

# OPTIMAL WIDELY-LINEAR DISTRIBUTED BEAMFORMING FOR RELAY NETWORKS

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## ABSTRACT

This paper presents a widely-linear (WL) distributed beamforming algorithm that takes advantage of strictly second-order (SO) non-circular source signals. We consider a single-antenna source-destination pair, which is assisted by multiple relays but suffers from strong interference. Assuming that perfect channel state information (CSI) is available, we design our algorithm based on the maximization of the signal-to-interference-plus-noise ratio (SINR) under a total relay power constraint after applying WL processing. We prove that in the case of no interference, the proposed WL distributed beamforming algorithm provides an SINR gain of 3 dB over its linear counterpart due to a virtual doubling of the number of relays. Also, the complexity is analyzed and simulations for the interference scenario show the performance gains in terms of the SINR and the bit error rate (BER).

**Index Terms**— Widely-linear processing, non-circular sources, distributed beamforming, ad-hoc relay networks.

## 1. INTRODUCTION

The establishment of reliable and energy-efficient transmissions in distributed ad-hoc networks of nodes has recently sparked a great interest in the field of wireless communications. One of the main concepts to improve the performance of a wireless network, i.e., its coverage, capacity, and reliability, is exploiting the cooperation between nodes, termed user cooperation diversity [1]–[6]. In such schemes, non-transmitting nodes assist each other by relaying source signals through multiple independent paths in the network, which are constructively combined at the destination. These relays create a virtual array of transmit antennas, and thereby provide spatial diversity in order to combat signal fading effects and interference without the need of multiple antennas at the users.

A very effective approach to obtain cooperative diversity is distributed relay beamforming [7]–[13] based on the amplify-and-forward (AF) relaying protocol [6], which is of special interest due to its simplicity. References [7]–[9] consider a single source-destination pair and compute the beamforming weights based on the assumption that the instantaneous channel state information (CSI) is available at the receiver. Thus, the weights are fed back to the relays. The studied strategies to determine the beamforming weights minimize the total relay power subject to a certain target signal-to-noise ratio (SNR) at the receiver [7], maximize the receiver SNR subject to certain power constraints [8], i.e., individual relay power constraints and a total relay power constraint, and minimize the mean squared error (MSE) at the destination [9]. Techniques that only rely on the statistics of the CSI were examined in [10] and [11], and extensions to multiple source-destination pairs were developed in [12] and [13].

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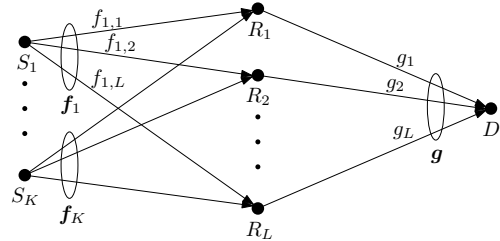


Fig. 1. A network of  $K$  sources,  $L$  relays, and one destination.

Recently, the concept of widely-linear (WL) processing has been applied to the conventional beamforming problem [14]–[17] to take advantage of the second-order (SO) non-circularity [18]–[20] of certain source signals. Examples of such signals are BPSK, Offset-QPSK, PAM, and ASK-modulated signals. It was shown that processing the non-circular data and its conjugate version separately leads to significant performance improvements over the traditional linear filtering [14]–[17]. However, WL processing has so far not been exploited in the design of cooperative diversity techniques, e.g., distributed beamforming, for ad-hoc relay networks.

In this paper, we propose an optimal WL distributed beamforming (WL-DB) algorithm that fully exploits the SO statistics of strictly non-circular (rectilinear) source signals. We consider a single-antenna source-destination pair that is subject to interference and assume that perfect CSI is available at the receiver. We maximize the signal-to-interference-plus-noise ratio (SINR) at the receiver under a total power constraint and prove that the presented WL-DB algorithm provides an SINR gain of 3 dB over the linear distributed beamforming (L-DB) algorithm if there is no interference. Moreover, we analyze the complexity and show simulations that illustrate the performance gains in terms of the maximum SINR and the bit error rate (BER). Extensions to other strategies of computing the beamforming weights, individual relay power constraints as well as to multiple source-destination pairs are straightforward.

