# MISO INTERFERENCE CHANNEL WITH IMPROPER GAUSSIAN SIGNALING

*Yong Zeng*[†] *Rui Zhang*[⋆] *Erry Gunawan*[†] *Yong Liang Guan*[†]

[†]School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore
[⋆]ECE Department, National University of Singapore
ze0003ng@e.ntu.edu.sg, elezhang@nus.edu.sg, {egunawan, eylguan}@ntu.edu.sg

## ABSTRACT

This paper studies the achievable rate region of the $K$-user Gaussian multiple-input single-output interference channel (MISO-IC) with interference treated as noise, when *improper* or circularly *asymmetric* complex Gaussian signaling is applied. By exploiting the separable rate expression with improper Gaussian signaling, we propose a separate covariance and pseudo-covariance optimization algorithm, which is guaranteed to improve the users' rates over the conventional proper or circularly symmetric complex Gaussian signaling. In particular, for the pseudo-covariance optimization, the semidefinite relaxation (SDR) technique is applied to provide a high-quality approximate solution. For the special case of two-user MISO-IC, the SDR technique yields the optimal pseudo-covariance solution.

***Index Terms***— Improper Gaussian signaling, MISO-IC, pseudo-covariance, semidefinite relaxation.

## 1. INTRODUCTION

The information-theoretical capacity of the general interference channel (IC) is a long-standing open problem [1]. Practically, a great deal of research on Gaussian ICs has focused on characterizing the achievable rate regions [2–9], under the assumption of employing single-user decoding (SUD) with the interference treated as noise at receivers. In particular, for the multiple-input single-output Gaussian IC (MISO-IC), all the Pareto-optimal rate-tuples can be achieved with beamforming, i.e., with rank-1 transmit covariance matrices [7–9].

The aforementioned works have all assumed proper or circularly symmetric complex Gaussian signaling. However, it was revealed in [10] that the more general improper or circularly asymmetric complex Gaussian signaling, together with symbol extension and interference alignment (IA), is able to improve the achievable degrees of freedom (DoF) for the sum-rate of a three-user single-input single-output Gaussian IC (SISO-IC). Later, it was shown in [11,12] that even for the two-user SISO-IC where IA is not applicable, the achievable rate region can still be enlarged with improper over proper Gaussian signaling. In [13], we have shown that with improper Gaussian signaling, the user's achievable rate in the general multiple-input multiple-output Gaussian IC (MIMO-IC) can be expressed as the summation of the rate achievable by the conventional proper Gaussian signaling, which depends on the users' transmit covariance matrices only, and an additional term, which is a function of both the covariance and pseudo-covariance matrices. Such a separable rate structure was exploited in [13] to optimize the covariance and pseudo-covariance separately so that the obtained improper Gaussian signaling strictly outperforms the conventional proper Gaussian signaling. However, the algorithm proposed in [13] is for the two-user SISO-IC and cannot be applied when there are more than two users and/or multiple transmitting antennas. This thus motivates our current work that extends the result in [13] to the more general $K$-user MISO-IC.

Similar to [7], we apply the rate-profile technique to characterize the achievable rate region of the MISO-IC with the interference treated as noise. However, unlike the case with proper Gaussian signaling, the resulting optimization problem with improper Gaussian signaling is non-convex and hence difficult to be solved optimally. By adopting a similar separate covariance and pseudo-covariance optimization approach as in [13], we develop an efficient algorithm that first solves the covariance optimization optimally and then the pseudo-covariance optimization approximately based on the celebrated semidefinite relaxation (SDR) technique [14]. For the special case of two-user MISO-IC, the SDR based solution is optimal for the pseudo-covariance optimization.

## 2. SYSTEM MODEL AND PROBLEM FORMULATION

This paper considers a $K$-user MISO-IC, where each transmitter is equipped with $M$ antennas and each receiver with one single antenna. All transmitters send independent information to their respective receivers at the same time and over the same frequency band, thus potentially interfering with each other at the receivers. The received baseband discrete-time signal for user $k$ is given by

$$y_k = \mathbf{h}_{kk}\mathbf{x}_k + \sum_{j\neq k}\mathbf{h}_{kj}\mathbf{x}_j + n_k,\ k = 1, \cdots, K, \quad (1)$$

This work was partially supported by Department of the Navy Grant N62909-12-1-7015 issued by Office of Naval Research Global.

