PERFORMANCE GAINS DUE TO IMPROPER SIGNALS IN MIMO BROADCAST CHANNELS WITH WIDELY LINEAR TRANSCEIVERS

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ABSTRACT

Proper Gaussian signals have been shown to be optimal in multiple-input multiple-output (MIMO) broadcast channels from an information theoretic point of view, i.e., capacity can be achieved with a strategy that transmits circularly symmetric complex Gaussian signals. In this work, we show that optimality of proper Gaussian signals does not necessarily hold if the transmit strategy is restricted to widely linear transceivers. The proof is performed by identifying a rate tuple that is achievable in a certain set of channels with widely linear transceivers and improper Gaussian signals, but lies outside the achievable rate region for widely linear transceivers and proper Gaussian signals.

Index Terms— Asymmetric complex signaling, broadcast channels, improper signals, multiuser MIMO systems, widely linear transceivers.

1. INTRODUCTION

It is well known that proper [1, 2], i.e., circularly symmetric, complex Gaussian signals achieve the capacity of single-user multiple-input multiple-output (MIMO) channels with additive proper complex Gaussian noise [3]. The same is true for multiple-input single-output (MISO) and multiple-input multiple-output (MIMO) broadcast channels (BC) since the capacity-achieving dirty paper coding (DPC) makes use of proper Gaussian signals [4, 5]. Even if DPC is not applied, the assumption of proper complex Gaussian transmit signals is generally adopted in the literature about Gaussian broadcast channels. However, recent investigations have suggested that in systems not applying DPC, gains can be achievable by employing improper signals [6].

As even approximate DPC (e.g. [7]) is prohibitively complex for implementation in practical systems, many researchers have studied so-called *linear transceivers* where nonlinear operations (encoding, detection, ...) are only applied to single data streams while all filtering operations that involve more than one data stream have to be linear (e.g., [8]). In order to adequately deal with improper signals, this concept has to be generalized to *widely linear transceivers* (cf. Section 2) by allowing the originally linear filters to be widely linear, i.e., the filter output depends linearly on the real and imaginary part of the input or, equivalently, on the input and its complex conjugate [2, 9]. As a complex-valued multiplication has to be implemented as four real-valued multiplications anyway, treating real and imaginary part separately, i.e., multiplying them by different coefficients, does not increase the computational complexity of the filtering operation. Therefore, widely linear filters are as suitable for low-complexity implementations as strictly linear filters.

In Section 3, we discuss that for proper Gaussian transmit signals, the optimal widely linear transceivers are (strictly) linear. Due to the combination of widely linear processing *and* improper signaling, we can achieve gains in comparison to transmission of proper Gaussian signals using the well-established linear transceivers or, equivalently, using widely linear transceivers.

Widely linear processing is usually applied to communication systems that have to employ improper signal constellations (for instance, to comply with existing specifications and standards), e.g., [10–16], and to systems that encounter improper noise, e.g., [17]. However, even if the *noise is proper* and the system specifications *do not enforce* the use of an improper signal constellation, applying widely linear processing to introduce artificial improperness can be beneficial. While this aspect is known for interference channels [18–20], it has not yet been studied for broadcast channels, apart from the following first discoveries in our recent work [6].

Therein, it was shown that in a MIMO broadcast channel with widely linear transceivers, some minimum rate requirements might be feasible with improper Gaussian signals, but infeasible even with arbitrarily high transmit power in the case of proper Gaussian signals. As such feasibility considerations only play a role for quality of service problems without time-sharing[1] in systems with more users than degrees of freedom, this result is rather specific, but it clearly motivates further research on possible gains by improper signals.

In this paper, we again consider a MIMO broadcast chan-

[1]Switching between several transmission modes and considering average per-user rates as well as the average sum transmit power.

nel with a restriction to widely linear transceivers, but we show a much more general result in Section 4: even if time-sharing is allowed, there can exist rate tuples that are achievable with a constraint on the sum transmit power only if the Gaussian transmit signals are improper. This means that the convex hull of the set of achievable rate tuples (rate region) for proper Gaussian signals can be a strict subset of the region for improper Gaussian signals. This implies possible gains by improper signals for various objective functions that take their optimal values on the Pareto boundary of the rate region.

Notation: We write $\mathbf{I}_N$ for the identity matrix of size N, $\boldsymbol{A}^{\mathrm{T}}$ for the transpose, $\boldsymbol{A}^{\mathrm{H}}$ for the conjugate transpose, and $\mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{C})$ for the circularly symmetric (proper) complex Gaussian distribution with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{C}$.