## 2. SYSTEM MODEL

Let us consider a single-antenna relay network with one source-destination pair,  $L$  relays and  $K - 1$  interfering source nodes, as illustrated in Fig. 1. There is no direct link between the  $K$  sources and the destination, all the relays work in half-duplex mode, and we have flat-fading channels. It is also assumed that the network is perfectly synchronized and that all the nodes operate in the same frequency band. Each transmission from the sources to the destination is implemented in two stages, e.g., in two consecutive time-slots. In the first time-slot, all the sources simultaneously broadcast their signals to the relays. In the second time-slot, the received signals at the relays are scaled by a complex beamforming weight to be designed,

and retransmitted to the destination.

The noisy mixture of source signals received by the relays can be modeled as

$$\mathbf{x} = \mathbf{F}\sqrt{\mathbf{P}}\mathbf{s} + \boldsymbol{\mu} \in \mathbb{C}^{L \times 1}, \quad (1)$$

where  $\mathbf{F} = [\mathbf{f}_1, \dots, \mathbf{f}_K] \in \mathbb{C}^{L \times K}$  is the channel matrix between the sources and the relays, and  $\mathbf{f}_i = [f_{i,1}, \dots, f_{i,L}]^T$ ,  $i = 1, \dots, K$ , contains the channel coefficients from the  $i$ -th source to all the relays. The vector  $\mathbf{s} = [s_1, \dots, s_K]^T \in \mathbb{C}^{K \times 1}$  represents the transmitted source signals,  $\mathbf{P} \in \mathbb{R}^{K \times K}$  is the matrix with the source powers on its diagonal, and  $\boldsymbol{\mu} \in \mathbb{C}^{L \times 1}$  is the additive noise at the relays.

Then, the retransmitted signal from the relays in the second step can be expressed as

$$\mathbf{r} = \mathbf{W}^H \mathbf{x} \in \mathbb{C}^{L \times 1}, \quad (2)$$

where  $\mathbf{W} = \text{diag}\{\mathbf{w}\}$  and  $\text{diag}\{\cdot\}$  places the elements of  $\mathbf{w}$  on the diagonal of  $\mathbf{W}$ , and  $\mathbf{w} = [w_1, \dots, w_L]^T \in \mathbb{C}^{L \times 1}$  contains the complex beamforming weights for each relay. Let us denote  $\mathbf{g} = [g_1, \dots, g_L]^T$  as the channel coefficient vector between the  $L$  relays and the destination. Using (1) and (2), the received signal at the destination can be written as

$$\begin{aligned} y &= \mathbf{g}^T \mathbf{r} + n \\ &= \underbrace{\sqrt{P_d} \mathbf{g}^T \mathbf{W}^H \mathbf{f}_d s_d}_{\text{desired signal}} + \underbrace{\mathbf{g}^T \mathbf{W}^H \sum_{k=1, k \neq d}^K \sqrt{P_k} \mathbf{f}_k s_k}_{\text{interference}} \\ &\quad + \underbrace{\mathbf{g}^T \mathbf{W}^H \boldsymbol{\mu} + n}_{\text{effective noise}}, \end{aligned} \quad (3)$$

where  $n$  is the zero-mean noise at the destination with variance  $\sigma_n^2$  and the subscript  $d$  denotes the desired signal component. Furthermore, we use the following assumptions:

- i) The relay noise is assumed to be zero-mean and i.i.d. with equal variance  $\sigma_\mu^2$  for each relay.
- ii) The source symbols are uncorrelated, i.e.,  $\mathbb{E}\{\mathbf{s}\mathbf{s}^H\} = \mathbf{I}_K$ .
- iii) The channels are time-invariant during the transmission.
- iv) The channel coefficients, the source symbols, and the noise at the relays and the destination are statistically independent.

