LOCALIZATION WITH DOPPLER BIASED TOAS: AN ILL-CONDITIONED PROBLEM

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ABSTRACT

This paper investigates moving target localization by the *biased* time-of-arrivals (TOAs), where an extracted TOA is biased by the unknown Doppler of the target. The phenomenon applies to radar and sonar systems that employ Doppler tolerant waveforms. While in principle those would allow the estimation of both position and velocity using position-only measurements, we unfortunately find that the Fisher information matrix is ill-conditioned. However, a Quasi-maximum likelihood estimator based on the misspecified model is suggested, and its performance is analyzed.

***Index Terms*—** Localization, estimation, TOA, measurement bias, maximum likelihood, misspecified model.

1. INTRODUCTION

A distributed system enables the localization of a target of interest with spatially complementary observations. Such system can be either passive or active; a typical realization for the former is a low-cost sensor network, while an example for the latter can be a multi-static radar or sonar system. Current available measurements include received signal strength [5], time-of-arrival (TOA) or range [3, 8, 9], time-difference-of-arrivals (TDOA) [4], angle-of-arrival (AOA) [6], and so forth. They can be individually or cooperatively utilized in target localization. In this paper, we are interested in localizing a single *moving* target with distributed TOAs.

TOA based localization has flourished with the development of wireless techniques, and it requires the knowledge of propagation time for spatially distinct transmitter-target-receiver paths [1]. Mathematically, localization in two dimensions involves inference of the intersection of multiple fuzzy TOA circles and ellipses, while that in three dimensions deals with multiple measurement balls and ellipsoids [7]. Many interesting works exist in the literature. They concentrate either on efficient localization algorithm design [1] or on dealing with challenging observation circumstances such as TOA origin uncertainty [7].

Many existing works assume that each measurement is from a stationary target and is *unbiased*. However, an extracted TOA can be biased by the Doppler of a target. Physically, many radar and sonar systems adopt Doppler tolerant waveforms (DTWs) to ascertain propagation delays from a moving target [3, 8, 9]. Those waveforms assure certain degree of robustness during acquisition of a moving target; however, the extracted TOA will be biased by the

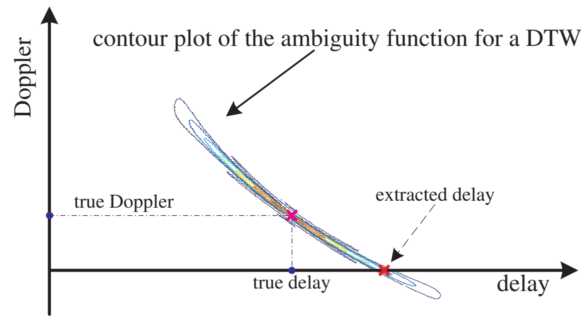


Fig. 1. An intuitive illustration of delay bias with a DTW. If the Doppler of a target is nonzero, the extracted delay from the zero Doppler axis (i.e., the maximum amplitude for a noise-free matched filter matched to zero-Doppler) will be biased by the Doppler. The delay bias is a function of Doppler and the ambiguity function ridge, which depends on system parameters.

Doppler as illustrated in Fig. 1. In the paper, we consider the localization of a moving target with Doppler biased TOAs, and our contributions are two-fold: *1)* we show that the Fisher information matrix of this problem is ill-conditioned, and thus the Cramér-Rao lower bound (CRLB) and maximum likelihood (ML) method are both invalid [10]. *2)* A Quasi-ML estimator with a misspecified model is suggested, and its asymptotic properties are discussed.

The rest of this paper is organized as follows: Section 2 introduces the biased TOA model, and Section 3 derives its localization CRLB. An approximate estimator based on a misspecified model is given in Section 4. Numerical results are shown in Section 5 and conclusions are drawn after that.

