One-Bit Quantization for Multi-Sensor GLRT Detection of Unknown Deterministic Signals

Xiaoying Li*, Jun Fang*, Yumeng Liu*, and Hongbin Li†
*National Key Laboratory on Communications, University of Electronic Science and Technology of China
Chengdu 611731, China Emails: 201021260101@std.uestc.edu.cn, JunFang@uestc.edu.cn
†Department of Electrical and Computer Engineering, Stevens Institute of Technology
Hoboken, NJ 07030 USA Email: Hongbin.Li@stevens.edu

***Abstract*— In this paper, we consider a decentralized detection problem in which a number of sensor nodes collaborate to detect the presence of an unknown deterministic signal. Due to stringent power/bandwidth constraints, each sensor quantizes its local observation into one bit of information. The binary data are then sent to the fusion center (FC), where a generalized likelihood ratio test (GLRT) detector is employed to make a global decision. In this context, we study one-bit quantizer design and analyze the asymptotic performance of the one-bit GLRT detector for cases where the quantized data are sent to the FC via perfect or imperfect channels. Simulation results are carried out to corroborate our theoretical analysis and to illustrate the performance of the proposed scheme.**

***Index Terms*— Decentralized detection, one-bit quantization, wireless sensor networks (WSNs).**

I. Introduction

The problem of decentralized detection in wireless sensor networks (WSNs) has attracted much interest over the past decade. A large amount of studies in decentralized detection [1]–[9] assumes that the knowledge of the probability density function (pdf) under either hypothesis is available. In this case, a local likelihood ratio test (LRT) can be conducted at each sensor and the local binary decision is sent to a fusion center (FC) to reach a global decision. The LRT has been proved to be the optimal local sensor decision for a binary hypothesis problem under both Bayesian and Neyman-Pearson criteria [1], [2]. Nevertheless, the search of optimal local detectors is still exponentially complex because the optimal local quantization thresholds are generally different and need to be jointly determined along with the global fusion rule [3], [5].

In this paper, we consider the problem of detecting the presence of an unknown deterministic signal. Due to the lack of signal knowledge, one cannot compute the local likelihood ratio at each sensor. A natural strategy in this case is to send sensor's original observations to the FC. A generalized likelihood ratio test (GLRT) is then conducted to make a final decision. Sending original observations to the FC, however, could be prohibitive for sensor networks whose bandwidth and energy are severely constrained. To meet stringent bandwidth/energy constraints, we consider the strategy where each sensor quantizes its local observation into one bit of information. A GLRT detector based on one-bit quantized data can be developed to form a global decision. In this framework, we examine the one-bit quantization design and analyze the asymptotic performance of the corresponding one-bit GLRT detector for both perfect channels and binary symmetric channels between sensors and the FC. Our analysis shows that, unlike the LRT fusion rule in which optimal local quantization thresholds are generally different, the optimal quantization thresholds for multi-sensor GLRT fusion are identical and should be equal to zero, irrespective of sensor observation disparities. In addition, when the optimal quantization thresholds are selected, the one-bit GLRT detector that uses only $\lceil N\pi/2 \rceil$ sensors, with each sensor sending one bit of information, can attain the same performance as the GLRT detector that requires original sensor observations of N sensors. Here $\lceil x \rceil$ denotes the ceiling operator that gives the smallest integer no smaller than x. Thus considerable bandwidth/energy savings can be achieved. Multi-sensor GLRT fusion based on quantized data was also studied in [10], [11] in the context of detecting a source with unknown locations and fusing dependent decisions. Nevertheless, optimal quantizer design and achievable asymptotic performance were not investigated. In some other studies [12], [13], decentralized detection of unknown deterministic signal based on quantized data were considered, but the fusion rule is ad hoc and not optimized.

