channels, and the receiver map dynamics. Although nonconvex, the formulated RA problem has zero duality gap, and it is optimally solved using a Lagrange dual approach[1].

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[1]*Notation:* $\mathbb{E}_{\mathbf{g}}[\cdot]$ denotes expectation with respect to the random process $\mathbf{g}$; $\Pr\{A\}$ the probability of event A; x^* the optimal value of x; $\mathbb{1}_{\{\cdot\}}$ the indicator function ($\mathbb{1}_{\{x\}} = 1$ if x is true, and zero otherwise); and, $[x]_a^b$ the projection of the scalar x onto $[a, b]$; that is, $[x]_a^b := \min\{\max\{x, a\}, b\}$.

1.1. Modeling and preliminaries

Consider an SU network in underlay access mode [1], with M transmitter-receiver pairs (indexed by m) deployed over an area $\mathcal{A} \subset \mathbb{R}^2$, sharing the spectrum with an incumbent PU system. Based on the output of the spectrum sensing scheme [2–4], SUs implement an adaptive RA to maximize their performance, while protecting the PU system from excessive interference.

When spectral resources are shared in a hierarchical setup, the CSI available to the SU network is generally heterogeneous; in fact, the channel information that can be acquired for a link may be more or less accurate depending on whether PUs or SUs are involved [5–8]. Provided the spectrum is available for the SUs to transmit, the SU-to-SU channels can be readily acquired by employing conventional training-based channel estimators. For this reason, the state of the SU-to-SU channels is considered perfectly known. The instantaneous power gains $\{g_m\}_{m=1}^M$ of the SU links are given by the squared magnitude of the small-scale fading realization scaled by the link average signal-to-interferference-plus-noise ratio (SINR).

Suppose now that PU transmitters communicate with Q PU *receivers* located at coordinates $\{\mathbf{x}^{(q)} \in \mathcal{A}\}_{q=1}^Q$. Let $h_{m,\mathbf{x}^{(q)}}$ denote the instantaneous channel gain between the SU transmitter m and the PU receiver q. Due to a lack of full PU-SU cooperation [1], training-based channel estimation cannot be employed in this case. Thus, even though the average link gain (path loss coefficients) can be obtained based on $\{\mathbf{x}^{(q)} \in \mathcal{A}\}_{q=1}^Q$, the instantaneous value of the primary link is *uncertain* due to random fading effects. Consequently, SU m cannot assess precisely the interference $p_m h_{m,\mathbf{x}^{(q)}}$ that it will cause to PU q, where p_m denotes the transmit-power. Hereafter, it is assumed that only the distribution of $h_{m,\mathbf{x}^{(q)}}$ is known to the SU network. Thus, given the maximum instantaneous interference power Γ tolerable by the PUs, the SUs can determine the probability of interfering a PU located at $\mathbf{x}^{(q)}$, which is denoted here as $\iota_{m,\mathbf{x}^{(q)}} := \Pr\{p_m h_{m,\mathbf{x}^{(q)}} > \Gamma\}$.

However, conventional spectrum sensing schemes can detect and localize active PU *sources*, and not "passive" PU *receivers* [2–4], which remain silent most of the time (and whose ACK/NACK messages may not be easily detected). As a consequence, the locations $\{\mathbf{x}^{(q)} \in \mathcal{A}\}_{q=1}^Q$ are generally unknown. Let $z_{\mathbf{x}}^{(q)}$ be a binary variable taking the value 1 if a PU receiver q is located at $\mathbf{x} \in \mathcal{A}$, and consider discretizing the PU coverage region into a set of grid points $\mathcal{G} := \{\mathbf{x}_g\}$ representing *potential* locations for the PU receivers. In lieu of $\{z_{\mathbf{x}_g}^{(q)}\}$, the idea is to use the set of probabilities $\beta_{\mathbf{x}}^{(q)} := \Pr\{z_{\mathbf{x}}^{(q)} = 1\}$, $\forall\, \mathbf{x} \in \mathcal{G}$, to identify areas where a PU receiver q is more likely to reside, and limit the interference accordingly. To this end, the following is assumed.

(as1) Processes $\{g_m\}$ and $\{h_{m,\mathbf{x}^{(q)}}\}$ are (mutually) independent.