### 3. WIDELY-LINEAR PROCESSING

In this section, we introduce the concept of WL processing to distributed relay beamforming. WL signal processing aims to take advantage of the SO non-circularity of the transmitted source signals by processing the non-circular data and its conjugate version separately. The statistics of non-circular signals are fully described by their SO moments, i.e., the covariance matrix and the pseudo-covariance matrix, whereas the latter is ignored in the linear processing [18]-[20]. Thus, by exploiting the additional information contained in the pseudo-covariance matrix, large gains in terms of the performance can be achieved [14]-[17]. In order to establish if this concept can also be applied to the distributed beamforming model from the previous section, we define the augmented relay vector  $\mathbf{x}_a = [\mathbf{x}^T, \mathbf{x}^H]^T \in \mathbb{C}^{2L \times 1}$  to be processed in the sequel. As shown in this section, it is evident that this WL transformation does not violate the assumptions i)-iv), hence, WL processing is directly applicable to the distributed beamforming design.

Due to the assumption of strictly SO non-circular source signals, where the complex symbol amplitudes lie on a line in the I/Q diagram, we can decompose the symbol vector in (1) as

$$\mathbf{s} = \boldsymbol{\Psi} \mathbf{s}_0, \quad (5)$$

where  $\mathbf{s}_0 \in \mathbb{R}^{K \times 1}$  is a real-valued symbol vector and  $\boldsymbol{\Psi} = \text{diag}\{e^{j\varphi_i}\}_{i=1}^K$  contains arbitrary complex phase shifts on its diagonal that are usually different for each signal.

Expanding the augmented relay vector  $\mathbf{x}_a \in \mathbb{C}^{2L \times 1}$  and inserting (5), yields

$$\mathbf{x}_a = \begin{bmatrix} \mathbf{x} \\ \mathbf{x}^* \end{bmatrix} = \begin{bmatrix} \mathbf{F}\sqrt{\mathbf{P}}\mathbf{s} \\ \mathbf{F}^* \sqrt{\mathbf{P}}\mathbf{s}^* \end{bmatrix} + \begin{bmatrix} \boldsymbol{\mu} \\ \boldsymbol{\mu}^* \end{bmatrix} \quad (6)$$

$$\begin{aligned} &= \begin{bmatrix} \mathbf{F} \\ \mathbf{F}^* \boldsymbol{\Psi}^* \boldsymbol{\Psi} \end{bmatrix} \sqrt{\mathbf{P}}\mathbf{s} + \begin{bmatrix} \boldsymbol{\mu} \\ \boldsymbol{\mu}^* \end{bmatrix} \\ &= \mathbf{F}_a \sqrt{\mathbf{P}}\mathbf{s} + \boldsymbol{\mu}_a, \end{aligned} \quad (7)$$

where  $\mathbf{F}_a = [\mathbf{f}_{a1}, \dots, \mathbf{f}_{aK}] \in \mathbb{C}^{2L \times K}$  and (7) is written similarly to (1) so that assumptions i)-iii) are still fulfilled. The extended dimensions of  $\mathbf{x}_a$  can be interpreted as a virtual doubling of the number of relays. According to (2), we have the augmented retransmitted vector

$$\mathbf{r}_a = \mathbf{W}_a^H \mathbf{x}_a \in \mathbb{C}^{2L \times 1}, \quad (8)$$

where  $\mathbf{W}_a = \text{diag}\{\mathbf{w}_a\}$  and  $\mathbf{w}_a \in \mathbb{C}^{2L \times 1}$  contains twice as many complex beamforming weights as in the linear case (2). The association of these weights to the  $L$  physical relays will be discussed in Section 4.2. Next, we define the augmented channel vector  $\mathbf{g}_a$  as

$$\mathbf{g}_a = [\mathbf{g}^T, \mathbf{g}^T]^T \in \mathbb{C}^{2L \times 1}, \quad (9)$$

where the complex conjugation as in (6) has to be omitted. In analogy to (4) and using (7)-(9), the received signal at the destination after the WL processing is obtained as