II. Problem Formulation

We consider a binary hypothesis testing problem in which a number of sensors collaborate to detect the presence of an unknown scalar deterministic signal θ. The binary hypothesis testing problem is formulated as follows:

$$\begin{aligned} H_0 : &\qquad x_n = w_n, \\ H_1 : &\qquad x_n = h_n\theta + w_n, \qquad n = 1, \dots, N \end{aligned} \tag{1}$$

where x_n denotes the nth sensor's observation, $h_n \in \mathbb{R}$ is the known observation coefficient defining the input/output relation of the nth sensor, w_n denotes the additive Gaussian noise with zero mean and variance σ_n^2, and the noise is assumed independent across the sensors. To meet stringent bandwidth/power budgets in wireless sensor networks (WSNs), each sensor quantizes its real-valued observation into one bit of information. We first assume an ideal channel between sensors

This work was supported in part by the National Science Foundation of China under Grant 61172114, and the National Science Foundation under Grant ECCS-0901066.

and the fusion center (FC) through which the date can be received without distortion. An imperfect link scenario where the binary data are sent through binary symmetric channels will be considered in Section V. For each sensor, given a quantization threshold τ_n, the binary data b_n is given by

$$b_n = \text{sgn}(x_n - \tau_n), \qquad \forall n = 1, \ldots, N \tag{2}$$

where $\text{sgn}(x) = 1$ if if $x > 0$, otherwise $\text{sgn}(x) = 0$. Upon receiving binary data $\{b_n\}_{n=1}^N$, the FC forms a final decision about the absence or presence of θ. The problem of interest is to determine the one-bit quantizer for each sensor, and to develop a detector to detect θ given $\{b_n\}_{n=1}^N$ for the FC.

III. GLRT DETECTOR

Suppose the quantization threshold τ_n is predetermined for each sensor. A generalized likelihood ratio test (GLRT), which replaces the unknown parameter with the maximum likelihood estimate (MLE), can be used to detect θ. For our case where there is no unknown parameter under H_0, the GLRT decides H_1 if

$$T_{\text{Q}}(\mathbf{b}) \triangleq \frac{p(\mathbf{b}|\hat{\theta}; H_1)}{p(\mathbf{b}|H_0)} > \eta \tag{3}$$

where the subscript 'Q' stands for the one-bit quantization scheme, $\mathbf{b} \triangleq [b_1 \; b_2 \; \ldots \; b_N]^T$, $\hat{\theta}$ is the MLE of θ, and η is a threshold determined by the specified false alarm probability. The MLE of θ can be computed by maximizing the log-likelihood function of θ

$$\hat{\theta} = \arg\max_{\theta} \; L(\theta) \tag{4}$$

where the log-likelihood function $L(\theta)$ can be written as

$$\begin{aligned} L(\theta) &\triangleq \log P(b_1, \ldots, b_N; \theta) \\ &= \sum_{n=1}^{N} \big\{ b_n \log\left[F_{w_n}(\tau_n - h_n\theta)\right] \\ &\quad + (1 - b_n) \log\left[1 - F_{w_n}(\tau_n - h_n\theta)\right] \big\} \end{aligned} \tag{5}$$

by noting that $\{b_n\}$ are independent and the probability mass function (PMF) of b_n is given by

$$P(b_n; \theta) = \left[F_{w_n}(\tau_n - h_n\theta)\right]^{b_n} \left[1 - F_{w_n}(\tau_n - h_n\theta)\right]^{1-b_n} \tag{6}$$

in which F_{w_n} denotes the complementary cumulative density function (CCDF) of w_n. It can be readily verified that $L(\theta)$ is a concave function for Gaussian noise [14]. Thus the ML estimation of θ is a well-behaved numerical problem and any gradient-based search starting from a random initial estimate is guaranteed to converge to the global maximum. Substituting $\hat{\theta}$ back to (3), we can compute the generalized likelihood ratio and make a final decision.

IV. OPTIMAL QUANTIZER DESIGN AND ASYMPTOTIC PERFORMANCE ANALYSIS

In this section, we study the optimal one-bit quantization design for each sensor and analyze the asymptotic performance of the GLRT detector. From [15], we know that the modified test statistic $2 \ln T_{\text{Q}}(\mathbf{b})$ asymptotically follows

$$2 \ln T_{\text{Q}}(\mathbf{b}) \overset{a}{\sim} \begin{cases} \chi_1^2 & \text{under } H_0 \\ \chi'^2_1(\lambda_{\text{Q}}) & \text{under } H_1 \end{cases} \tag{7}$$