2. SYSTEM MODEL AND DUAL UPLINK

Data transmission in a MIMO broadcast channel with M transmit antennas and N_k receive antennas at user $k \in \{1, \dots, K\}$ can be described by

$$\boldsymbol{y}_k = \boldsymbol{H}_k \left(\sum_{k'=1}^{K} \boldsymbol{x}_{k'} \right) + \boldsymbol{\eta}_k. \tag{1}$$

The transmit signal vector $\boldsymbol{x}_{k'}$ for user k' has M entries, $\boldsymbol{H}_k \in \mathbb{C}^{N_k \times M}$ is a channel matrix, and $\boldsymbol{\eta}_k \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{C}_{\boldsymbol{\eta}_k})$ is a proper complex noise vector with N_k entries.

Since widely linear filtering corresponds to linear filtering of the real and imaginary part of a signal, we introduce an equivalent real-valued broadcast channel with $2M$ antennas at the base station and $2N_k$ antennas at the user terminal k:

$$\boldsymbol{y}_{k,\text{real}} = \boldsymbol{H}_{k,\text{real}} \left(\sum_{k'=1}^{K} \boldsymbol{x}_{k',\text{real}} \right) + \boldsymbol{\eta}_{k,\text{real}} \tag{2}$$

where $\boldsymbol{A}_{\text{real}}$ and $\boldsymbol{a}_{\text{real}}$ denote the real-valued counterparts

$$\boldsymbol{A}_{\text{real}} = \begin{bmatrix} \Re(\boldsymbol{A}) & -\Im(\boldsymbol{A}) \\ \Im(\boldsymbol{A}) & \Re(\boldsymbol{A}) \end{bmatrix} \text{ and } \boldsymbol{a}_{\text{real}} = \begin{bmatrix} \Re(\boldsymbol{a}) \\ \Im(\boldsymbol{a}) \end{bmatrix} \tag{3}$$

of a matrix $\boldsymbol{A}$ and a vector $\boldsymbol{a}$, respectively, and $\Re$ and $\Im$ are used to denote real and imaginary part, respectively. For $\boldsymbol{a} \in \mathbb{C}^L$ to be circularly symmetric, the ith and $(L+i)$th entry of $\boldsymbol{a}_{\text{real}}$ have to be independent and identically distributed (i.i.d.), which implies that they both need to have half the power of the ith entry of $\boldsymbol{a}$. When calculating Shannon rates in the real-valued representation of the broadcast channel, a factor of $\frac{1}{2}$ has to be present in front of the logarithm, while this factor is not present in the complex-valued representation [21, Ch. 5].

A broadcast channel can also be studied in the dual uplink [4, 22], where data is transmitted from the users through channels $(\boldsymbol{C}_{\boldsymbol{\eta}_k}^{-\frac{1}{2}} \boldsymbol{H}_k)^{\mathrm{H}}$ to a base station, and proper complex Gaussian noise $\boldsymbol{\eta} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_M)$ is added. The uplink downlink duality for MISO broadcast channels from [4] was extended to a rate duality for MIMO broadcast channels with linear transceivers in [22]. Applying the reasoning of [22, Section IV] to the real-valued formulation (2), we directly obtain the following lemma.

Lemma 1. *In a MIMO broadcast channel with sum transmit power P, the rates $[r_1, \dots, r_K]^{\mathrm{T}}$ are achievable with widely linear transceivers if and only if they are also achievable in the dual uplink with widely linear transceivers.*

3. PROPER GAUSSIAN TRANSMIT SIGNALS

If we restrict the transmit signals $\boldsymbol{x}_k$ to be proper and Gaussian, we get the following relation between (strictly) linear transceivers and widely linear transceivers.