where $\mathbf{h}_{kk} \in \mathbb{C}^{1\times M}$ denotes the direct channel from transmitter $k$ to receiver $k$, while $\mathbf{h}_{kj}, j \neq k$, denotes the interference channel from transmitter $j$ to receiver $k$; $n_k$ represents the circularly symmetric complex Gaussian (CSCG) noise with zero mean and variance $\sigma^2$, denoted by $n_k \sim \mathcal{CN}(0, \sigma^2)$; and $\mathbf{x}_k \in \mathbb{C}^{M\times 1}$ is the transmitted signal vector from transmitter $k$. Unlike the conventional proper Gaussian signaling, this paper considers the more general improper Gaussian signaling at the transmitters. For background knowledge of improper (Gaussian) random vectors (RVs), the readers may refer to [13] and references therein. Given a zero-mean RV $\mathbf{x}_k$, we denote its covariance and pseudo-covariance matrices as $\mathbf{C}_{\mathbf{x}_k}$ and $\widetilde{\mathbf{C}}_{\mathbf{x}_k}$, respectively, as follows:

$$\mathbf{C}_{\mathbf{x}_k} \triangleq \mathbb{E}\{\mathbf{x}_k\mathbf{x}_k^H\},\ \widetilde{\mathbf{C}}_{\mathbf{x}_k} \triangleq \mathbb{E}\{\mathbf{x}_k\mathbf{x}_k^T\}, \quad (2)$$

where $(\cdot)^H$ and $(\cdot)^T$ denote Hermitian transpose and transpose, respectively. For the conventional proper Gaussian signaling, the pseudo-covariance matrices for all transmitters $\widetilde{\mathbf{C}}_{\mathbf{x}_k}$'s are set to zero matrices, and thus are not included for the transmit optimization. However, for the more general improper Gaussian signaling, the additional degrees of freedom given by the pseudo-covariance matrices provide a further opportunity for rate maximization. $\mathbf{C}_{\mathbf{x}_k}$ and $\widetilde{\mathbf{C}}_{\mathbf{x}_k}$ are a valid pair of covariance and pseudo-covariance matrices if and only if their corresponding augmented covariance matrix $\underline{\mathbf{C}}_{\mathbf{x}_k}$ is positive semidefinite [15], i.e.,

$$\underline{\mathbf{C}}_{\mathbf{x}_k} \triangleq \begin{bmatrix} \mathbf{C}_{\mathbf{x}_k} & \widetilde{\mathbf{C}}_{\mathbf{x}_k} \\ \widetilde{\mathbf{C}}^*_{\mathbf{x}_k} & \mathbf{C}^*_{\mathbf{x}_k} \end{bmatrix} \succeq \mathbf{0}, \quad (3)$$

where $(\cdot)^*$ denotes the complex conjugate. For the MISO-IC with single-antenna receivers, the covariance and pseudo-covariance of the received signal $y_k$ can be written as

$$C_{y_k} = \sum_{j=1}^{K} \mathbf{h}_{kj}\mathbf{C}_{\mathbf{x}_j}\mathbf{h}_{kj}^H + \sigma^2, \widetilde{C}_{y_k} = \sum_{j=1}^{K} \mathbf{h}_{kj}\widetilde{\mathbf{C}}_{\mathbf{x}_j}\mathbf{h}_{kj}^T. \quad (4)$$

Denote the interference-plus-noise term at receiver $k$ by $s_k$, i.e., $s_k = \sum_{j\neq k} \mathbf{h}_{kj}\mathbf{x}_j + n_k$. Then we have

$$C_{s_k} = \sum_{j\neq k} \mathbf{h}_{kj}\mathbf{C}_{\mathbf{x}_j}\mathbf{h}_{kj}^H + \sigma^2, \widetilde{C}_{s_k} = \sum_{j\neq k} \mathbf{h}_{kj}\widetilde{\mathbf{C}}_{\mathbf{x}_j}\mathbf{h}_{kj}^T. \quad (5)$$