2. PROBLEM STATEMENT

We are interested in active target localization, where an active transmitter and several passive sensors collaborate to monitor a region of interest (ROI). Suppose that a target invades the ROI. Passive sensors will claim a local detection and extract the arrival of the reflection with a matched filter. Physically, the true delay between the transmitter and the $i$the passive sensor is

$$\tau_i(\boldsymbol{\theta}) = \frac{1}{c}\big(||\boldsymbol{\theta} - \boldsymbol{s}_i|| + ||\boldsymbol{\theta} - \boldsymbol{u}||\big) \tag{1}$$

for a distributed configuration, where

- $\boldsymbol{\theta}$ denotes the unknown target location vector;
- $\boldsymbol{s}_i$ represents the location vector of the ith receiver;

X. Song, P. Willett and S. Zhou were supported by the U.S. Office of Naval Research under Grants N00014-09-10613 and N00014-10-10412.

G. Wang was supported by the National Natural Science Foundation of China under Grant 61201099 and the Program for Ningbo Municipal Technology Innovation Team under Grant 2011B81002.

- $\boldsymbol{u}$ stands for the location vector of the transmitter; and
- c denotes the signal propagation speed.

The dimension of location vectors can be either two or three, and we will not specify it here for generality.

In active exploration, many real systems including radar and sonar employ DTWs, such as linear frequency modulation (LFM) and hyperbolic frequency modulation (HFM), to assure detection robustness against a moving target. For those systems, however, the extracted delay will be affected by the target's Doppler as well as the usual noise. Specifically, the measured delay for a certain bistatic transmitter-receiver pair is written as [3, 8, 9]

$$\tilde{\tau}_i = \tau_i(\boldsymbol{\theta}) + b_i(\boldsymbol{\theta}, \boldsymbol{v}) + w_i, \tag{2}$$

where

- w_i represents the measurement noise, and it is modeled as independently and identically distributed (i.i.d.) zero-mean white Gaussian noise with variance σ^2;
- $b_i(\boldsymbol{\theta}, \boldsymbol{v})$ refers to TOA *bias* due to the fact that

$$\mathbb{E}\{\tilde{\tau}_i - \tau_i(\boldsymbol{\theta})\} = b_i(\boldsymbol{\theta}, \boldsymbol{v}) \not\equiv 0, \tag{3}$$

where $\boldsymbol{v}$ represents the velocity vector of the target.

Mathematically, the measurement bias is a function of target location and target velocity, and it is modeled as [3, 8, 9]

$$b_i(\boldsymbol{\theta}, \boldsymbol{v}) = \begin{cases} \frac{\dot{r}_i(\boldsymbol{\theta}, \boldsymbol{v})\lambda}{c - 2\dot{r}_i(\boldsymbol{\theta}, \boldsymbol{v})}, & \text{HFM} \\ \frac{\dot{r}_i(\boldsymbol{\theta}, \boldsymbol{v})\lambda}{c}, & \text{LFM}, \end{cases} \tag{4}$$

where

- $\lambda = \frac{T(f_1+f_2)}{2(f_2-f_1)} \neq 0$ is a system related constant, and it depends on waveform pulse width T, waveform start frequency f_1, and the end frequency f_2.
- $\dot{r}_i(\boldsymbol{\theta}, \boldsymbol{v})$ denotes the range-rate, and it is calculated as

$$\begin{aligned} \dot{r}_i(\boldsymbol{\theta}, \boldsymbol{v}) &= \frac{\partial}{\partial t}\Big[||\boldsymbol{\theta} + t\boldsymbol{v} - \boldsymbol{s}_i|| + ||\boldsymbol{\theta} + t\boldsymbol{v} - \boldsymbol{u}||\Big]\Big|_{t=0} \\ &= \frac{(\boldsymbol{\theta} - \boldsymbol{s}_i)^T\boldsymbol{v}}{||\boldsymbol{\theta} - \boldsymbol{s}_i||} + \frac{(\boldsymbol{\theta} - \boldsymbol{u})^T\boldsymbol{v}}{||\boldsymbol{\theta} - \boldsymbol{u}||}. \end{aligned} \tag{5}$$

