FEASIBILITY OF INTERFERENCE NEUTRALIZATION IN RELAY-AIDED MIMO INTERFERENCE BROADCAST CHANNEL WITH PARTIAL CONNECTIVITY

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ABSTRACT

In this paper, we study the feasibility of interference neutralization (*IN*) in partially connected relay-aided multi-input-multi-output (MIMO) interference broadcast channel (IBC), where each relay only communicates with users in the same cell. Due to partial connectivity, there is no need to exchange channel information among the relays in different cells. We first present the necessary and sufficient condition for *IN* feasibility using linear transceiver. We then provide the minimum relay configuration required to support the maximal number of data streams without interference. Finally, the achievable degrees-of-freedom region is obtained.

***Index Terms*—** Partially connected, relay-aided interference broadcast channel, degrees of freedom, spatial resources

1. INTRODUCTION

Besides improving the transmission coverage and reliability, relays are introduced to assist interference management recently [1–3]. While *interference alignment (IA)* is promising to achieve high degrees of freedom (DoF) of interference channel (IC) and interference broadcast channel (IBC) [4], *interference neutralization (IN)* is an essential interference management mechanism in relay-aided interference network [3]. When there are more than one propagation paths from a source to its interfering destination, which is a natural scenario for relay systems, multiple copies of interference reaching each user can sum up to zero. In other words, the interferences are eliminated over the air, i.e., *neutralized*.

According to the connectivity of base stations (BSs), relays and users, the relay-aided interference system can be categorized into two setups of particular interest. In a fully connected system, relays have connectivity to all the BSs and users in the system. This models cellular system with relays located in the middle of the BS and user in one cell. In this setup, the achievable DoF of a B-cell single-input-single-output (SISO)-IC with single relay or multiple two-hop relays were obtained in [5, 6]. For multi-input-multi-output (MIMO)-IC, the study on achievable DoF focused on the two-cell case [7, 8]. For more than two-cell MIMO-IC, a DoF upper bound was obtained in [9], whose tightness was not analyzed. In a partially connected system, relays have connectivity to all the BSs but only to the users within the same cell. This models the cellular system with low power relays located near the users. In this setup, the DoF of relay aided SISO-IC was analyzed in [10] by converting the system into an IC with two symbol extensions.

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The partially connected relay-aided MIMO-IBC is a less well studied scenario in aforementioned prior work yet a practical setup in cellular system. To understand its potential, we study the feasibility condition of *IN* without symbol extensions in this paper, from which the achievable DoF with linear transceiver can be obtained. We consider partially connected setup with a single full-duplex amplify-and-forward (AF) relay in each cell, which only communicates with the users within the same cell. Instead of converting the relay-aided system to an IC and apply the results of IA as in [10], we directly derive the necessary and sufficient conditions for *IN* feasibility, then find the minimum relay configuration required to support the maxinmum number of data streams without interference. Finally, the achievable DoF region is obtained as a consequence.

Notations: Transpose and conjugate transpose are represented by $(.)^T$ and $(.)^H$, respectively. $\otimes$ is the Kronecker product operator, $vec(\boldsymbol{A})$ is the vectorization operator that concatenates the columns of matrix $\boldsymbol{A}$ into a column vector.

2. SYSTEM MODEL

Consider a B-cell MIMO-IBC, where K_b users are located in the bth cell, and the kth user in cell b, user b_k, intends to receive d_{b_k} data streams from BS b, $b = 1, \cdots, B$. The total number of data streams in cell b is $D_b = \sum_{k=1}^{K_b} d_{b_k}$. To decouple the impact of *IN* and *IA*, we consider a scenario where neither the BSs nor the users are able to deal with inter-cell interference (ICI). Then, the number of antennas at BS b and user b_k are respectively D_b and d_{b_k}. Note that in this setting, the users suffer from severe interference directly from BSs in adjacent cells, where the relays are deployed to help eliminate

ICIs. We consider that a single full-duplex AF relay is located near the users in each cell, so that their forwarded signals can be only received by users in the same cell. In this way, the users receive signals both through the direct links from the BSs and the relay links within the same cell. This scenario is different from the two-hop relay interference networks considered in literature [2, 5–8]. The number of antennas at each relay is M_b. Such a partially connected relay-aided MIMO-IBC is denoted as $\prod_{b=1}^{B}\left(K_b, \sum_{k=1}^{K_b} d_{b_k}, M_b\right)$, and a partially connected symmetric relay-aided MIMO-IBC is denoted as $(K, Kd, M)^B$.