Under the assumptions of Gaussian inputs and that the interference is treated as Gaussian noise at receivers, and by applying the result in [13] to the MISO-IC setup, the achievable rate at receiver $k$ can be expressed as

$$R_k = \underbrace{\log\left(1 + \frac{\mathbf{h}_{kk}\mathbf{C}_{\mathbf{x}_k}\mathbf{h}_{kk}^H}{\sigma^2 + \sum_{j\neq k}\mathbf{h}_{kj}\mathbf{C}_{\mathbf{x}_j}\mathbf{h}_{kj}^H}\right)}_{\triangleq R_k^{\text{proper}}(\{\mathbf{C}_{\mathbf{x}_j}\})} + \frac{1}{2}\log\frac{1 - C_{y_k}^{-2}|\widetilde{C}_{y_k}|^2}{1 - C_{s_k}^{-2}|\widetilde{C}_{s_k}|^2}. \quad (6)$$

It is observed from (6) that with improper Gaussian signaling, each user's achievable rate is a summation of the rate achievable by the conventional proper Gaussian signaling, denoted by $R_k^{\text{proper}}(\{\mathbf{C}_{\mathbf{x}_j}\})$, and an additional term, which is a function of both the users' transmit covariance and pseudo-covariance matrices. Therefore, for a given set of transmit covariance matrices obtained by any proper Gaussian signaling scheme, the achievable rates in MISO-IC can be improved with improper Gaussian signaling by choosing the pseudo-covariance matrices that make the second term in (6) strictly positive. The achievable rate region $\mathcal{R}$ for the $K$-user MISO-IC is defined as the set of rate-tuples that can be simultaneously achieved by all users under a given set of transmit power constraints for each transmitter, denoted by $P_k, k = 1, ..., K$. With $R_k$ given in (6), we thus have

$$\mathcal{R} \triangleq \bigcup_{\substack{\mathrm{Tr}\{\mathbf{C}_{\mathbf{x}_k}\}\leq P_k,\\ \underline{\mathbf{C}}_{\mathbf{x}_k}\succeq \mathbf{0},\forall k}} \Big\{(r_1, \cdots, r_K) : 0 \leq r_k \leq R_k, \forall k\Big\},$$

where the constraint $\underline{\mathbf{C}}_{\mathbf{x}_k} \succeq \mathbf{0}$ follows from (3). To characterize the Pareto boundary of $\mathcal{R}$, we adopt the rate-profile method as in [7]. Specifically, any Pareto-optimal rate-tuple on the boundary of the rate region can be obtained by solving the following optimization problem with a given rate-profile vector denoted by $\boldsymbol{\alpha} = (\alpha_1 \cdots \alpha_K)$.

$$\begin{aligned} \text{(P1):} \quad & \max_{\{\mathbf{C}_{\mathbf{x}_k}\},\{\widetilde{\mathbf{C}}_{\mathbf{x}_k}\},R} \quad R \\ & \text{s.t.} \quad R_k \geq \alpha_k R,\ \forall k, \\ & \qquad \mathrm{Tr}\{\mathbf{C}_{\mathbf{x}_k}\} \leq P_k,\ \forall k, \\ & \qquad \begin{bmatrix} \mathbf{C}_{\mathbf{x}_k} & \widetilde{\mathbf{C}}_{\mathbf{x}_k} \\ \widetilde{\mathbf{C}}^*_{\mathbf{x}_k} & \mathbf{C}^*_{\mathbf{x}_k} \end{bmatrix} \succeq \mathbf{0},\ \forall k, \end{aligned}$$

where $\alpha_k$ denotes the target ratio between user $k$'s achievable rate and the users' sum-rate, $R$. Without loss of generality, we assume that $\alpha_k > 0, \forall k$, and $\sum_{k=1}^{K} \alpha_k = 1$. Denote the optimal value of (P1) as $R^\star$. Then the rate-tuple $R^\star \cdot \boldsymbol{\alpha}$ must be on the Pareto boundary corresponding to the rate-profile given by $\boldsymbol{\alpha}$. Thereby, by solving (P1) with different rate-profile vectors of $\boldsymbol{\alpha}$, the complete Pareto boundary of $\mathcal{R}$ can be found [7].

## 3. SEPARATE COVARIANCE AND PSEUDO-COVARIANCE OPTIMIZATION

(P1) is a non-convex optimization problem, and thus it is difficult to achieve the global optimum efficiently. In this section, we propose a separate covariance and pseudo-covariance optimization algorithm by utilizing the rate expression given in (6) to obtain an efficient suboptimal solution for (P1).