where χ_ν^2 denotes a central chi-squared distribution with ν degrees of freedom, and $\chi'^2_\nu(\lambda)$ denotes a non-central chi-squared distribution with ν degrees of freedom and noncentrality parameter λ. The noncentrality parameter λ_{Q} can be computed as

$$\lambda_{\text{Q}} = (\theta_1 - \theta_0)^T I(\theta_0)(\theta_1 - \theta_0) \tag{8}$$

where $\theta_0 = 0$ and $\theta_1 = \theta$ denote the value of θ under H_0 and H_1 respectively, and $I(\theta)$ denotes the Fisher information (FI) which is given by

$$\begin{aligned} I(\theta) &= -E\left[\frac{\partial^2 L(\theta)}{\partial \theta^2}\right] \\ &= \sum_{n=1}^{N} \frac{h_n^2 p_{w_n}^2(\tau_n - h_n\theta)}{F_{w_n}(\tau_n - h_n\theta)[1 - F_{w_n}(\tau_n - h_n\theta)]} \end{aligned} \tag{9}$$

and $p_{w_n}(x)$ denotes the probability density function (pdf) of w_n. We see that the noncentrality parameter is a function of the quantization thresholds $\{\tau_n\}_{n=1}^N$. Clearly, given a specified false alarm probability, a larger noncentrality parameter λ results in better detection performance. Therefore the optimal quantization thresholds are those that maximize the noncentrality parameter λ_{Q}:

$$\max_{\{\tau_n\}} \quad \lambda_{\text{Q}} = \theta^2 \sum_{n=1}^{N} \frac{h_n^2 p_{w_n}^2(\tau_n)}{F_{w_n}(\tau_n)[1 - F_{w_n}(\tau_n)]} \tag{10}$$

The above optimization can be decoupled into a set of independent quantization threshold design problems

$$\max_{\tau_n} \quad g(\tau_n) \triangleq \frac{p_{w_n}^2(\tau_n)}{F_{w_n}(\tau_n)[1 - F_{w_n}(\tau_n)]} \quad \forall n \tag{11}$$

For the Gaussian random variable w_n, the function $g(\tau_n)$ is a unimodal, positive and symmetric function attaining its maximum when $\tau_n = 0$ [14]. Therefore the optimal quantization threshold for each sensor is given by

$$\tau_n^* = 0 \qquad \forall n \tag{12}$$

Substituting the optimal quantization thresholds back into (10), the largest achievable noncentrality parameter of one-bit GLRT detector is given by

$$\lambda_{\text{Q}} = \frac{2\theta^2}{\pi} \sum_{n=1}^{N} \frac{h_n^2}{\sigma_n^2} \tag{13}$$

Note that the optimal quantization thresholds given in (12) holds valid irrespective of observation disparities across sensors. This is different from the likelihood ratio test (LRT)-based fusion rule in which optimal local quantization thresholds are generally different and functions of observation and channel parameters. Optimal one-bit quantization design was also studied in the context of decentralized estimation. Nevertheless, it turns out that, for decentralized estimation (e.g. [14]), the optimal quantizer design requires the knowledge of the unknown signal, which makes the acquisition of optimal quantization thresholds impossible. Our result, in contrast, is very encouraging in that the optimal quantization thresholds are independent of the unknown signal to be detected.

A. Comparison With The Clairvoyant GLRT Detector

It is interesting to examine the performance of the one-bit GLRT detector as compared with a GLRT detector that has full access to sensors' original observations (also referred to as the clairvoyant GLRT detector). The latter detector provides a bound on the achievable performance of all rate-constrained methods. For the clairvoyant detector, its modified test statistic asymptotically follows

$$2\ln T_{\text{NQ}}(\mathbf{x}) \overset{a}{\sim} \begin{cases} \chi_1^2 & \text{under } H_0 \\ \chi_1'^2(\lambda_{\text{NQ}}) & \text{under } H_1 \end{cases} \tag{14}$$

where the subscript 'NQ' represents no quantization, and λ_{NQ} can be easily computed and given as

$$\lambda_{\text{NQ}} = \theta^2 \sum_{n=1}^{N} \frac{h_n^2}{\sigma_n^2} \tag{15}$$