Lemma 2. *When using proper Gaussian transmit signals in a MIMO broadcast channel with sum transmit power P, the rates $[r_1, \dots, r_K]^{\mathrm{T}}$ are achievable with widely linear transceivers if and only if they are also achievable with strictly linear transceivers.*

Proof of Lemma 2. One direction is trivial since every strictly linear filter is also widely linear. To prove the converse direction, we first note that a proper signal with any covariance matrix $\boldsymbol{Q}_k$ can be created by means of a strictly linear filter. It remains to be shown that also the receive filters can be chosen to be strictly linear without a loss in performance. As a consequence of the reasoning in [22, Section IV], the rates achievable with widely linear transceivers for given covariance matrices $\boldsymbol{Q}_k$ can be achieved by a strategy that applies linear MMSE receive filters in the equivalent real-valued scenario (2). This corresponds to widely linear MMSE receive filters (cf. [9]) in the complex-valued formulation (1). However, if all transmit signals are proper, the MSE-optimal widely linear filters are strictly linear [9]. This completes the proof. ☐

Thus, existing literature on optimizing linear transceivers in MIMO broadcast channels can be directly applied to obtain optimal widely linear transceivers if a restriction to proper Gaussian transmit signals is present. As this also allows us to apply the duality of [22] directly to the complex-valued formulation in the case of proper Gaussian signals, we have the following corollary.

Corollary 1. *In a MIMO broadcast channel with sum transmit power P and widely linear transceivers, the rates $[r_1, \dots, r_K]^{\mathrm{T}}$ are achievable with proper Gaussian transmit signals if and only if they are also achievable with proper Gaussian transmit signals in the dual uplink channel with widely linear transceivers.*

4. GAINS DUE TO IMPROPER SIGNALS

The main result of this paper is stated in the following theorem. As the aim of this section is not only to prove that gains by improper Gaussian signals are possible, but also to give some intuition about why this phenomenon occurs, some additional explanations are given as well.

Theorem 1. *In MIMO broadcast channels with widely linear transceivers, proper Gaussian transmit signals are not always optimal.*

Proof of Theorem 1. We provide a proof by construction. We first choose system parameters and derive the minimum sum power needed to achieve a given rate tuple with proper Gaussian signals. Then, we show that there exists a strategy with improper signals achieving higher rates with the same power.

Note that to prove the statement, it suffices to identify a single configuration for which improper signals yield a gain. Thus, we can restrict the following considerations to a system with small dimensions and a very simple channel realization without loss of generality. Similar proof techniques have previously been applied in other contexts, e.g., in [23] and [24].

Consider a broadcast channel with $M = 2$ transmit antennas, $K = 3$ receivers, and $N_k = 1$ antenna at each receiver k.[2] In this case, the noise covariance matrices are scalars, which we choose to be $\sigma_k^2 = 1\ \forall k$, and the channel matrices are row vectors $\boldsymbol{h}_k^{\mathrm{H}}$. We choose the channel realization[3]

$$\boldsymbol{h}_1^{\mathrm{H}} = \begin{bmatrix}1 & 0\end{bmatrix} \quad \boldsymbol{h}_2^{\mathrm{H}} = \begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix} \quad \boldsymbol{h}_3^{\mathrm{H}} = \begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{\mathrm{j}}{\sqrt{2}}\end{bmatrix}. \quad (4)$$

We first derive the minimal transmit power needed to achieve the rates $\rho_k = \rho = 1\ \ \forall k$ with proper Gaussian signals. Due to Lemma 2 and Corollary 1, we can apply results previously obtained for linear transceivers in [4, 22, 25] directly to the complex-valued formulation (1), and the real-valued formulation (2) is not needed in this part of the proof.

The following derivation of the optimal strategy with proper Gaussian signals is driven by intuition and based on symmetry arguments, but the optimality can be verified numerically by implementing the power minimization algorithm from [25], which can compute the globally optimal solution with time-sharing up to an arbitrarily small error tolerance.

Since we only have two degrees of freedom, we either serve one user exclusively during each of three equally long time slots (for reasons of symmetry), or a pair of users is served in each slot as shown in Fig. 1. One of these two possibilities achieves the optimal power.

In the first case, the sum power can be easily calculated as

$$\tilde{P} = \left(2^{3\rho} - 1\right) = 7. \quad (5)$$

In the second case, the average power P is the power needed to serve a user pair (k, j) at the rates $r_k = r_j = \frac{3}{2}\rho$, and is distributed equally among the users k and j (for reasons of symmetry). Using a dual uplink formulation [4, 22], the rate of user k can be expressed as

	Time Slot 1	Time Slot 2	Time Slot 3
User 1	$r_1 = \frac{3}{2}\rho$, $p_1 = \frac{1}{2}P$	$r_1 = \frac{3}{2}\rho$, $p_1 = \frac{1}{2}P$	
User 2	$r_2 = \frac{3}{2}\rho$, $p_2 = \frac{1}{2}P$		$r_2 = \frac{3}{2}\rho$, $p_2 = \frac{1}{2}P$
User 3		$r_3 = \frac{3}{2}\rho$, $p_3 = \frac{1}{2}P$	$r_3 = \frac{3}{2}\rho$, $p_3 = \frac{1}{2}P$