$$\begin{aligned} y_a &= \underbrace{\sqrt{P_d} \mathbf{g}_a^T \mathbf{W}_a^H \mathbf{f}_{a,d} s_d}_{\text{augm. desired signal}} + \underbrace{\mathbf{g}_a^T \mathbf{W}_a^H \sum_{k=1, k \neq d}^K \sqrt{P_k} \mathbf{f}_{a,k} s_k}_{\text{augm. interference}} \\ &\quad + \underbrace{\mathbf{g}_a^T \mathbf{W}_a^H \boldsymbol{\mu}_a + n}_{\text{augm. effective noise}}. \end{aligned} \quad (10)$$

Note that also assumption iv) still holds after applying WL processing. Therefore, we base our further developments on (10).

### 4. SINR MAXIMIZATION

In this section, we derive a WL-DB algorithm based on the SINR maximization criterion. Using the model in (10), we compute the  $2L$  beamforming weights such that the SINR at the destination in the WL case is maximized subject to a total relay power constraint. The presented development is inspired by the one in [10] that uses linear processing. In contrast to [10], we incorporate WL processing, consider an interference scenario as shown in Fig. 1, and avoid the computationally expensive eigendecomposition to solve the resulting generalized eigenvector problem.

#### 4.1. WL Relay Weight Computation

The optimization problem is stated as

$$\begin{aligned} &\max_{\mathbf{w}_a} \quad \text{SINR}_{\text{WL}} \\ &\text{subject to} \quad P_r \leq P_{\text{max}}. \end{aligned} \quad (11)$$

Here,  $P_r$  is the total relay power,  $P_{\text{max}}$  is the maximum allowable total transmit power, and  $\text{SINR}_{\text{WL}}$  is the SINR in the WL case, which is defined as

$$\text{SINR}_{\text{WL}} = \frac{P_s}{P_i + P_n}, \quad (12)$$

where  $P_s$ ,  $P_i$ , and  $P_n$  represent the power of the desired signal, the interference power, and the noise power at the receiver, respectively.

Similar to [10]–[13], we next derive the expressions for the powers required in (11) and (12). For the total relay power  $P_r$ , we have

$$P_r = \mathbb{E}\{\|\mathbf{r}_a\|^2\} = \mathbf{w}_a^H \mathbf{D}_a \mathbf{w}_a, \quad (13)$$

where  $\mathbf{w}_a = \text{diag}\{\mathbf{W}_a\} \in \mathbb{C}^{2L \times 1}$  and  $\text{diag}\{\cdot\}$  extracts the diagonal from  $\mathbf{W}_a$ ,  $\mathbf{D}_a \in \mathbb{R}^{2L \times 2L}$  is a diagonal matrix with  $[\mathbf{D}_a]_{ll} = [\mathbf{T}_a]_{ll}$ ,  $l = 1, \dots, 2L$ , and the augmented covariance matrix  $\mathbf{T}_a$  can be expressed as

$$\mathbf{T}_a = \mathbb{E}\{\mathbf{x}_a \mathbf{x}_a^H\} = \mathbf{F}_a \mathbf{P} \mathbf{F}_a^H + \sigma_\mu^2 \mathbf{I}_{2L}, \quad (14)$$

where we have used assumptions i)–iv). From (10) and assumptions ii) and iii), we obtain the power of the desired signal component as

$$P_s = \mathbb{E}\{|\sqrt{P_d} \mathbf{g}_a^T \mathbf{W}_a^H \mathbf{f}_{a_d} s_d|^2\} = \mathbf{w}_a^H \mathbf{R}_a \mathbf{w}_a \quad (15)$$

where  $\mathbf{R}_a = P_d \mathbf{h}_{a_d} \mathbf{h}_{a_d}^H \in \mathbb{C}^{2L \times 2L}$  with  $\mathbf{h}_{a_d} = \mathbf{g}_a \odot \mathbf{f}_{a_d}$  and  $\odot$  is the Schur-Hadamard (element-wise) matrix product. Using assumptions ii), iii), and the model (10), the interference power can be written as