(as2) Processes $z_{\mathbf{x}}^{(q)}$ and $z_{\mathbf{x}}^{(v)}$, $q \neq v$, are independent.

Assumption *(as2)* implies that each PU receiver has its own mobility pattern, while *(as1)* presupposes that the uncertain component of $\{h_{m,\mathbf{x}^{(q)}}\}$ is spatially uncorrelated. This is the case when e.g., spatial correlation of shadowing in negligible [4, 9], or path loss and shadowing are accurately acquired as in, e.g. [2].

1.2. Relation to prior work

Power control for underlay CRs under channel uncertainty is considered in [10, 11], for the case of one PU link and one SU link. Assuming that the distribution of the SU-to-PU link is available, the instantaneous interference is limited by means of probabilistic constraints. Instantaneous and average interference constraints are compared in [5] for the same setup. Extensions to multiple SU links can be found in, e.g., [6, 9]. However, assuming that the statistics of the SU-to-PU links are known, tacitly implies that the PU receiver locations are known and are used to compute path loss coefficients (i.e., the mean of the channel gains). The distribution of SU-to-PU links is estimated online in [8], based on channel measurements.

Once the locations of PU transmitters are acquired through sensing, a prudent way to protect the PU system is to estimate the PU coverage region [2, 4], and ensure that the interference does not exceed a prescribed level at any point of the boundary of the PU coverage region [4, 7]. However, this solution may lead to sub-optimal operation of the SU network, especially if the PU receivers are actually located far from the PU region boundary.

The contribution of the present paper is twofold: *i)* a novel RA scheme is formulated, where the SU transmissions are scheduled and their powers are computed based on the secondary CSI $\mathbf{g} := \{g_m\}$ and statistical primary state information (PSI) $\mathbf{s} := \{\iota_{m,\mathbf{x}}\} \cup \{\beta_{\mathbf{x}}^{(q)}\}$, which models uncertainties of *both* channels *and* PU locations (Section 2); and, *ii)* a recursive Bayesian estimator is developed to optimally estimate the receiver maps $\{\beta_{\mathbf{x}}^{(q)}\}$ based on a single-bit message received when a PU is interfered (Section 3).

2. RA UNDER PRIMARY STATE UNCERTAINTY

To simplify notation and exposition, suppose that the SU system operates over a single primary band. Define a binary scheduling variable w_m taking the value 1 if the mth SU is scheduled to transmit to its intended receiver, and 0 otherwise. Secondary transmissions are assumed orthogonal and, thus, $\sum_m w_m(\mathbf{g},\mathbf{s}) \leq 1$. When $\sum_m w_m(\mathbf{g},\mathbf{s}) = 0$, no user transmits either because the quality of all SU-to-SU channels is poor, or, excessive interference is inflicted to the PU. Under bit error rate or capacity constraints, instantaneous rate and transmit power variables are coupled, and this rate-power coupling is modeled here using Shannon's capacity formula $r_m(g_m, p_m) = \log(1 + g_m p_m/\kappa_m)$, where κ_m represents the coding scheme-dependent SINR gap [12]. Channels in $\mathbf{g}$ vary across time due to fading. Thus, the SU network operates in a time-slotted setup, where the duration of each slot (indexed by t) corresponds to the coherence time of the small-scale fading process.

Let $\bar{p}_m$ and $\bar{r}_m$ denote the average power and rate transmitted by SU m. These can be expressed as $\bar{p}_m(\mathbf{x}) := \mathbb{E}_{\mathbf{g},\mathbf{s}}[w_m(\mathbf{g},\mathbf{s})p_m(\mathbf{g},\mathbf{s})]$ and $\bar{r}_m(\mathbf{x}) := \mathbb{E}_{\mathbf{g},\mathbf{s}}[w_m(\mathbf{g},\mathbf{s})r_m(g_m, p_m(\mathbf{g},\mathbf{s}))]$, where $\mathbf{y} := \{w_m(\mathbf{g},\mathbf{s}), p_m(\mathbf{g},\mathbf{s}), \forall m, \mathbf{g}, \mathbf{s}\}$. The metric to be optimized will be designed to encourage high average transmission rates, while discouraging high average power consumptions. To this end, let $U_m(\bar{r}_m)$ denote a concave, non-decreasing, utility function quantifying the reward associated with the rate $\bar{r}_m$, and $J_m(\bar{p}_m)$ denote a convex, increasing, function representing the cost incurred by using the average transmit-power of $\bar{p}_m$. Further, assume that $U_m(\bar{r}_m)$ and $J_m(\bar{p}_m)$ are differentiable. The metric to be optimized is then

$$f(\{\bar{r}_m\}, \{\bar{p}_m\}) := \sum_m U_m(\bar{r}_m) - J_m(\bar{p}_m)\,. \tag{1}$$