### 3.1. Covariance Optimization

When restricted to proper Gaussian signaling by setting $\widetilde{\mathbf{C}}_{\mathbf{x}_k} = \mathbf{0}, \forall k$, (P1) reduces to

$$\text{(P1.1)}: \max_{r, \{\mathbf{C}_{\mathbf{x}_k}\}} \quad r$$
$$\text{s.t.} \log\left(1 + \frac{\mathbf{h}_{kk}\mathbf{C}_{\mathbf{x}_k}\mathbf{h}_{kk}^H}{\sigma^2 + \sum_{j\neq k}\mathbf{h}_{kj}\mathbf{C}_{\mathbf{x}_j}\mathbf{h}_{kj}^H}\right) \geq \alpha_k r, \ \forall k,$$
$$\mathrm{Tr}\{\mathbf{C}_{\mathbf{x}_k}\} \leq P_k, \ \mathbf{C}_{\mathbf{x}_k} \succeq \mathbf{0}, \ \forall k.$$

Denote the optimal value of (P1.1) as $r^\star$, then the rate-tuple $r^\star \cdot \boldsymbol{\alpha}$ is on the Pareto boundary of the achievable rate region with proper Gaussian signaling. It has been shown in [7–9] that all the Pareto-optimal rate-tuples with proper Gaussian signaling can be achieved by transmit beamforming, i.e., with rank-1 transmit covariance matrices. Therefore, without loss of optimality for (P1.1), we can assume

$$\mathbf{C}_{\mathbf{x}_k} = \mathbf{t}_k\mathbf{t}_k^H, \ \forall k, \tag{7}$$

where $\mathbf{t}_k$ is the transmit beamforming vector for user $k$. Then for any fixed target rate $r$, the feasibility problem related to (P1.1) can be formulated as

$$\text{(P1.2)}: \text{Find } \{\mathbf{t}_k\}$$
$$\text{s.t. } \sigma^2 + \sum_{j=1}^{K}|\mathbf{h}_{kj}\mathbf{t}_j|^2 \leq \left(1 + \frac{1}{e^{\alpha_k r} - 1}\right)(\mathbf{h}_{kk}\mathbf{t}_k)^2, \ \forall k,$$
$$\Im\{\mathbf{h}_{kk}\mathbf{t}_k\} = 0, \ \|\mathbf{t}_k\|^2 \leq P_k, \ \forall k,$$

where without loss of generality, we have assumed that for all $k$, $\mathbf{h}_{kk}\mathbf{t}_k$ is a nonnegative real number [16]. (P1.2) is a second-order cone programming (SOCP) problem, which can be efficiently solved [17]. Then (P1.1) can be optimally solved by iteratively solving (P1.2) with different values of $r$, and applying a bisection search over $r$ [17].

### 3.2. Pseudo-Covariance Optimization

Denote the optimal solution to (P1.1) by $\{r^\star, \mathbf{C}^\star_{\mathbf{x}_k} = \mathbf{t}_k\mathbf{t}_k^H\}$. By fixing the transmit covariance matrices as $\{\mathbf{C}^\star_{\mathbf{x}_k} = \mathbf{t}_k\mathbf{t}_k^H\}$, (P1) is further optimized over the pseudo-covariance matrices $\{\widetilde{\mathbf{C}}_{\mathbf{x}_k}\}$. The resulting problem is

$$\text{(P1.3)}: \max_{R, \{\widetilde{\mathbf{C}}_{\mathbf{x}_k}\}} \quad R$$
$$\text{s.t. } \alpha_k r^\star + \frac{1}{2}\log\frac{1 - C_{y_k}^{-2}|\widetilde{C}_{y_k}|^2}{1 - C_{s_k}^{-2}|\widetilde{C}_{s_k}|^2} \geq \alpha_k R, \ \forall k,$$
$$\begin{bmatrix} \mathbf{t}_k\mathbf{t}_k^H & \widetilde{\mathbf{C}}_{\mathbf{x}_k} \\ \widetilde{\mathbf{C}}^*_{\mathbf{x}_k} & (\mathbf{t}_k\mathbf{t}_k^H)^* \end{bmatrix} \succeq \mathbf{0}, \ \forall k, \tag{8}$$

where $C_{y_k}$ and $C_{s_k}$ are fixed covariances given the previously optimized transmit covariance matrices $\{\mathbf{C}^\star_{\mathbf{x}_k}\}$. (P1.3) can be equivalently written as a minimum-weighted-rate maximization (MinWR-Max) problem as follows.