$$P_i = \mathbb{E}\left\{\left|\mathbf{g}_a^T \mathbf{W}_a^H \sum_{k=1, k \neq d}^K \sqrt{P_k} \mathbf{f}_{a_k} s_k\right|^2\right\} = \mathbf{w}_a^H \mathbf{Q}_{i_a} \mathbf{w}_a, \quad (16)$$

where  $\mathbf{Q}_{i_a} = \sum_{k=1, k \neq d}^K P_k \mathbf{h}_{a_k} \mathbf{h}_{a_k}^H \in \mathbb{C}^{2L \times 2L}$  with  $\mathbf{h}_{a_k} = \mathbf{g}_a \odot \mathbf{f}_{a_k}$ . The expression for the noise power at the receiver can be written as

$$P_n = \mathbb{E}\{|\mathbf{g}_a^T \mathbf{W}_a^H \boldsymbol{\mu}_a + n|^2\} = \mathbf{w}_a^H \mathbf{Q}_{n_a} \mathbf{w}_a + \sigma_n^2, \quad (17)$$

where  $\mathbf{Q}_{n_a} = \sigma_\mu^2 \text{diag}\{\mathbf{g}_a \mathbf{g}_a^H\} \in \mathbb{R}^{2L \times 2L}$  and we have used assumptions i) and iv). Whereas  $\mathbf{D}_a$  and  $\mathbf{Q}_{n_a}$  are real-valued diagonal matrices, the augmented covariance matrices  $\mathbf{R}_a$  and  $\mathbf{Q}_{i_a}$  possess the block structure

$$\mathbf{R}_a = \begin{bmatrix} \mathbf{R} & \mathbf{C} \\ \mathbf{C}^* & \mathbf{R}^* \end{bmatrix} \quad \text{and} \quad \mathbf{Q}_{i_a} = \begin{bmatrix} \mathbf{R}_i & \mathbf{C}_i \\ \mathbf{C}_i^* & \mathbf{R}_i^* \end{bmatrix}, \quad (18)$$

where the covariance matrices  $\mathbf{R}$  and  $\mathbf{R}_i$  of size  $L \times L$ , and the pseudo-covariance matrices  $\mathbf{C}$  and  $\mathbf{C}_i$  of size  $L \times L$  appear in conjugate pairs. The additional information  $\mathbf{C}$  and  $\mathbf{C}_i$  contained in the signal and the interference component respectively, will be exploited by the proposed WL-DB algorithm.

Finally, using (13), (15), (16), and (17), the WL-DB optimization problem (11) can be rewritten as

$$\begin{aligned} \max_{\mathbf{w}_a} \quad & \frac{\mathbf{w}_a^H \mathbf{R}_a \mathbf{w}_a}{\mathbf{w}_a^H (\mathbf{Q}_{i_a} + \mathbf{Q}_{n_a}) \mathbf{w}_a + \sigma_n^2} \\ \text{subject to} \quad & \mathbf{w}_a^H \mathbf{D}_a \mathbf{w}_a \leq P_{\max}. \end{aligned} \quad (19)$$

Note that the virtual doubling of the relay weights due to the WL processing at the receiver does not affect the maximum available relay transmit power  $P_{\max}$  used for the linear processing [10]. Thus, the power is merely allocated across twice as many virtual relays.

In order to simplify the optimization problem in (19), we find that at optimality the inequality constraint has to be satisfied with equality [10], i.e.,  $\mathbf{w}_a^H \mathbf{D}_a \mathbf{w}_a = P_{\max}$ . Inserting the equality constraint into the objective function in (19), the original problem can be reformulated as the unconstrained optimization problem

$$\max_{\mathbf{w}_a} \quad \frac{\mathbf{w}_a^H \mathbf{R}_a \mathbf{w}_a}{\mathbf{w}_a^H (\mathbf{Q}_{i_a} + \mathbf{Q}_{n_a} + \frac{\sigma_n^2}{P_{\max}} \mathbf{D}_a) \mathbf{w}_a}. \quad (20)$$