To account for the interference caused to the PU system [1, 13], consider first the binary random variable

$$i^{(q)}(\{p_m\}, \mathbf{s}) := \max_{\mathbf{x}\in\mathcal{G}} z_{\mathbf{x}}^{(q)} \mathbb{1}_{\{\sum_m w_m(\mathbf{g},\mathbf{s})p_m(\mathbf{g},\mathbf{s})h_{m,\mathbf{x}^{(q)}} > \Gamma\}}, \tag{2}$$

which equals 1 if the PU receiver q is interfered. Further, define the binary random variable $i(\{p_m\}, \mathbf{s}) := 1 - \prod_{q=1}^{Q}(1 - i^{(q)}(\{p_m\}, \mathbf{s}))$, which is 1 if one or more PU receivers are interfered. Since $w_m(\mathbf{g},\mathbf{s}) \in \{0, 1\}$ and at most one SU transmits per time slot, $i(\mathbf{g},\mathbf{s})$ can be rewritten as $i(\{p_m(\mathbf{g},\mathbf{s})\}, \mathbf{s}) = \sum_m w_m(\mathbf{g},\mathbf{s})\, i_m(p_m(\mathbf{g},\mathbf{s}), \mathbf{s})$, where

$$i_m(p_m(\mathbf{g},\mathbf{s}), \mathbf{s}) := 1 - \prod_{q=1}^{Q}\left(1 - \max_{\mathbf{x}\in\mathcal{G}} z_{\mathbf{x}}^{(q)} \mathbb{1}_{\{p_m(\mathbf{g},\mathbf{s})h_{m,\mathbf{x}^{(q)}} > \Gamma\}}\right) \tag{3}$$

depends only on the transmit-power of SU m. Let $i^{\max} \in (0, 1)$ denote the maximum long-term probability (rate) of interference. Then, the following constraint must hold

$$\mathbb{E}_{\mathbf{g},\mathbf{s}}\left[\sum\nolimits_m w_m(\mathbf{g},\mathbf{s})\, i_m(p_m(\mathbf{g},\mathbf{s}), \mathbf{s})\right] \leq i^{\max}\,. \tag{4}$$

Based on (1) and (4), the optimal RA subject to ("s. to") interference constraints is obtained as the solution of:

$$u^* := \max_{\{\bar{p}_m \geq 0\}, \{\bar{r}_m \geq 0\}, \mathbf{y}} \sum_{m=1}^{M} U_m(\bar{r}_m) - J_m(\bar{p}_m) \tag{5a}$$

$$\text{s.to}: \quad \mathbb{E}_{\mathbf{g},\mathbf{s}}\left[w_m(\mathbf{g},\mathbf{s})p_m(\mathbf{g},\mathbf{s})\right] \leq \bar{p}_m \tag{5b}$$

$$\mathbb{E}_{\mathbf{g},\mathbf{s}}\left[w_m(\mathbf{g},\mathbf{s})r_m(p_m(\mathbf{g},\mathbf{s}))\right] \geq \bar{r}_m \tag{5c}$$

$$\mathbb{E}_{\mathbf{g},\mathbf{s}}\left[\sum\nolimits_m w_m(\mathbf{g},\mathbf{s})\, i_m(p_m(\mathbf{g},\mathbf{s}), \mathbf{s})\right] \leq i^{\max} \tag{5d}$$

$$w_m(\mathbf{g},\mathbf{s}) \in \{0, 1\}, \text{ and } \sum\nolimits_m w_m(\mathbf{g},\mathbf{s}) \leq 1 \tag{5e}$$

where the constraints (5b) and (5c) have been relaxed from equalities to inequalities without loss of optimality, and the non negativity of $\{p_m\}$ is left implicit. Problem (5) is nonconvex because *i)* $\{w_m\}$ are binary variables; *ii)* the monomials $w_m p_m$ and $w_m r_m(p_m)$ are not *jointly* convex in w_m and p_m; and, *iii)* the interference constraint (5d) is nonconvex. Nevertheless, an *optimal* solution can be obtained by following the next three steps.