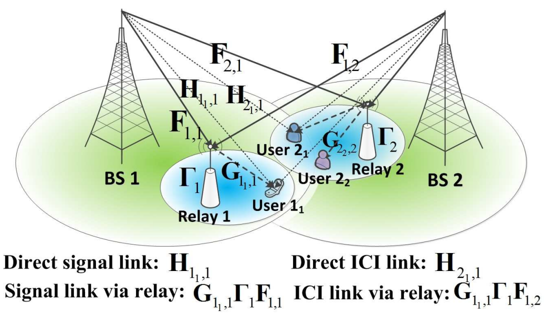

Fig. 1. Example of a partially connected relay-aided MIMO IBC, $B = 2, K_1 = 1, K_2 = 2$

Denote $\boldsymbol{H}_{b_k,b'} \in \mathbb{C}^{d_{b_k}\times D_{b'}}$ as the channel matrix from BS b' to user b_k, $\boldsymbol{F}_{b,b'} \in \mathbb{C}^{M_b\times D_{b'}}$ as the channel matrix from BS b' to relay b, $\boldsymbol{G}_{b_k,b} \in \mathbb{C}^{d_{b_k}\times M_b}$ as the channel matrix from relay b to user b_k, where $b, b' = 1, \cdots, B$, and $b_k = 1, \cdots, K_b$. All elements in the channel matrices are assume as identically and independent distributed ($i.i.d.$) random variables.

User b_k receives its desired signals through two types of propagation links, which are *direct signal link* and *signal link via relay*. Similarly, it receives ICI through two types of links, which are *direct ICI link* and *ICI link via relay*. The received signal at user b_k can be expressed as

$$\begin{aligned}
\boldsymbol{Y}_{b_k} = & \underbrace{\boldsymbol{U}_{b_k}^H\left(\boldsymbol{H}_{b_k,b} + \boldsymbol{G}_{b_k,b}\boldsymbol{\Gamma}_b\boldsymbol{F}_{b,b}\right)\boldsymbol{V}_{b_k}\boldsymbol{x}_{b_k}}_{\text{Desired signal}} \\
& + \sum_{k'=1,k'\neq k}^{K_b} \underbrace{\boldsymbol{U}_{b_k}^H\left(\boldsymbol{H}_{b_k,b} + \boldsymbol{G}_{b_k,b}\boldsymbol{\Gamma}_b\boldsymbol{F}_{b,b}\right)\boldsymbol{V}_{b_{k'}}\boldsymbol{x}_{b_{k'}}}_{\text{Multi-user Interference}} \\
& + \sum_{b'=1,b'\neq b}^{B} \underbrace{\boldsymbol{U}_{b_k}^H\left(\boldsymbol{H}_{b_k,b'} + \boldsymbol{G}_{b_k,b}\boldsymbol{\Gamma}_b\boldsymbol{F}_{b,b'}\right)\boldsymbol{V}_{b'}\boldsymbol{x}_{b'}}_{\text{Inter-cell Interference}} \\
& + \underbrace{\boldsymbol{U}_{b_k}^H\left(\boldsymbol{n}_{b_k} + \boldsymbol{G}_{b_k,b}\boldsymbol{\Gamma}_b\boldsymbol{n}_b\right)}_{\text{Effective noise}},
\end{aligned} \tag{1}$$