This type of optimization problem is known to be a generalized eigenvector problem [10]. Hence, if  $\bar{\mathbf{w}}_a$  is the solution to (20), the objective function is globally maximized when  $\bar{\mathbf{w}}_a$  is chosen as

$$\bar{\mathbf{w}}_a = \mathcal{P} \left\{ \left( \mathbf{Q}_{i_a} + \mathbf{Q}_{n_a} + \frac{\sigma_n^2}{P_{\max}} \mathbf{D}_a \right)^{-1} \mathbf{R}_a \right\}, \quad (21)$$

where  $\mathcal{P}\{\cdot\}$  is the normalized principal eigenvector operator. Note here that  $\mathbf{R}_a$  is of rank one and, therefore, any matrix multiplication with  $\mathbf{R}_a$  is also rank-one, which gives

$$\bar{\mathbf{w}}_a = \mathcal{P} \left\{ P_d \left( \mathbf{Q}_{i_a} + \mathbf{Q}_{n_a} + \frac{\sigma_n^2}{P_{\max}} \mathbf{D}_a \right)^{-1} \mathbf{h}_{a_d} \mathbf{h}_{a_d}^H \right\} \quad (22)$$

$$= \mathcal{P} \left\{ P_d \mathbf{v} \mathbf{h}_{a_d}^H \right\}. \quad (23)$$

The eigendecomposition in (23) provides only one non-zero eigenvalue whose corresponding eigenvector is a scaled version of  $\mathbf{v}$ . Thus, it can be avoided and we can express (21) after the normalization directly as

$$\bar{\mathbf{w}}_a = \frac{\mathbf{v}}{\|\mathbf{v}\|}. \quad (24)$$

However, the beamforming vector that solves (19) still needs to be scaled properly to satisfy the power constraint, i.e., the final solution is given by

$$\mathbf{w}_a = \sqrt{\frac{P_{\max}}{\bar{\mathbf{w}}_a^H \mathbf{D}_a \bar{\mathbf{w}}_a}} \bar{\mathbf{w}}_a. \quad (25)$$

## 4.2. WL Relay Weight Association

Having obtained the WL beamforming weights for the  $2L$  virtual relays from the WL optimization problem (19), these weights have to be associated to the  $L$  physical relays. By decomposing  $\mathbf{w}_a = [\mathbf{w}_1^T, \mathbf{w}_2^T]^T$ , where  $\mathbf{w}_n \in \mathbb{C}^{L \times 1}$  and  $n = 1, 2$ , defining  $\mathbf{W}_n = \text{diag}\{\mathbf{w}_n\}$ , and using (6) and (9), we rewrite (10) as

$$\begin{aligned} y_a &= \mathbf{g}_a^T \mathbf{W}_a^H \mathbf{x}_a + n = \mathbf{g}^T \mathbf{W}_1^H \mathbf{x} + \mathbf{g}^T \mathbf{W}_2^H \mathbf{x}^* + n \\ &= \mathbf{g}^T \mathbf{r}_{\text{WL}} + n \end{aligned} \quad (26)$$

such that the physical relays transmit the widely-linear combination

$$\mathbf{r}_{\text{WL}} = \mathbf{W}_1^H \mathbf{x} + \mathbf{W}_2^H \mathbf{x}^*. \quad (27)$$

Note that  $\mathbf{w}_1$  and  $\mathbf{w}_2$  are not complex conjugate versions of each other as in conventional WL receive beamforming [14]–[17]. This is due to the stacking in (9), which ensures that both terms in (27) pass through the same channel in (26).