$$\text{(P1.4)}: \max_{\{\widetilde{\mathbf{C}}_{\mathbf{x}_k}\}} \min_{k=1,\cdots,K} \frac{1}{2\alpha_k}\log\frac{1 - C_{y_k}^{-2}|\widetilde{C}_{y_k}|^2}{1 - C_{s_k}^{-2}|\widetilde{C}_{s_k}|^2}$$
$$\text{s.t. } \begin{bmatrix} \mathbf{t}_k\mathbf{t}_k^H & \widetilde{\mathbf{C}}_{\mathbf{x}_k} \\ \widetilde{\mathbf{C}}^*_{\mathbf{x}_k} & (\mathbf{t}_k\mathbf{t}_k^H)^* \end{bmatrix} \succeq \mathbf{0}, \ \forall k. \tag{9}$$

**Lemma 1.** *The positive semidefinite constraint in* (9) *is satisfied if and only if*

$$\widetilde{\mathbf{C}}_{\mathbf{x}_k} = Z_k\widetilde{\mathbf{t}}_k\widetilde{\mathbf{t}}_k^T, \ k = 1, \cdots, K, \tag{10}$$

*where $Z_k$ is a complex scalar variable with constraint $|Z_k| \leq \|\mathbf{t}_k\|^2$, and $\widetilde{\mathbf{t}}_k = \mathbf{t}_k/\|\mathbf{t}_k\|$.*

The proof is omitted due to the space limitation. Lemma 1 shows the optimality of rank-1 pseudo-covariance matrices if rank-1 transmit covariance matrices are applied. By substituting (10) into (4), we have

$$\widetilde{C}_{y_k} = \sum_{j=1}^{K}(\mathbf{h}_{kj}\widetilde{\mathbf{t}}_j)^2 Z_j, \ \forall k. \tag{11}$$

By defining $\mathbf{m}_k \triangleq C_{y_k}^{-1}\begin{bmatrix}(\mathbf{h}_{k1}\widetilde{\mathbf{t}}_1)^2 & \cdots & (\mathbf{h}_{kK}\widetilde{\mathbf{t}}_K)^2\end{bmatrix}^H$, $\mathbf{z} \triangleq \begin{bmatrix}Z_1 & \cdots & Z_K\end{bmatrix}^T$, we have

$$C_{y_k}^{-2}\left|\widetilde{C}_{y_k}\right|^2 = |\mathbf{m}_k^H\mathbf{z}|^2 = \mathbf{z}^H\mathbf{M}_k\mathbf{z}, \tag{12}$$

where $\mathbf{M}_k = \mathbf{m}_k\mathbf{m}_k^H$. Similarly, by defining $\mathbf{w}_k \triangleq C_{s_k}^{-1} \begin{bmatrix}\cdots & (\mathbf{h}_{k(k-1)}\widetilde{\mathbf{t}}_{k-1})^2 & 0 & (\mathbf{h}_{k(k+1)}\widetilde{\mathbf{t}}_{k+1})^2 & \cdots\end{bmatrix}^H$, then

$$C_{s_k}^{-2}|\widetilde{C}_{s_k}|^2 = |\mathbf{w}_k^H\mathbf{z}|^2 = \mathbf{z}^H\mathbf{W}_k\mathbf{z}, \tag{13}$$

where $\mathbf{W}_k = \mathbf{w}_k\mathbf{w}_k^H$. Therefore, (P1.4) can be written as

$$\text{(P1.5)}: \max_{\mathbf{z}\in\mathbb{C}^K} \min_{k=1,\cdots,K} \frac{1}{2\alpha_k}\log\frac{1 - \mathbf{z}^H\mathbf{M}_k\mathbf{z}}{1 - \mathbf{z}^H\mathbf{W}_k\mathbf{z}}$$
$$\text{s.t. } |\mathbf{e}_k^H\mathbf{z}|^2 \leq \left\|\mathbf{t}_k\right\|^4, \ \forall k, \tag{14}$$

where $\mathbf{e}_k$ is a unit vector where the $k$th entry is one and all the other entries are zero. Next, we show that an approximate solution to (P1.5) can be obtained by applying the SDR technique [14]. With the identity $\mathbf{x}^H\mathbf{A}\mathbf{x} = \mathrm{Tr}(\mathbf{A}\mathbf{x}\mathbf{x}^H)$, the SDR problem of (P1.5) is given by