where $\boldsymbol{x}_{b_k} \in \mathbb{C}^{d_{b_k}\times 1}$ is the symbol vector for user b_k satisfying $E\left\{\boldsymbol{x}_{b_k}^H\boldsymbol{x}_{b_k}\right\} = Pd_{b_k}$, P is the transmit power per symbol, $\boldsymbol{x}_b = \left[\ \boldsymbol{x}_{b_1}^T, \cdots, \boldsymbol{x}_{b_{K_b}}^T\ \right]^T$ is the symbol vector for the K_b users in cell b, $\boldsymbol{V}_{b_k} \in \mathbb{C}^{D_b\times d_{b_k}}$ is the transmit matrix for user b_k satisfying $Tr\left\{\boldsymbol{V}_{b_k}^T\boldsymbol{V}_{b_k}\right\} = 1$, $\boldsymbol{V}_b = \left[\ \boldsymbol{V}_{b_1}, \cdots, \boldsymbol{V}_{b_K}\ \right] \in \mathbb{C}^{D_b\times D_b}$ is the transmit matrix for the K_b users in cell b, $\boldsymbol{U}_{b_k} \in \mathbb{C}^{d_{b_k}\times d_{b_k}}$ is the receive matrix at user b_k, $\boldsymbol{\Gamma}_b \in \mathbb{C}^{M_b\times M_b}$ is the processing matrix at the relay in cell b, and $\boldsymbol{n}_b \in \mathbb{C}^{M_b\times 1}$ and $\boldsymbol{n}_{b_k} \in \mathbb{C}^{d_{b_k}\times 1}$ are the additive white Gaussian noise at relay b and user b_k, respectively.

In order to ensure a total number of $\sum_{b=1}^{B} D_b$ data streams to be transmitted without interference, the linear transceivers $\boldsymbol{V}_{b_k}$, $\boldsymbol{\Gamma}_b$ and $\boldsymbol{U}_{b_k}$ should satisfy the following interference-free transmission equations

$$rank\left\{\boldsymbol{U}_{b_k}^H\left(\boldsymbol{H}_{b_k,b} + \boldsymbol{G}_{b_k,b}\boldsymbol{\Gamma}_b\boldsymbol{F}_{b,b}\right)\boldsymbol{V}_{b_k}\right\} = d_{b_k}, \tag{2a}$$

$$\boldsymbol{U}_{b_k}^H\left(\boldsymbol{H}_{b_k,b} + \boldsymbol{G}_{b_k,b}\boldsymbol{\Gamma}_b\boldsymbol{F}_{b,b}\right)\boldsymbol{V}_{b_{k'}} = \mathbf{0}, k' \neq k, \tag{2b}$$

$$\boldsymbol{U}_{b_k}^H\left(\boldsymbol{H}_{b_k,b'} + \boldsymbol{G}_{b_k,b}\boldsymbol{\Gamma}_b\boldsymbol{F}_{b,b'}\right)\boldsymbol{V}_{b'} = \mathbf{0}, b' \neq b, \tag{2c}$$

where (2a) is the data transmission constraint to ensure that each user receives d_{b_k} desired data streams, and (2b) and (2c) are the zero-forcing constraints to eliminate multi-user interference (MUI) and ICI.

3. INTERFERENCE NEUTRALIZATION FEASIBILITY

In the following, we first show that in relay-aided MIMO-IBC, the interference-free transmission equations reduce to an *IN equation*. Then we derive the necessary and sufficient condition by studying the solvability of the *IN equation*.

3.1. IN equation

From the viewpoint of each BS, data transmission constraint (2a) and MUI-free constraint (2b) ensure that BS b transmit D_b data streams to all the K_b users in cell b without interference. It indicates that the transmit matrix $\boldsymbol{V}_b$ and the receive matrix $\boldsymbol{U}_{b_k}^H$ are full rank, and the effective channel matrix from BS b to all the users in cell b is full rank. Therefore, the interference-free transmission equations (2a)-(2c) are equivalent to the following two equations

$$rank\left(\boldsymbol{H}_{b,b} + \boldsymbol{G}_{b,b}\boldsymbol{\Gamma}_b\boldsymbol{F}_{b,b}\right) = D_b, \tag{3a}$$

$$\bar{\boldsymbol{H}}_b + \boldsymbol{G}_{b,b}\boldsymbol{\Gamma}_b\bar{\boldsymbol{F}}_{b,b} = \mathbf{0}, \tag{3b}$$