## 4.3. Achievable SINR Performance

In order to analyze the SINR performance of the WL-DB algorithm as compared to its linear counterpart, we consider the non-interference case, i.e.,  $K = 1$ , for simplicity and formulate the following theorem:

**Theorem 1.** *The WL-DB algorithm for  $K = 1$  provides a 3 dB gain over the L-DB algorithm in terms of the maximum SINR.*

*Proof.* The SINR expression for  $K = 1$  in (19) reduces to

$$\text{SINR}_{\text{WL}} = \frac{\mathbf{w}_a^H \mathbf{R}_a \mathbf{w}_a}{\mathbf{w}_a^H \mathbf{Q}_{n_a} \mathbf{w}_a + \sigma_n^2} \quad \text{for} \quad \mathbf{w}_a^H \mathbf{D}_a \mathbf{w}_a = P_{\max}. \quad (28)$$

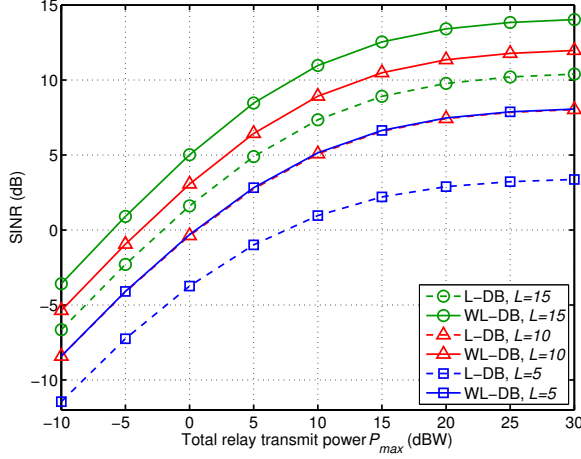


Fig. 2. Maximum SINR versus the maximum total relay power  $P_{\max}$  for  $K = 5$  and SNR = 0 dB at the relays and the destination.

Let us define  $\mathbf{w} \in \mathbb{C}^{L \times 1}$  as the solution to the linear SINR maximization problem such that

$$\text{SINR}_L = \frac{\mathbf{w}^H \mathbf{R} \mathbf{w}}{\mathbf{w}^H \mathbf{Q}_n \mathbf{w} + \sigma_n^2} \quad \text{for } \mathbf{w}^H \mathbf{D} \mathbf{w} = P_{\max}, \quad (29)$$

where  $\mathbf{R}$  and the diagonal matrices  $\mathbf{D}$  and  $\mathbf{Q}_n$  are the first  $L \times L$  block matrices of  $\mathbf{R}_a$ ,  $\mathbf{D}_a$  and  $\mathbf{Q}_{n_a}$ , respectively. As  $\mathbf{D}$  is real-valued and contained in both diagonal  $L \times L$  blocks of  $\mathbf{D}_a$ , we have  $\|\mathbf{w}\| = \|\mathbf{w}_a\|$  but the total relay power  $P_{\max}$  is divided equally into  $\mathbf{w}_1$  and  $\mathbf{w}_2$  and we obtain the symmetry  $|\mathbf{w}_m|^2 = |\mathbf{w}_{1_m}|^2 + |\mathbf{w}_{2_m}|^2$ ,  $m = 1, \dots, L$ . The same property applies to  $\mathbf{Q}_n$  and  $\mathbf{Q}_{n_a}$  so that the denominators of (28) and (29) are equal. It can be shown that due to the structure of the rank-one matrix  $\mathbf{R}_a$  in (15) and (18), and the aforementioned symmetry, the result of each of the four quadratic forms of the block matrices in  $\mathbf{R}_a$  is equal to  $\frac{1}{2} \cdot \mathbf{w}^H \mathbf{R} \mathbf{w}$  in the numerator of (29). Thus, we can conclude that

$$\text{SINR}_{\text{WL}} = 2 \cdot \text{SINR}_L, \quad (30)$$

which completes the proof.  $\square$

We have observed that in the case of interference, i.e.,  $K > 1$ , the SINR gain may increase.