$$\text{(P1.5-SDR)}: \max_{\mathbf{Z}\succeq\mathbf{0}} \min_{k=1,\cdots,K} \frac{1}{2\alpha_k}\log\frac{1 - \mathrm{Tr}(\mathbf{M}_k\mathbf{Z})}{1 - \mathrm{Tr}(\mathbf{W}_k\mathbf{Z})}$$
$$\text{s.t. } \mathrm{Tr}(\mathbf{E}_k\mathbf{Z}) \leq \left\|\mathbf{t}_k\right\|^4, \ \forall k, \tag{15}$$

where $\mathbf{E}_k = \mathbf{e}_k\mathbf{e}_k^H$. It is easy to see that (P1.5) is equivalent to (P1.5-SDR) with the additional constraint $\mathrm{rank}(\mathbf{Z}) = 1$, in which case $\mathbf{Z}$ can be written as $\mathbf{Z} = \mathbf{z}\mathbf{z}^H$. Therefore, the optimal value of (P1.5-SDR), $\tau_{\text{sdr}}$, provides an upper bound on that of (P1.5). Since $\mathbf{Z} = \mathbf{0}$ is feasible for (P1.5-SDR), we have $\tau_{\text{sdr}} \geq 0$.

**Theorem 1.** *For any matrix $\mathbf{Z}$ that is feasible to (P1.5-SDR), the following inequalities hold:*

$$1 - \mathrm{Tr}(\mathbf{W}_k\mathbf{Z}) > 0,\ 1 - \mathrm{Tr}(\mathbf{M}_k\mathbf{Z}) > 0,\ \forall k. \tag{16}$$

The proof is omitted due to the space limitation. With Theorem 1 and given $\tau_{\text{sdr}} \geq 0$, it can be shown that (P1.5-SDR) is a quasi-convex problem [17]. For any given $\tau$, consider the following problem.

$$\begin{aligned}\text{(P1.6): } &\min_{\mathbf{Z}\succeq\mathbf{0}}\ \mathrm{Tr}(\mathbf{E}_1\mathbf{Z})\\ &\text{s.t. } 1 - \mathrm{Tr}(\mathbf{M}_k\mathbf{Z}) \geq e^{2\alpha_k\tau}(1 - \mathrm{Tr}(\mathbf{W}_k\mathbf{Z})),\ \forall k,\\ &\qquad \mathrm{Tr}(\mathbf{E}_k\mathbf{Z}) \leq \left\|\mathbf{t}_k\right\|^4,\ k = 2, \cdots, K.\end{aligned}$$

(P1.6) is a semidefinite programming (SDP) problem, which minimizes the left hand side (LHS) of (15) corresponding to $k = 1$. Denote the optimal value of (P1.6) as $f(\tau)$. If $f(\tau) \leq \|\mathbf{t}_1\|^4$, then $\tau_{\text{sdr}} \geq \tau$; otherwise, $\tau_{\text{sdr}} < \tau$. Therefore, (P1.5-SDR) can be optimally solved by solving (P1.6) together with a bisection search over $\tau$.

Denote the solution to (P1.5-SDR) by $\mathbf{Z}^\star$. If $\mathrm{rank}(\mathbf{Z}^\star) = 1$, i.e., $\mathbf{Z}^\star = \mathbf{z}\mathbf{z}^H$, then $\mathbf{z}$ is the optimal solution to (P1.5). In this case, SDR is tight; otherwise, we apply the following Gaussian randomization procedure customized to our case to find an approximate solution to (P1.5) [14]:

**Algorithm 1** Gaussian Randomization Procedure for (P1.5)

**Input:** The solution $\mathbf{Z}^\star$ to (P1.5-SDR) and the number of randomizations $L$.

**for** $l = 1, \cdots, L$ **do**

Generate $\boldsymbol{\xi}_l \sim \mathcal{CN}(\mathbf{0}, \mathbf{Z}^\star)$, and construct a feasible point $\mathbf{z}_l$ to (P1.5) as follows:

$$[\mathbf{z}_l]_k = \kappa_k[\boldsymbol{\xi}_l]_k, \text{with } \kappa_k = \min\left\{1, \frac{\|\mathbf{t}_k\|^2}{|[\boldsymbol{\xi}_l]_k|}\right\}, \forall k,$$

where $[\cdot]_k$ is the $k$th entry.