where $\boldsymbol{U}_{b_k}^H$ and $\boldsymbol{V}_b$ are eliminated because they are invertible, $\boldsymbol{H}_{b,b} = \left[\boldsymbol{H}_{b_1,b}^T \ \cdots \ \boldsymbol{H}_{b_{K_b},b}^T\right]^T$ is the channel matrix composed of all the *direct signal links* in cell b, $\boldsymbol{G}_{b,b} = \left[\boldsymbol{G}_{b_1,b}{}^T \ \cdots \ \boldsymbol{G}_{b_{K_b},b}{}^T\right]^T$ is composed of the channel matrix from relay b to all the users in cell b, $\bar{\boldsymbol{H}}_b = \left[\boldsymbol{H}_{b,1} \ \cdots \ \boldsymbol{H}_{b,b-1} \ \boldsymbol{H}_{b,b+1} \ \cdots \ \boldsymbol{H}_{b,B}\right]$ is the channel matrix of all the *direct ICI links* to the users in cell b, and $\bar{\boldsymbol{F}}_{b,b} = \left[\boldsymbol{F}_{b,1} \ \cdots \ \boldsymbol{F}_{b,b-1} \ \boldsymbol{F}_{b,b+1} \ \cdots \ \boldsymbol{F}_{b,B}\right]$ is the channel matrix from all the BSs except BS b to relay b.

It is seen from (3b) that the relay processing matrix $\boldsymbol{\Gamma}_b$ is a function of the channel matrices $\bar{\boldsymbol{H}}_b$, $\boldsymbol{G}_{b,b}$ and $\bar{\boldsymbol{F}}_{b,b}$. Therefore, the second term in (3a) is statistically independent of the first term $\boldsymbol{H}_{b,b}$. Since the elements of $\boldsymbol{H}_{b,b}$ are $i.i.d.$, the first term in (3a) is full rank almost surely. Then, it is not hard to show that the constraint (3a) is satisfied almost surely. As a result, the feasibility of interference-free transmission only depends on the solvability of (3b). After applying the property of Kronecker product $vec\,(\boldsymbol{AXB}) = \left(\boldsymbol{B}^T \otimes \boldsymbol{A}\right) vec\,(\boldsymbol{X})$, we obtain the *IN equation* as

$$\boldsymbol{C}_b \boldsymbol{\gamma}_b = -\boldsymbol{h}_b, \tag{4}$$

where $\boldsymbol{C}_b \triangleq \bar{\boldsymbol{F}}_{b,b}^T \otimes \boldsymbol{G}_{b,b} \in \mathbb{C}^{D_b\left(\sum_{b' \neq b} D_{b'}\right) \times M_b^2}$ is the effective coefficient matrix, $\boldsymbol{\gamma}_b = vec(\boldsymbol{\Gamma}_b)$ consists of the unknown variables in the processing matrix of relay b, and $\boldsymbol{h}_b = vec(\bar{\boldsymbol{H}}_b)$ consists of the channel coefficients of the *direct ICI links*.

Each element in vector $\boldsymbol{h}_b$ is the channel coefficient from one transmit antenna at BS b' to one receive antenna at a user in cell b, where $b' \neq b$. Each element in $\boldsymbol{C}_b \boldsymbol{\gamma}_b$ is the effective channel coefficient of the *ICI link via relay* b between the same pair of transmit and receive antennas. Therefore, the intuitive meaning of the *IN equation* is as follows. The ICIs received at each user through the direct ICI link and through the relay should have the same amplitude but opposite signs, such that the two copies of the ICI can be neutralized.

3.2. Necessary and sufficient condition for IN feasibility

From the definitions of the matrices defined above, we see that no matter whether the streams of each BS are destined to multiple users or to one user, $\bar{\boldsymbol{H}}_b$ and $\boldsymbol{G}_{b,b}$ have the same form. In other words, as long as $D_b = \sum_{k=1}^{K_b} d_{b_k}$ data streams are transmitted in cell b, the IN equation is the same regardless of how the data streams are distributed within the cell. This means that we can analyze the feasibility of relay-aided MIMO-IBC and MIMO-IC in a unified way. We thus have the following theorem presenting the necessary and sufficient condition for IN feasibility of the $\prod_{b=1}^{B}(K_b, D_b, M_b)$ system which is determined by the solvability of the *IN equation* (4). This result sheds lights on how relay resources should be configured to neutralize ICIs.