Fig. 1. Visualization of the symmetric time-sharing solution.

$$\begin{aligned}\frac{3}{2} = \frac{3}{2}\rho = r_k(P) &= \\ \log_2\left(1 + \frac{P}{2}\boldsymbol{h}_k^{\mathrm{H}}\left(\mathbf{I}_M + \frac{P}{2}\boldsymbol{h}_j\boldsymbol{h}_j^{\mathrm{H}}\right)^{-1}\boldsymbol{h}_k\right) &= \\ \log_2\left(1 + \frac{\frac{P}{2} + \frac{P^2}{8}}{1 + \frac{P}{2}}\right)\end{aligned} \quad (6)$$

where j is the user that is scheduled together with user k in the respective time slot. The third line is due to the matrix inversion lemma[4] and due to (4). Solving this for P, we get $P \approx 5.8249 < 7 = \tilde{P}$. Thus, the solution with two users per time slot is optimal here. Due to the optimality of P, the rate vector $\boldsymbol{\rho} = [1, 1, 1]^{\mathrm{T}}$ lies on the Pareto boundary of the rate region for widely linear transceivers with proper Gaussian transmit signals and sum power P.

Let us now consider the case of improper Gaussian transmit signals making use of the real-valued formulation (2). To prove the theorem, it is sufficient to derive a suboptimal strategy with improper transmit signals that achieves a strictly larger rate tuple with the same sum transmit power. As will be shown in the following, this can even be done without making use of the possibility of time-sharing.

The equivalent real-valued channels are given by

$$\boldsymbol{H}_{1,\text{real}} = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}, \quad (7)$$

$$\boldsymbol{H}_{2,\text{real}} = \begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0\\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix}, \quad (8)$$

$$\boldsymbol{H}_{3,\text{real}} = \begin{bmatrix}\frac{1}{\sqrt{2}} & 0 & 0 & -\frac{1}{\sqrt{2}}\\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0\end{bmatrix}. \quad (9)$$

[2] Note that this is a (yet very simple) special of a MIMO broadcast channel and, thus, a valid system to prove the theorem.

[3] This channel realization has similarities with the example used in [24] to prove the inseparability of parallel MIMO broadcast channels with linear transceivers, but the theorem to be proven is a completely different one here.

[4] $(\boldsymbol{A} + \boldsymbol{BCD})^{-1} = \boldsymbol{A}^{-1} - \boldsymbol{A}^{-1}\boldsymbol{B}\left(\boldsymbol{C}^{-1} + \boldsymbol{DA}^{-1}\boldsymbol{B}\right)^{-1}\boldsymbol{DA}^{-1}$ (e.g., [26])

We propose a possibly suboptimal strategy with one real-valued data stream per user, so that the widely linear receive filters have a real valued representation

$$\boldsymbol{v}_k^{\mathrm{T}} = \begin{bmatrix}\cos\varphi_k & \sin\varphi_k\end{bmatrix} \tag{10}$$

where $\boldsymbol{v}_k^{\mathrm{T}}\boldsymbol{v}_k = 1$ is without loss of generality. We obtain an effective vector broadcast channel with channel vectors

$$\tilde{\boldsymbol{h}}_k^{\mathrm{T}} = \boldsymbol{v}_k^{\mathrm{T}}\boldsymbol{H}_{k,\mathrm{real}} \tag{11}$$

which include the receive filters. For given values of φ_k and given per-user powers p_k, the rates of all users can be calculated as

$$r_k = \frac{1}{2}\log_2\!\left(1 + p_k\tilde{\boldsymbol{h}}_k^{\mathrm{T}}\left(\frac{1}{2}\mathbf{I}_{2M} + \sum_{j\neq k} p_j\tilde{\boldsymbol{h}}_j\tilde{\boldsymbol{h}}_j^{\mathrm{T}}\right)^{-1}\tilde{\boldsymbol{h}}_k\right) \tag{12}$$

in the dual uplink [4, 22] of the effective real-valued vector broadcast channel. Note that we have accounted for the fact that the equivalent real-valued noise has variance $\frac{1}{2}$ in each component, and we have used $\frac{1}{2}$ as pre-log factor since only real-valued symbols are transmitted (cf. Section 2).