Comparing (13) with (15), we quickly reach that

$$\lambda_{\text{Q}} = \frac{2}{\pi}\lambda_{\text{NQ}} \tag{16}$$

When $h_n = h$ and $\sigma_n^2 = \sigma^2$ for all n, the above relationship implies that the performance loss of the one-bit scheme due to quantization can be compensated by slightly increasing the number of sensors by a factor of $\pi/2$. In other words, to meet the same detection performance, the number of sensors required by the one-bit GLRT detector is $\pi/2$ times the number of sensors used by the clairvoyant GLRT detector. The one-bit scheme, however, may still be considered more efficient than the clairvoyant detector in a rate distortion sense since it only needs to transmit a total number of $\lceil N\pi/2 \rceil$ bits, in which $\lceil x \rceil$ denotes the ceiling operator that gives the smallest integer no smaller than x, while the clairvoyant detector requires sending N real-valued messages to the FC.

For the general case where sensors' local signal-to-noise ratios (SNRs) $\{h_n^2/\sigma_n^2\}$ are different, we cannot guarantee that the one-bit GLRT scheme with $\lceil N\pi/2 \rceil$ sensors achieves the same performance as the clairvoyant GLRT detector using N sensors. Nevertheless, if sensors in the network are uniformly distributed and the number of sensors is sufficiently large, then we can expect that the percentage of sensors corresponding to a certain SNR remains fixed. Therefore increasing the number of sensors by a scaling factor would result in an increase in the noncentrality parameter by approximately a same factor. In this case, our argument made in the last paragraph remains valid.

V. Extension To The Imperfect Channel Case

The previous section assumes that the one-bit binary data can be transmitted to the FC without any distortion. In this section, we consider an imperfect link scenario where the one-bit quantized data are sent to the FC over binary symmetric channels (BSC), i.e.

$$y_n = \begin{cases} b_n & \text{with probability } 1-p \\ 1-b_n & \text{with probability } p \end{cases}$$

where y_n denotes the received data and p is the crossover probability of the BSC channel. Our objective is to detect θ based on received data $\{y_n\}$. Again, the modified test statistic for the GLRT detector asymptotically follows the same distributions as that of (7), except with a different noncentrality parameter under the alternative hypothesis. To evaluate the detection performance in the presence of transmission errors, we need to compute the FI and the corresponding noncentrality parameter. For the channel model being considered here, the PMF of the received data y_n is given by

$$\begin{aligned} P(y_n;\theta) &= P(y_n = b_n)\cdot(1-p) + P(y_n = 1-b_n)\cdot p \\ &= (1-p)\cdot[F_{w_n}(\tau_n - h_n\theta)]^{y_n}[1-F_{w_n}(\tau_n - h_n\theta)]^{1-y_n} \\ &\quad + p\cdot[F_{w_n}(\tau_n - h_n\theta)]^{1-y_n}[1-F_{w_n}(\tau_n - h_n\theta)]^{y_n} \end{aligned} \tag{17}$$

The likelihood function is a product of the PMFs associated with $\{y_n\}$. The FI can be computed by taking the second-order derivative of the likelihood function

$$I_{\text{Q-BSC}} = \sum_{n=1}^{N} \left\{ \frac{(1-2p)^2 h_n^2 p_{w_n}^2(\tau_n - h_n\theta)}{[p + (1-2p)F_{w_n}(\tau_n - h_n\theta)]} \times \frac{1}{[1-p-(1-2p)F_{w_n}(\tau_n - h_n\theta)]} \right\} \tag{18}$$

where the subscript 'Q-BSC' stands for the one-bit quantization scheme with its quantized data transmitted through BSC channels. From (8), the noncentrality parameter can be computed as

$$\lambda_{\text{Q-BSC}} = \theta^2(1-2p)^2 \sum_{n=1}^{N} \left\{ \frac{h_n^2 p_{w_n}^2(\tau_n)}{[p + (1-2p)F_{w_n}(\tau_n)]} \times \frac{1}{[1-p-(1-2p)F_{w_n}(\tau_n)]} \right\} \tag{19}$$