First, constraints $w_m(\mathbf{g},\mathbf{s}) \in \{0, 1\}$ can be relaxed as $w_m(\mathbf{g},\mathbf{s}) \in [0, 1]$. Next, to cope with the non convexity of $w_m p_m$ and $w_m r_m(p_m)$, consider introducing the dummy variables $\tilde{p}_m := w_m p_m$. Despite these changes, it can be shown that the solution of the reformulated problem is the same than that of (5); see e.g., [14] for details. Although the non-convexity of (5d) cannot be easily addressed, it turns out that it is possible to leverage the results of [15, Thm. 1] to show that the duality gap is *zero*. Consequently, a Lagrange dual approach can be adopted without loss of optimality.

Consider then dualizing the average constraints (5b), (5c) and (5d), and let $\{\pi_m\}$, $\{\rho_m\}$ and θ denote the Lagrange multipliers associated with (5b), (5c) and (5d), respectively. Exploiting the separability of the resultant Lagrangian across SU links, as well as the per-fading state separability [15, Thm. 4], the optimal variables $\{\bar{p}_m^*\}$, $\{\bar{r}_m^*\}$, $\{w_m^*\}$ and $\{p_m^*\}$ can be found as follows.

- *Average rate and power.* With $\{\pi_m^*\}$, $\{\rho_m^*\}$ denoting the optimal dual variables, $\bar{p}_m^*, \bar{r}_m^*$ are obtained as

$$\bar{p}_m^* := \arg\min_{p_m \geq 0} J_m(\bar{p}_m) - \pi_m^* \bar{p}_m \tag{6a}$$

$$\bar{r}_m^* := \arg\max_{r_m \geq 0} U_m(\bar{r}_m) - \rho_m^* \bar{r}_m \,. \tag{6b}$$

These are scalar convex optimization problems that can be efficiently solved. Further, when functions $\{U_m(\cdot)\}$ and $\{J_m(\cdot)\}$ are differentiable with invertible derivatives, it follows that $\bar{p}_m = \left[(\dot{J}_m)^{-1}(\pi_m^*)\right]_+$ and $\bar{r}_m = \left[(\dot{U}_m)^{-1}(\rho_m^*)\right]_+$, where $(\dot{U}_m)^{-1}(\cdot)$ and $(\dot{J}_m)^{-1}(\cdot)$ are the inverse of the derivative of $U_m(\cdot)$ and $J_m(\cdot)$.
- *Per-fading state scheduling and powers.* Defining the function

$$\varphi_m(g_m[t], p) := \rho_m^* r_m(g_m[t], p) - \pi_m^* p - \theta^* \mathbb{E}_{\mathbf{s}[t]}\left[i_m(p, \mathbf{s}[t])\right] \tag{6c}$$

the optimal $w_m^*(\mathbf{g}, \mathbf{s})$ and $p_m^*(\mathbf{g}, \mathbf{s})$ for all m and $\mathbf{g}, \mathbf{s}$ are given by

$$p_m^*[t] \quad := \left[\arg\max_{p} \ \varphi_m(g_m[t], p)\right]_0^\infty \tag{6d}$$

$$w_m^*[t] \quad := \mathbb{1}_{\{(m=\arg\max_n \varphi_n((p_n^*[t]))) \text{ and } (\varphi_m(p_n^*[t])>0)\}} . \tag{6e}$$

Key to understanding the solution of (5) is the definition of $\varphi_m(\cdot)$. Intuitively, (6c) can be interpreted as an instantaneous user-quality indicator where the transmit rate is rewarded with a price ρ_m^*, the transmit power is penalized with a price π_m^*, and the interference $\mathbb{E}_{\mathbf{s}[t]}\left[i_m(p, \mathbf{s}[t])\right]$ is penalized with a price θ^* [8]. Analytically, $\varphi_m(x)$ represents the contribution of user m to the Lagrangian of (5), when $p_m[t] = x$ and $w_m[t] = 1$. Using *(as1)*–*(as2)*, the interference term in (6c) can be simplified as $\mathbb{E}_{\mathbf{s}[t]}\left[i_m(p, \mathbf{s}[t])\right] = 1 - \prod_{q=1}^{Q}(1 - \sum_{\mathbf{x}\in\mathcal{G}} \iota_{m,\mathbf{x}} \beta_{\mathbf{x}}^{(q)})$. Further, if $h_{m,\mathbf{x}}$ is Rayleigh-distributed, it follows that $\iota_{m,\mathbf{x}} = e^{-\Gamma/(p_m \gamma_{m,\mathbf{x}})}$ with $\gamma_{m,\mathbf{x}}$ denoting the path loss between SU m and grid point $\mathbf{x}$ [12]. In any case, (6d) turns out to be generally nonconvex; however, since only one single scalar variable (the transmit power p) is involved, efficient methods can be employed to find $p_m^*[t]$.