Choosing $\varphi_1 = 0$, $\varphi_2 = -\frac{5\pi}{12}$, $\varphi_3 = -\frac{7\pi}{12}$, and $p_k = P/3\ \forall k$, we achieve the rates $r_k \approx 1.1072\ \forall k$ with improper Gaussian signals. Due to Lemma 1, the same rates are achievable in the downlink with sum transmit power P. Clearly, the rate vector $\boldsymbol{r} \approx 1.1072\boldsymbol{\rho}$ lies outside of the rate region for proper Gaussian transmit signals with sum power P. □

Instead of increasing the rates, the advantage of improper signals can also be used to reduce the transmit power. After choosing the angles φ_k, the minimal transmit power needed to achieve the rate vector $\boldsymbol{\rho}$ in the effective vector broadcast channel can be computed numerically using, e.g., the method proposed in [27]. In the case discussed above, this would lead to a sum transmit power $P_{\text{improper}} \approx 4.7834 < P$.

Applying the matrix inversion lemma in the second line of (6) and in (12), we can see that the achievable rates depend only on the per-stream powers p_k and on the inner products between the channels $\boldsymbol{h}_k$ or between the effective real-valued vector channels $\tilde{\boldsymbol{h}}_k$, respectively. To better understand why improper Gaussian signals have an advantage in this setting, we study these inner products by adopting the notion of the *Hermitian angle* [28] between two complex unit-norm vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, given by $\theta = \arccos|\boldsymbol{a}^{\mathrm{H}}\boldsymbol{b}|$. In the original complex-valued broadcast channel, we have[5]

$$\arccos\left|\boldsymbol{h}_k^{\mathrm{H}}\boldsymbol{h}_j\right| = \arccos\frac{1}{\sqrt{2}} = 45^\circ \quad \forall k \neq j \tag{13}$$

but our particular choice of φ_k yields much larger angles between the effective real-valued channel vectors:

$$\arccos\left|\tilde{\boldsymbol{h}}_k^{\mathrm{T}}\tilde{\boldsymbol{h}}_j\right| = \arccos\frac{\cos\frac{5\pi}{12}}{\sqrt{2}} \approx 79.5^\circ \quad \forall k \neq j. \tag{14}$$

[5]Note that $\boldsymbol{h}_k^{\mathrm{H}}\boldsymbol{h}_k = 1$ and $\tilde{\boldsymbol{h}}_k^{\mathrm{T}}\tilde{\boldsymbol{h}}_k = 1$ holds in the considered example.

Thus, data can be transmitted over very dissimilar effective channels in the higher-dimensional real-valued formulation. This advantage obviously outweighs the loss that results from conveying information only in the real part (a real-valued dimension corresponds to only half a degree of freedom).

5. DISCUSSION

If no interference cancellation such as dirty paper coding (DPC) is performed, the proper complex Gaussian distribution is no longer the optimal input distribution of MIMO broadcast channels. Even though this result might be surprising at the first glance, there is an intuitive explanation.

Due to the interference cancellation, a MIMO broadcast channel with DPC inherits some elementary properties of point-to-point MIMO systems, such as the optimality of proper Gaussian transmit signals [4, 5] and the fact that carriers can be treated separately in a multicarrier-system [29]. The reason for this inheritance is that there is one user that does not see any interference. For this user's optimal transmit signal, the same rules as in a point-to-point system have to apply. This has consequences for the properties of the effective noise (including interference) experienced by the next user, and the same reasoning can be applied recursively [24].

Without nonlinear interference cancellation, i.e., using (widely) linear transceivers, no user in a broadcast channel is in the same situation as in a point-to-point channel, and such properties are not inherited. Instead, the broadcast channel becomes an interference-limited scenario, which has at least some similarities with an interference channel. For instance, treating carriers separately can be suboptimal in multicarrier broadcast channels without DPC [24], which had been shown before for interference channels in [23], and also the gains by improper signaling observed in this paper had been discovered first in interference channels [18]. Note, however, that improper signals and coding across carriers can increase the degrees of freedom (DoF) of an interference channel while the gains in broadcast channels apply to achievable rates in the finite-SNR regime since linear transceivers and proper signals achieve the full DoF of a broadcast channel anyway.

Developing the thought further, we have to raise the question whether there is an input distribution for MIMO broadcast channels with (widely) linear transceivers that can perform even better than improper Gaussian inputs. Indeed, we are not aware of any paper that proves optimality of Gaussian inputs for MIMO broadcast channels without DPC. This is another similarity to Gaussian interference channels.

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