As expected, the noncentrality parameter $\lambda_{\text{Q-BSC}}$ not only depends on the quantization thresholds $\{\tau_n\}$, but also on the crossover probability p. Given a specified p, the optimal quantization threshold for each sensor can be obtained by solving

$$\max_{\tau_n} \frac{p_{w_n}^2(\tau_n)}{[p + (1-2p)F_{w_n}(\tau_n)][1-p-(1-2p)F_{w_n}(\tau_n)]} \tag{20}$$

The above optimization can be re-expressed as

$$\min_{\tau_n} \frac{\Delta + F_{w_n}(\tau_n)(1 - F_{w_n}(\tau_n))}{p^2_{w_n}(\tau_n)} \tag{21}$$

where $\Delta \triangleq (p - p^2)/(1 - 2p)^2$ is a positive value. Since both $\Delta/p^2_{w_n}(\tau_n)$ and $F_{w_n}(\tau_n)(1 - F_{w_n}(\tau_n))/p^2_{w_n}(\tau_n)$ attain their minima when $\tau_n = 0$, the optimal quantization threshold is independent of the probability p and equal to zero, i.e.

$$\tau_n^* = 0 \qquad \forall n \tag{22}$$

When the optimal quantization thresholds are adopted, the noncentrality parameter $\lambda_{\text{Q-BSC}}$ achieves its maximum:

$$\lambda_{\text{Q-BSC}} = \frac{2\theta^2(1-2p)^2}{\pi} \sum_{n=1}^{N} \frac{h_n^2}{\sigma_n^2} \tag{23}$$

Observing (13) and (23), we have

$$\lambda_{\text{Q-BSC}} = (1-2p)^2 \lambda_{\text{Q}} \tag{24}$$

This relationship quantifies the performance loss due to imperfect channel links. Additionally, it tells us how many more sensors are needed in order to achieve the same performance as the error-free one-bit GLRT detector and the clairvoyant detector.

VI. Simulation Results

We provide simulation results to corroborate our analysis and to illustrate the performance of the proposed one-bit GLRT scheme. We compare the one-bit GLRT scheme with the clairvoyant detector that has full access to sensors' original observations. In our simulations, we assume a homogeneous scenario where all sensors have identical observation qualities with $h_n = 1$ and $\sigma_n^2 = 1$ for all n. Optimal quantization thresholds are selected for the proposed schemes in our experiments, i.e. $\tau_n = 0, \forall n$. Fig. 1 plots the detection probabilities of the clairvoyant GLRT detector and the one-bit GLRT detector (with perfect/imperfect links between sensors and the FC) as a function of the number of sensors. The crossover probability for the BSCs is set to be 0.2, i.e. $p = 0.2$. In the figure, solid lines represent the theoretical asymptotic performance, while the plus marks, $+$, represent the performance of the Monte Carlo experiments obtained by averaging over 10^5 independent runs. From Fig. 1, we see that the theoretical asymptotic analysis provides a good approximation of the experimental performance, even when the number of sensors is small. In addition, it can be observed that to achieve the same detection probability, say, $P_{\text{D}} = 0.8$, the one-bit GLRT detector (with perfect links) requires about 40 sensors, which is approximately $\pi/2$ times the number of sensors needed by the clairvoyant detector (the clairvoyant detector requires about 25 sensors to achieve $P_{\text{D}} = 0.8$). The one-bit GLRT detector incurs considerable performance degradation due to imperfect transmissions. From (24), we know that to attain a same detection rate, the number of sensors required in the presence of transmission errors is $1/(1-2p)^2$ times that for the error-free case. Hence to attain a detection probability of 0.8, the required number of sensors is $40/(1-2p)^2 \approx 111$. As observed from the figure, this theoretical prediction coincides with our simulation result very well.

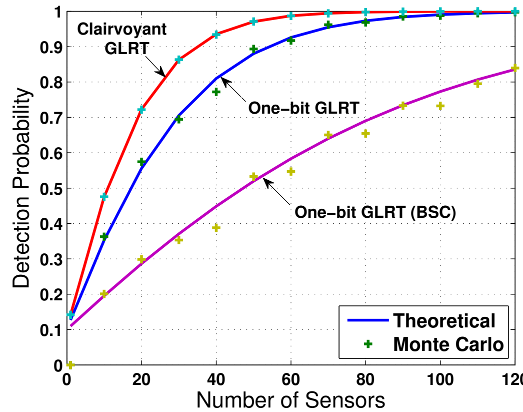

Fig. 1. Detection probability vs. number of sensors for one-bit GLRT detector and the clairvoyant detector, $P_{\text{FA}} = 0.1$.