2.1. Estimating the optimum Lagrange multipliers

Finding the optimal multipliers $\{\rho_m^*, \pi_m^*, \theta^*\}$ may be computational challenging because: a) they have to be found numerically by averaging over all possible realizations of $\mathbf{g}$ and $\mathbf{s}$; and, b) if either the channel statistics or the number of PUs/SUs change, $\{\pi_m^*, \rho_m^*, \theta^*\}$ must be recomputed. An alternative consists in resorting to stochastic approximation iterations [16,17], whose goal is to obtain samples $\{\pi_m[t]\}$, $\{\rho_m[t]\}$ and $\theta[t]$, $t = 1, 2, \ldots$ that are nevertheless sufficiently close to the optimal dual variables $\{\pi_m^*, \rho_m^*, \theta^*\}$. Specifically, with $\mu_\pi > 0$, $\mu_\rho > 0$ and $\mu_\theta > 0$ denoting the stepsizes, the following iterations yield the desired multipliers $\forall t$:

$$\pi_m[t+1] = [\pi_m[t] - \mu_\pi(\bar{p}_m^*(\pi_m[t] - w_m^*[t] p_m^*[t]))]_0^\infty \tag{7}$$

$$\rho_m[t+1] = [\rho_m[t] + \mu_\rho(\bar{r}_m^* - w_m^*[t] r_m(h_m[t], p_m^*[t]))]_0^\infty \tag{8}$$

$$\theta[t+1] = [\theta[t] - \mu_\theta(i^{\max} - i[t])]_0^\infty \,. \tag{9}$$

Notice that besides being computationally affordable, these techniques allow one to cope with non-stationary channels and PU occupancies. Updates (7)-(9) form an *unbiased* stochastic subgradient of the dual function of (5); see e.g., [18]. Using also that the updates in (7)-(9) are bounded, it can be shown that the sample average of the stochastic RA *i)* is feasible; and, *ii)* it incurs minimal performance loss relative to the optimal solution of (5). Rigorously stated, define $\mu := \max\{\mu_\pi, \mu_\theta, \mu_\vartheta\}$; $\bar{u}[t] := \frac{1}{t}\sum_m \sum_{l=1}^{t} U_m(w_m^*[l] r_m(p_m^*[l])) - J_m(p_m^*[l])$; and, $\bar{i}[t] := \frac{1}{t}\sum_{l=1}^{t} i[l]$. Then, it holds with probability one that as $t \to \infty$: *i)* $\bar{i}[t] = i^{\max}$; and, *ii)* $\bar{u}[t] \geq u^* - \delta(\mu)$, where $\delta(\mu) \to 0$ as $\mu \to 0$ (see, e.g. [17]).

3. RECEIVER MAP ESTIMATION

The SU network relies on the interference probabilities $\{\iota_{m,\mathbf{x}}\}$ and PU receivers' spatial distribution $\{\beta_{\mathbf{x}}^{(q)}\}$ to schedule SU transmissions and limit their powers based on the expected probability of interference $\mathbb{E}_{\mathbf{s}[t]}\left[i_m(p, \mathbf{s}[t])\right]$ [cf. (6c)]. Once the virtual grid $\mathcal{G}$ is chosen, $\{\iota_{m,\mathbf{x}}\}$ can be computed as a function of the transmit-powers p_m. The aim here is to develop an online Bayesian estimator for $\{\beta_{\mathbf{x}}^{(q)}\}$ based on the following assumptions.
(as3) The PU system notifies the SUs if disruptive interference occurred to one or more PU receivers.
(as4) An estimate (or an upper bound) of the number of PU receivers Q is available.
A *one*-bit message $i[t] = 1$ is sufficient to notify the SU system that the event $p_{m^*}^* h_{m^*,\mathbf{x}^{(q)}} > \Gamma$ occurred to at least one PU receiver; thus, *(as3)* entails just a minimal PU-SU message passing. This goes along the lines of [9,19] (see also references therein), where the PU's ARQs are assumed to be either exchanged or eavesdropped by the SU transceivers. Moreover, Section 4 will illustrate that *(as4)* is not very restrictive, since just an upper-bound on Q suffices to carry out the localization task. More sophisticated schemes that jointly estimate and track Q and $\{\beta_{\mathbf{x}}^{(q)}[t]\}$ will be the subject of future research.