#### 4.4. Complexity Analysis

The computational complexity of the proposed WL-DB algorithm is dominated by the computation of the rank-one matrices in (19) and the matrix inversion in (22). Thus, the computational cost is  $\mathcal{O}((2L)^3 + (2L)^2)$ , where the factor 2 is due to the virtual doubling of the number of relay nodes caused by the WL processing. The proposed algorithm only requires a slight increase of the necessary mathematical operations by a single factor compared to its linear counterpart whose computational complexity is  $\mathcal{O}(L^3 + L^2)$ .

### 5. SIMULATION RESULTS

In this section, we show simulations that demonstrate the performance of the proposed WL-DB algorithm based on the SINR maximization under the total relay power constraint. As a comparison, its linear counterpart (L-DB) is used to illustrate the maximum gain achieved by the WL processing. In the simulations, we assume Rayleigh flat-fading channels with unit-variance channel coefficients. The variances of the relay and the destination noise are

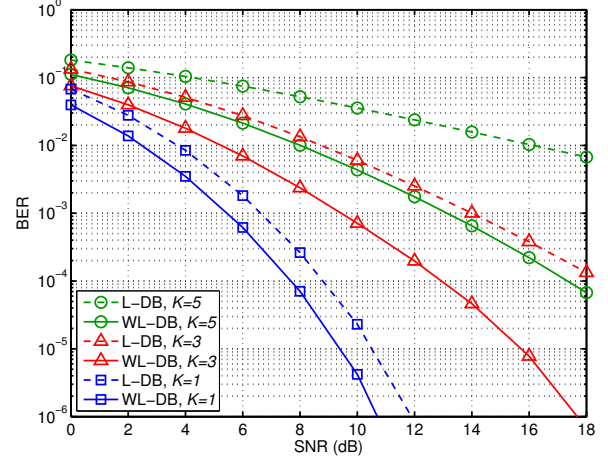


Fig. 3. BER versus the SNR at the relays and the destination for  $L = 5$  and  $P_{\max} = 1$  dBW.

equal to each other and the desired user as well as the  $K - 1$  interferers transmit with the same power of 0 dBW. Moreover, we assume that all the sources use the binary phase shift keying (BPSK) modulation for their transmission. All the curves are obtained by averaging over 1000 Monte Carlo trials.

In Fig. 2, we display the maximum achievable SINR as a function of the maximum total relay transmit power. We have fixed the number of sources to  $K = 5$ , the SNR at the relays and the destination is 0 dB, and we vary the number of relay nodes  $L$ . The non-circularity phases  $\varphi_i$  contained in  $\Psi$  are given by  $\varphi = [0, \pi/2, \pi/8, \pi/4, \pi/16]$ . It is evident that the WL-DB algorithm provides significant gains of about 3 dB over its linear counterpart for different numbers of relays  $L$ . Note that the curve of the WL-DB with  $L = 5$  matches the one of the L-DB with  $L = 10$ , which is due to the virtual doubling of the relay nodes.

Fig. 3 depicts the BER as a function of the SNR, which is the same at the relays and the destination. To obtain the BER curves, a symbol-by-symbol maximum likelihood (ML) decoder is used at the receiver. Here, the number of relays is fixed to  $L = 5$ , the total relay transmit power is  $P_{\max} = 1$  dBW, and we vary the number of interferers  $K - 1$ . The non-circularity phases of the sources are separated by  $\pi/4$  starting from 0. It can be seen that the WL-DB algorithm achieves a lower BER than its linear counterpart and that the gain increases as the number of interferers grows.

### 6. CONCLUSION

In this paper, we have presented a WL-DB algorithm that fully exploits the SO statistics of strictly non-circular sources in a network, which consists of a single-antenna source-destination pair, multiple relays and multiple interferers. The WL processing procedure virtually doubles the number of relays so that twice as many beamforming weights need to be designed. The weight computation is based on the SINR maximization under a total relay power constraint after applying WL processing, assuming that perfect CSI is available at the receiver. We have shown that in a non-interference scenario, the proposed WL-DB algorithm provides a 3 dB SINR gain over its linear counterpart, requiring only a slight increase in the computational complexity. Simulations demonstrate the significant performance gains in terms of the SINR and the BER.

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