Theorem: *For the partially connected asymmetric relay-aided MIMO-IBC $\prod_{b=1}^{B}(K_b, D_b, M_b)$, a total number of $\sum_{b=1}^{B} D_b$ interference-free data streams can be transmitted iff $\boldsymbol{C}_b$ is full row rank, i.e.,*

$$rank\,(\boldsymbol{C}_b) = D_b \left(\sum_{b' \neq b} D_{b'} \right). \tag{5}$$

Proof: According to linear algebra, (4) is solvable iff

$$rank\,(\boldsymbol{C}_b) = rank\,([\boldsymbol{C}_b, -\boldsymbol{h}_b]). \tag{6}$$

Since the elements of $\boldsymbol{h}_b$ are $i.i.d.$, they are statistically independent of the elements of $\boldsymbol{C}_b$. If $\boldsymbol{C}_b$ is not full row rank, adding the column vector $\boldsymbol{h}_b$ to the matrix $\boldsymbol{C}_e$ increases its rank with probability one, which means (6) can not be satisfied. As a result, $\boldsymbol{C}_e$ should be full row rank. ■

The theorem implies that we need to configure the relay resources in a way such that the ICI at each user is "resolvable": the effective channel coefficients of the *ICI links via relay* from one transmit antenna at the BS to one receive antenna at the user should be statistically independent. This is because (5) ensures that the matrix $\boldsymbol{C}_b$ is full row rank, i.e., its rows are linearly independent among each other. Then, after multiplying the relay processing vector $\boldsymbol{\gamma}_b$, the effective channel coefficients of the *ICI link via relay* b are statistically independent. Since the channels of the *direct ICI links* are $i.i.d.$, the ICIs generated by each data stream can be neutralized by adjusting the relay amplifying coefficients.

Remark 1: It is shown from (4) that relay b needs to know $\boldsymbol{G}_{b,b}$ and $\bar{\boldsymbol{F}}_{b,b}^T$, which are the channels of the links connected to itself. Apart from that, relay b also needs to know $\bar{\boldsymbol{H}}_b$, the channels of the direct ICI links of the users in cell b, which can be fed back by the users together with the relay-user channel $\boldsymbol{G}_{b,b}$. This way, the channel information required by relay b can be obtained without channel exchange among cells. After all the ICIs are neutralized by the relays, the B-cell network degrades to B parallel multi-user MIMO (MU-MIMO) systems. Therefore, each BS only needs to have the channel information of its serving users and employ single-cell MU-MIMO precoding to avoid MUI.

4. MINIMUM RELAY CONFIGURATION AND MAXIMUM ACHIEVABLE DOF REGION

4.1. Minimum relay configuration

In this section, we present a corollary from the theorem in section 3.2 , which gives the minimal number of relay antennas required to support a maximum of $\sum_{b=1}^{B} D_b$ interference-free data streams in the B-cell system.

Corollary: *For the partially connected asymmetric relay-aided MIMO-IBC $\prod_{b=1}^{B}(K_b, D_b, M_b)$, a total number of $\sum_{b=1}^{B} D_b$ interference-free data streams can be transmitted iff*

$$M_b \geq max \left\{ D_b, \sum_{b' \neq b} D_{b'} \right\}. \tag{7}$$

Proof: According to the property of Kronecker product, the rank of the coefficient matrix $\boldsymbol{C}_b$ in (4) is given as

$$rank\,(\boldsymbol{C}_b) = rank\left(\bar{\boldsymbol{F}}_{b,b}^T\right) rank\,(\boldsymbol{G}_{b,b}), \tag{8}$$

where the dimension of $\bar{\boldsymbol{F}}_{b,b}^T$ and $\boldsymbol{G}_{b,b}$ are respectively $\sum_{b' \neq b} D_{b'} \times M_b$ and $D_b \times M_b$. From the structure of these matrices in (3a) and (3b), we know that their elements are all $i.i.d.$. Consequently, the ranks of $\bar{\boldsymbol{F}}_{b,b}^T$ and $\boldsymbol{G}_{b,b}$ are determined by the minimum number of their rows and columns.

When (7) does not hold, either $\bar{\boldsymbol{F}}_{b,b}^T$ or $\boldsymbol{G}_{b,b}$ does not have full row rank. In this case, $rank\,(\boldsymbol{C}_b) < D_b(\sum_{b' \neq b} D_{b'})$, which does not satisfy condition (5) in the theorem.