To account for PU mobility, $z_{\mathbf{x}}^{(q)}[t]$ is modeled as a first-order (spatiotemporal) Markov process characterized by the transition probabilities $\phi_{\mathbf{x},\mathbf{x}'}^{(q)}[t] := \Pr\{z_{\mathbf{x}}^{(q)}[t] = 1 | z_{\mathbf{x}'}^{(q)}[t-1] = 1\}$. Such transition probabilities are assumed to be non-zero only if $\mathbf{x}' \in \mathcal{G}_{\mathbf{x}}$, where the set $\mathcal{G}_{\mathbf{x}}$ contains $\mathbf{x}$ and its neighbors. Collect in the set $\mathcal{I}_t := \{i[\tau], \tau = 1, \ldots, t\}$ the interference notifications up to time slot t, and define further the sets $\mathcal{H}_t := \mathcal{I}_t \cup \{p_m^*[\tau], w_m^*[\tau], \tau = 1, \ldots, t\}$ and $\tilde{\mathcal{H}}_t := \mathcal{H}_{t-1} \cup \{p_m^*[t], w_m^*[t], \forall m\}$. Since the elements of $\mathcal{I}_t$ constitute the observed states of a Hidden Markov Model (HMM), a recursive Bayesian estimator can be implemented to acquire (and track) the posterior probability mass function of $\{z_{\mathbf{x}}^{(q)}\}$. To this end, let $\beta_{\mathbf{x}}^{(q)}[t|t-1] := \Pr\{z_{\mathbf{x}}^{(q)}[t] = 1 | \mathcal{H}_{t-1}\}$ and $\beta_{\mathbf{x}}^{(q)}[t|t] := \Pr\{z_{\mathbf{x}}^{(q)}[t] = 1 | \mathcal{H}_t\}$ denote the instantaneous beliefs given $\mathcal{H}_{t-1}$ and $\mathcal{H}_t$, respectively. Thus, the receiver maps can be recursively updated by performing the following steps per grid point $\mathbf{x}$ and PU receiver q (see e.g., [20]).
Prediction step:

$$\beta_{\mathbf{x}}^{(q)}[t|t-1] = \textstyle\sum_{\mathbf{x}' \in \mathcal{G}_{\mathbf{x}}} \phi_{\mathbf{x},\mathbf{x}'}^{(q)}[t] \beta_{\mathbf{x}}^{(q)}[t-1|t-1] \,. \tag{10}$$

Correction step:

$$\beta_{\mathbf{x}}^{(q)}[t|t] = \frac{\Pr\{i[t] = o | z_{\mathbf{x}}^{(q)}[t] = 1, \mathcal{H}_{t-1}\} \beta_{\mathbf{x}}^{(q)}[t|t-1]}{\Pr\{i[t] = o | \tilde{\mathcal{H}}_t\}} \tag{11}$$

where $o \in \{0, 1\}$ denotes the value observed for $i[t]$. To simplify (11), notice that $z_{\mathbf{x}}^{(q)}[t] = 1$ implies that $z_{\mathbf{x}'}^{(q)}[t] = 0$ for the grid points $\mathbf{x}' \in \mathcal{G} \backslash \{\mathbf{x}\}$. Thus, it follows that $\Pr\{i[t] = 1 | z_{\mathbf{x}}^{(q)}[t] = 1, \mathcal{H}_{t-1}\}$ is given by

$$\Pr\{i^{(q)}[t] = 1 | z_{\mathbf{x}}^{(q)}[t] = 1, \mathcal{H}_{t-1}\} = \iota_{m,\mathbf{x}}[t] \,. \tag{12}$$