**end for**

determine $l^\star = \arg\max_{l=1,\cdots,L}\ \min_{k=1,\cdots,K} \frac{1}{2\alpha_k}\log\frac{1-\mathbf{z}_l^H\mathbf{M}_k\mathbf{z}_l}{1-\mathbf{z}_l^H\mathbf{W}_k\mathbf{z}_l}$

**Output:** $\hat{\mathbf{z}} = \mathbf{z}_{l^\star}$ as an approximate solution for (P1.5).

It is worth noting that when $K = 2$, (P1.6) is a complex-valued SDP problem with three linear constraints. It is known that if such a problem is feasible, there is always a rank-1 solution [18]. Therefore, for $K = 2$, the pseudo-covariance subproblem (P1.3) is optimally solved.

## 4. NUMERICAL RESULTS

In this section, we assume that all the transmitters have the same power constraint $P$, i.e., $P_k = P$, $\forall k$. The average signal-to-noise ratio (SNR) is defined as $P/\sigma^2$. For Algorithm 1, $L = 1000$ is used. The channel coefficients are generated from the independent and identically distributed (i.i.d.) CSCG random variables with zero-mean and unit-variance. Denote $\hat{\tau}$ as the objective value of (P1.5) with $\hat{\mathbf{z}}$ obtained by Algorithm 1. We first evaluate the ratio $\tau^{\text{sdr}}/\hat{\tau}$, which gives an upper bound for the true approximation ratio $\tau^\star/\hat{\tau}$. In particular, if $\tau^{\text{sdr}}/\hat{\tau} = 1$, then the obtained SDR-based solution is optimal. We consider the maximum of the minimum (max-min) achievable rates of all users by setting the rate-profile in (P1) as $\boldsymbol{\alpha} = 1/K\mathbf{1}$, where $\mathbf{1}$ is an all-one vector. Table 1 summarizes the mean and the standard deviation of the ratio $\tau_{\text{sdr}}/\hat{\tau}$ at SNR = 10 dB with different pairs of values for $M$ and $K$, based on simulation results over 1000 random channel realizations. It is observed that for all the setups considered, the mean values of the ratios are between 1 and 1.64, which demonstrates the high-quality approximate solution by the SDR technique.

**Table 1**: Mean and standard deviation (std) of the ratio $\tau_{\text{sdr}}/\hat{\tau}$.

| | $K$ | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $M = 1$ | mean | 1.0 | 1.032 | 1.138 | 1.267 | 1.391 |
| | std | 0 | 0.092 | 0.245 | 0.350 | 0.441 |
| $M = 2$ | mean | 1.0 | 1.012 | 1.162 | 1.401 | 1.640 |
| | std | 0 | 0.068 | 0.388 | 0.621 | 0.691 |

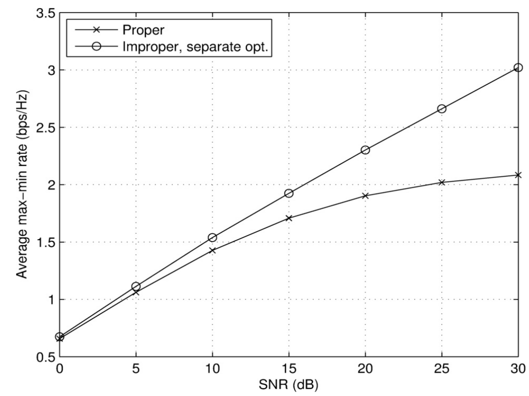


**Fig. 1**: Average max-min rate with $K = 3$ and $M = 2$.

The max-min rates obtained with the optimal proper and the proposed improper Gaussian signaling are plotted in Fig. 1. It is observed that a significant gain is achieved by the proposed scheme due to the pseudo-covariance optimization.

## 5. CONCLUSION

This paper studies the transmit optimization for the $K$-user MISO-IC with interference treated as Gaussian noise. By exploiting the separable achievable rate structure by improper Gaussian signaling, a separate transmit covariance and pseudo-covariance optimization algorithm is proposed. For the pseudo-covariance optimization subproblem, an approximate solution is obtained based on the SDR technique. Simulation results show the promising rate improvement by the proposed improper Gaussian signaling design over the conventional proper signaling counterpart for the Gaussian MISO-IC.