VII. Conclusions

We studied multi-sensor GLRT detection fusion based on one-bit quantized data. The optimal quantization thresholds for sensors are shown independent of the unknown signal to be detected, and are equal to zero for both perfect links and imperfect binary symmetric channels between sensors and the FC. Our analysis indicates that the proposed one-bit GLRT scheme can achieve the same detection performance as a clairvoyant detector by slightly increasing the number of sensors by a factor of $\pi/2$. Simulation results were provided to corroborate our analysis and to illustrate the performance of the proposed scheme.

References

[1] A. R. Reibman and L. W. Nolte, "Optimal detection and performance of distributed sensor systems," *IEEE Transactions on Aerospace and Electronic Systems*, vol. AES-23, no. 1, pp. 24–30, Jan. 1987.

[2] I. Y. Hoballah and P. K. Varshney, "Distributed bayesian signal detection," *IEEE Transactions on Information Theory*, vol. 35, pp. 995–1000, Sept. 1989.

[3] J. N. Tsistsiklis, "Decentralized detection," in *Advances in statistical signal processing, signal detection*, H. V. Poor and J. B. Thomas, Eds. Greenwich, CT: JAI, 1993, vol. 2.

[4] C. Rago, P. Willett, and Y. Bar-Shalom, "Censoring sensors: a low-communication-rate scheme for distributed detection," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 32, no. 2, pp. 554–568, April 1996.

[5] R. Viswanathan and P. K. Varshney, "Distributed detection with multiple sensors: Part I - fundamentals," *Proceedings of the IEEE*, vol. 85, no. 1, pp. 54–63, January 1997.

[6] R. S. Blum, S. A.Kassam, and H. V. Poor, "Distributed detection with multiple sensors: Part II - advanced topics," *Proceedings of the IEEE*, vol. 85, no. 1, pp. 64–79, January 1997.

[7] J. Chamberland and V. Veeravalli, "Decentralized detection in sensor networks," *IEEE Transactions on Signal Processing*, vol. 51, no. 2, pp. 407–416, February 2003.

[8] B. Chen and P. K. Willett, "On the optimality of the likelihood-ratio test for local sensor decision rules in the presence of nonideal channels," *IEEE Transactions on Information Theory*, vol. 51, no. 2, pp. 693–699, Feb. 2005.

[9] J.-F. Chamberland and V. V. Veeravalli, "How dense should a sensor network be for detection with correlated observations," *IEEE Transactions on Information Theory*, vol. 52, no. 11, pp. 5099–5106, Nov. 2006.

[10] R. Niu and P. K. Varshney, "Joint detection and localization in sensor networks based on local decisions," in *49th Asilomar Conference on Signals, Systems, and Computers*, Pacific Grove, CA, 2006.

[11] S. G. Iyengar, R. Niu, and P. K. Varshney, "Fusing dependent decisions for hypothesis testing with heterogeneous sensors," *IEEE Transactions on Signal Processing*, vol. 60, no. 9, pp. 4888–4897, Sept. 2012.

[12] V. A. Aalo and R. Viswanathan, "Multilevel quantisation and fusion scheme for the decentralised detection of an unknown signal," *IEE Proc.-Radar, Sonar, Navig.*, vol. 141, no. 1, pp. 37–44, February 1994.

[13] R. Niu and P. K. Varshney, "Performance analysis of distributed detection in a random sensor field," *IEEE Transactions on Signal Processing*, vol. 56, no. 1, pp. 339–349, Jan. 2008.

[14] A. Ribeiro and G. B. Giannakis, "Bandwidth-constrained distributed estimation for wireless sensor networks - Part I: Gaussian PDF," *IEEE Transactions on Signal Processing*, vol. 54, no. 3, pp. 1131–1143, March 2006.

[15] S. M. Kay, *Fundamentals of Statistical Signal Processing: Detection Theory*. Upper Saddle River, NJ: Prentice Hall, 1998.