Clearly, $\Pr\{i^{(q)}[t] = 0 | z_{\mathbf{x}}^{(q)}[t] = 1, \mathcal{H}_{t-1}\} = 1 - \iota_{m,\mathbf{x}}[t]$. Furthermore, using *(as1)*, *(as2)*, and *(as4)*, the denominator of (11) is

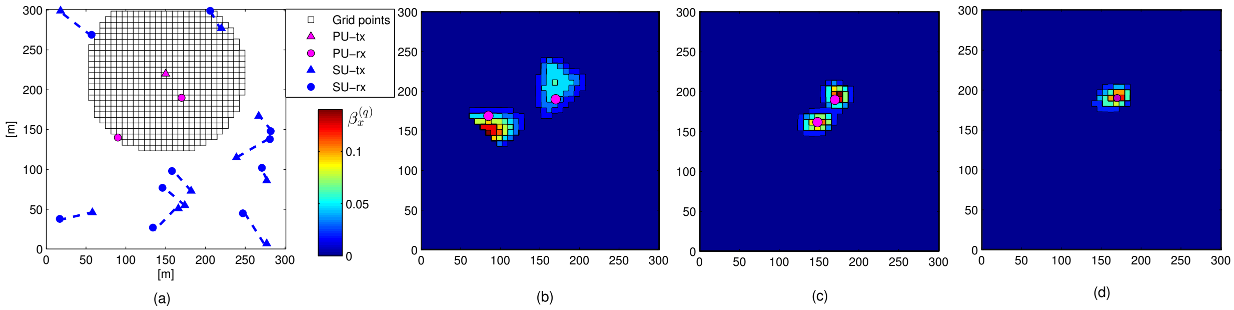

Fig. 1. (a) Scenario. (b) Receiver map at $t = 100$, purple dots represent the actual PU locations. (c) Map at $t = 2000$. (d) Map at $t = 5000$.

simplified as

$$\Pr\{i[t] = 1|\mathcal{H}_{t-1}\} = 1 - \prod_{q=1}^{Q} \left(1 - \sum_{\mathbf{x}\in\mathcal{G}} \iota_{m,\mathbf{x}}[t]\beta_{\mathbf{x}}^{(q)}[t|t-1]\right) \quad (13)$$

while $\Pr\{i[t] = 0|\mathcal{H}_{t-1}\}$ is computed in the obvious way. With a slight increase on complexity, the correction step can be modified to account for errors in $i[t]$.

Thus, the proposed joint RA and map estimation algorithm amounts to implementing the following steps at each time slot t.

[S1] Perform the prediction step (10).

[S2] Based on $\{\beta_{\mathbf{x}}^{(q)}[t|t-1]\}$, find $\{\bar{r}_m[t], \bar{p}_m[t], w_m[t], p_m[t]\}$ via (6), where the multipliers are obtained using (7)-(9) and the term $\mathbb{E}_{\mathbf{s}[t]}[i_m(p, \mathbf{s}[t])]$ in (6c) is replaced with the belief $\mathbb{E}_{\mathbf{s}[t]}[i_m(p, \mathbf{s}[t]) |\mathcal{H}_{t-1}] = 1 - \prod_{q=1}^{Q}(1 - \sum_{\mathbf{x}\in\mathcal{G}} \iota_{m,\mathbf{x}}(p)\beta_{\mathbf{x}}^{(q)}[t|t-1])$.

[S3] Acquire $i[t]$, and run the correction step (11).

4. NUMERICAL RESULTS

Consider the scenario depicted in Fig. 1(a), where $M = 10$ SU transmitters are deployed in a 300×300 m area. A PU source (marked with a purple triangle) communicates with 2 PU receivers (marked with purple circles). The first PU receiver is: located at $\mathbf{x}^{(1)} = (x = 190, y = 170)$; static; and served by the PU source during the entire simulation interval $t \in [1, 8000]$. The second PU is: located at $\mathbf{x}^{(2)} = (90, 140)$, mobile, with $\phi_{\mathbf{x},\mathbf{x}'}^{(q)}[t] = 0.05$ $\forall \mathbf{x}' \in \mathcal{G}_{\mathbf{x}}$; and communicates with the source only during the interval $[1, 4000]$. The PU system is protected by setting $\Gamma = -70$ dB [cf. (2)] and $i^{\max} = 0.05$ [cf. (4)].