## 6. REFERENCES


[1] R. Etkin, D. Tse, and H. Wang, "Gaussian interference channel capacity to within one bit," *IEEE Trans. Inf. Theory*, vol. 54, no. 1, pp. 5534–5562, Dec. 2008.

[2] M. Charafeddine, A. Sezgin, and A. Paulraj, "Rates region frontiers for $n$-user interference channel with interference as noise," in *Proc. Allerton Conference*, Sept. 2007.

[3] L. Liu, R. Zhang, and K. C. Chua, "Achieving global optimality for weighted sum-rate maximization in the $K$-user Gaussian interference channel with multiple antennas," *IEEE Trans. Wireless Commun.*, vol. 11, no. 5, pp. 1933–1945, May 2012.

[4] W. Utschick and J. Brehmer, "Monotonic optimization framework for coordinated beamforming in multicell networks," *IEEE Trans. Signal Process.*, vol. 60, no. 4, pp. 1899–1909, Apr. 2012.

[5] E. Björnson, G. Zheng, M. Bengtsson, and B. Ottersten, "Robust monotonic optimization framework for multi-cell MISO systems," *IEEE Trans. Signal Process.*, vol. 60, no. 5, pp. 2508–2523, May 2012.

[6] E. Jorswieck, E. Larsson, and D. Danev, "Complete characterization of the Pareto boundary for the MISO interference channel," *IEEE Trans. Signal Process.*, vol. 56, no. 10, pp. 5292–5296, Oct. 2008.

[7] R. Zhang and S. Cui, "Cooperative interference management with MISO beamforming," *IEEE Trans. Signal Process.*, vol. 58, no. 10, pp. 5450–5458, Oct. 2010.

[8] X. Shang, B. Chen, and H. V. Poor, "Multiuser MISO interference channles with single-user detection: optimality of beamforming and the achievable rate region," *IEEE Trans. Inf. Theory*, vol. 57, no. 7, pp. 4255 – 4273, July 2011.

[9] R. Mochaourab and E. A. Jorswieck, "Optimal beamforming in interference networks with perfect local channel information," *IEEE Trans. Signal Process.*, vol. 59, no. 3, pp. 1128–1141, Mar. 2011.

[10] V. R. Cadambe, S. A. Jafar, and C. Wang, "Interference alignment with asymmetric complex signaling - settling the Host-Madsen-Nosratinia conjecture," *IEEE Trans. Inf. Theory*, pp. 4552 – 4565, Sept. 2010.

[11] S. H. Park, H. Park, and I. Lee, "Coordinated SINR balancing techniques for multi-cell downlink transmission," in *Proc. VTC 2010-fall*, 2010.

[12] Z. K. M. Ho and E. Jorswieck, "Improper Gaussian signaling on the two-user SISO interference channel," *IEEE Trans. Wireless Commun.*, vol. 11, no. 9, pp. 3194 – 3203, Sept. 2012.

[13] Y. Zeng, C. M. Yetis, E. Gunawan, Y. L. Guan, and R. Zhang, "Transmit optimization with improper Gaussian signaling for interference channels," to appear in *IEEE Trans. Signal Process.*, ArXiv pre-print at http://arxiv.org/abs/1207.5206.

[14] Z.-Q. Luo, W. K. Ma, A. M. So, Y. Ye, and S. Zhang, "Semidefinite relaxation of quadratic optimization problems," *IEEE Signal Processing Mag*, vol. 27, no. 3, pp. 20–34, May 2010.

[15] P. J. Schreier and L. L. Scharf, *Statistical Signal Processing of Complex-Valued Data: The Theory of Improper and Noncircular Signals*, Cambridge Univ. Press, 2010.

[16] A. Wiesel, Y. C. Eldar, and S. Shamai (Shitz), "Linear precoding via conic optimization for fixed MIMO receivers," *IEEE Trans. Signal Process.*, vol. 54, no. 1, pp. 161–176, Jan. 2006.

[17] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge, U.K.: Cambridge Univ. Press, 2004.

[18] Y. Huang and D. P. Palomar, "Rank-constrained separable semidefinite programming with applications to optimal beamforming," *IEEE Trans. Signal Process.*, vol. 58, no. 2, pp. 664–678, Feb. 2010.