When (7) holds, $\bar{\boldsymbol{F}}_{b,b}^{T}$ and $\boldsymbol{G}_{b,b}$ both have full row rank, therefore, according to (8), $\boldsymbol{C}_b$ has full row rank. According to the theorem, this means that (7) is the necessary and sufficient condition for supporting a total number of $\sum_{b=1}^{B} D_b$ interference-free data streams. ■

The corollary indicates that in order to make ICI resolvable, the $\sum_{b'\neq b} D_{b'}$ ICIs received at one antenna at each user should be resolvable, and the ICIs generated from one data stream from one BS to all the D_b receive antennas of the users in cell b should also be resolvable. The former is ensured by $M_b \geq \sum_{b'\neq b} D_{b'}$, which makes ICI statistically independent at the relay. The latter is ensured by $M_b \geq D_b$, such that relay b provides at least D_b statistically independent copies of the same ICI.

Remark 2: For the partially connected symmetric relay-aided MIMO-IBC $(K, D, M)^B$, the minimum number of relay antennas in each cell is $M = (B-1)D$. When there is no direct link between BSs and users, the system reduces to a concatenation of MIMO-IBC (BS-relay) and MIMO broadcast channel (relay-users). In this case, each relay needs to eliminate all ICI meanwhile transmit all data streams to the users within the same cell, then the required number of antennas at each relay is BD. This gives rise to an interesting observation: although the direct links from BSs to users cause ICI from the BSs to the users, these links can help neutralize the ICI resulting in the reduced number of relay antennas.

4.2. Maximum achievable DoF region

The minimum number of relay antennas in (7) directly leads to the achievable DoF region of the system when given the number of relay antennas in each cell, which is

$$D_b \leq M_b, \tag{10}$$

$$\sum_{b'=1, b'\neq b}^{B} D_{b'} \leq M_b. \tag{11}$$

From (7), we know that $M_b \geq \sum_{b'\neq b} D_{b'}$. Therefore, the total number of relay antennas required in the system is

$$\sum_{b=1}^{B} M_b \geq \sum_{b=1}^{B} \sum_{b'\neq b} D_{b'} \geq (B-1) \sum_{b=1}^{B} D_b. \tag{12}$$

Since the minimum number of antennas at each relay in symmetric system is $M = (B-1)D$, the total number of relay antennas is $BM = (B-1)BD$. We know from (12) that when the total DoFs of the symmetric and asymmetric systems are identical, i.e., $\sum_{b=1}^{B} D_b = BD$, the total number of relay antennas in an asymmetric system is always greater than or equal to that of a symmetric system. Conversely, this means that with the same total numbers of relay antennas, the DoF of a symmetric system is always no less than that of an asymmetric system.

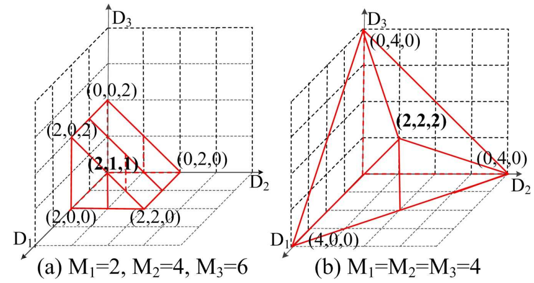

Fig. 2. DoF region of a partially connected three-cell relay-aided MIMO-IBC, $M_1 + M_2 + M_3 = 12$. (a) asymmetric system, (b) symmetric system.

In Fig. 2, we show the DoF region of a partially connected three-cell relay-aided MIMO-IBC with different numbers of antennas at relays. As illustrated in Fig. 2(a), the maximum achievable DoF in each cell is restricted by the relay with minimal number of antennas, i.e., $M_1 = 2$. In Fig. 2(b), the maximum achievable DoF in the symmetric system is $(D_1, D_2, D_3) = (2, 2, 2)$, and the total DoF achieves the maximum value of the partially connected relay-aided MIMO-IBC with $M_1 + M_2 + M_3 = 12$.

5. CONCLUSION

In this paper, we studied IN feasibility in partially connected relay-aided MIMO-IBC. We provided and proved the necessary and sufficient condition for IN feasibility. Then, the minimum relay configuration required to support a given number of non-interfering data streams and the maximal DoF achieved by linear transceiver were obtained. The result shows that the required number of relay antennas is reduced with the direct link between BSs and users, and the achievable total DoF is maximized when the system is symmetric.

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