From the sensing phase, the SU system acquires an estimate of the location of the PU source, and of its coverage region (see e.g., [2–4]). The PU coverage region is discretized using uniformly spaced grid points, each one covering an area of 8×8 m. The multipliers are initialized as $\rho_m[0] = 0.1$, $\pi_m[0] = 0.03$, and $\theta[0] = 20$, while the step sizes are set to $\mu_\rho = 0.03$, $\mu_\pi = 0.005$, and $\mu_\theta = 0.2$. To assess robustness of the proposed framework to model mismatches, it is assumed that: *i)* the SUs have imperfect knowledge of the PU transition probabilities, which are supposed to be $\hat{\phi}_{\mathbf{x},\mathbf{x}'}^{(q)} = 0.01$ for both receivers; and, *ii)* the presumed number of PUs is always $Q = 2$, even in the interval $[4000, 8000]$, where the second PU receiver is not active. The per-SU utilities are set to $U_m(\bar{r}_m) = \log(\bar{r}_m)$ and $J_m(\bar{p}_m) = \bar{p}_m^2/4$. Finally, $k_m = 1$ for all m, the path loss obeys the model $\gamma_{m,\mathbf{x}} = \|\mathbf{x}_m - \mathbf{x}\|_2^{-3.5}$, and the small-scale fading is Rayleigh distributed.

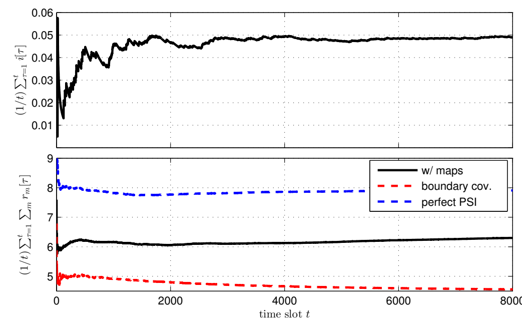

Fig. 2. Convergence and performance of the RA.

Pictorially, performance of the receiver localization scheme can be assessed through the maps shown in Figs. 1(b), (c) and (d). The value (color) of a point in the map represents the sum of the beliefs $\sum_q \beta_{\mathbf{x}}^{(q)}[t|t]$ at the corresponding grid point $\mathbf{x}$ [cf. (11)]. The chromatic scale uses blue for low (belief) values and red for high ones. Each of the maps corresponds to a different time instant, namely $t = 200$, $t = 2000$ and $t = 5000$. A uniform distribution across the east and the west halves of the region is used for $\beta_{\mathbf{x}}^{(1)}[0|0]$ and $\beta_{\mathbf{x}}^{(2)}[0|0]$, respectively. It can be seen that after 100 time slots it is already possible to unveil the areas where the PU receivers are likely to reside. Clearly, as time goes by, the localization accuracy improves as corroborated by Fig. 1(c). Recall that only one PU receiver is served by the PU source when $t > 4000$. Indeed, as shown in Fig. 1(d), the receiver map peaks at only the actual location of the PU receiver. Furthermore, the numerical results reveal that the two beliefs $\beta_{\mathbf{x}}^{(1)}[t|t]$ and $\beta_{\mathbf{x}}^{(2)}[t|t]$ are approximately the same for all $\mathbf{x}$, thus indicating that just an upper bound on Q is sufficient.

In Fig. 2, convergence and feasibility of the RA schemes are tested. The upper subplot demonstrates that the running average $\bar{i}[t]$ approaches its limit $i^{\max}$ around $t \approx 3500$; then, after a transient period that begins at $t = 4000$, it converges again around $t \approx 5000$. The running average of the sum rate $(1/t)\sum_m \sum_\tau r_m[\tau]$ is depicted in the lower subplot of Fig. 2. The rates achieved by the proposed scheme are compared with those obtained by: S1) a scheme where the beliefs are set to 1 for grid points in the boundary of the PU coverage region [4, 7], and, S2) a scheme where perfect PSI (including that of SU-to-PU instantaneous links) is available. The results illustrate that our scheme clearly outperforms S1, motivating the additional complexity required to implement the map estimator.

Additional numerical results, along with RA schemes robust to interference notification errors are reported in [21].

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