Let $\boldsymbol{v} = \boldsymbol{0}$, and then we have $\dot{r}_i(\boldsymbol{\theta}, \boldsymbol{v}) = b_i(\boldsymbol{\theta}, \boldsymbol{v}) = \boldsymbol{0}$. Thus, the stationary localization model in [1] can be treated as a special case of (2). Note that [4] also emphasizes *bias* in target localization. However, its bias refers to algorithm rather than measurement, and the mathematical model in [4] is completely different from (2).

3. THE CRLB

3.1. Derivation

Suppose that the transmitter and all receivers are properly synchronized and that their coordinates are known. Therefore, distributed TOAs can be extracted to infer the target location. Let the likelihood function of $\tilde{\tau}_i$ be

$$f(\tilde{\tau}_i|\boldsymbol{\Theta}) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(- \frac{\big(\tilde{\tau}_i - \tau_i(\boldsymbol{\theta}) - b_i(\boldsymbol{\theta}, \boldsymbol{v})\big)^2}{2\sigma^2}\right), \tag{6}$$

where $\boldsymbol{\Theta} = [\boldsymbol{\theta}^T, \boldsymbol{v}^T]^T$ collects the unknown target location and velocity vectors. Let $Z = \{\tilde{\tau}_1, \cdots, \tilde{\tau}_N\}$ represent the measurement set, where N denotes the number of receivers, and thus we have

$$f(Z|\boldsymbol{\Theta}) = \prod_{i=1}^{N} f(\tilde{\tau}_i|\boldsymbol{\Theta}) \tag{7}$$

as the w_i's are independent. Suppose that $\hat{\boldsymbol{\Theta}}$ is an unbiased estimate of $\boldsymbol{\Theta}$. The estimate's covariance matrix will be bounded by [11]

$$\mathbb{E}\big\{(\hat{\boldsymbol{\Theta}} - \boldsymbol{\Theta})(\hat{\boldsymbol{\Theta}} - \boldsymbol{\Theta})^T\big\} \succeq \boldsymbol{J}^{-1}, \tag{8}$$

where $\boldsymbol{J}$ is the Fisher information matrix (FIM) defined as

$$\begin{aligned} \boldsymbol{J} &= -\mathbb{E}\left\{\frac{\partial}{\partial\boldsymbol{\Theta}}\left[\frac{\partial}{\partial\boldsymbol{\Theta}} \ln f(Z|\boldsymbol{\Theta})\right]^T\right\} \\ &= -\sum_{i=1}^{N}\mathbb{E}\left\{\frac{\partial}{\partial\boldsymbol{\Theta}}\left[\frac{\partial^T}{\partial\boldsymbol{\Theta}} \ln f(\tilde{\tau}_i|\boldsymbol{\Theta})\right]^T\right\}. \end{aligned} \tag{9}$$

With the fact that

$$\begin{aligned} \frac{\partial \ln f(\tilde{\tau}_i|\boldsymbol{\Theta})}{\partial\boldsymbol{\Theta}} &= -\frac{\partial}{\partial\boldsymbol{\Theta}}\left[\frac{\big(\tilde{\tau}_i - \tau_i(\boldsymbol{\theta}) - b_i(\boldsymbol{\theta}, \boldsymbol{v})\big)^2}{2\sigma^2}\right] \\ &= \frac{\tilde{\tau}_i - \tau_i(\boldsymbol{\theta}) - b_i(\boldsymbol{\theta}, \boldsymbol{v})}{\sigma^2}\left[\frac{\partial\tau_i(\boldsymbol{\theta})}{\partial\boldsymbol{\Theta}} + \frac{\partial b_i(\boldsymbol{\theta}, \boldsymbol{v})}{\partial\boldsymbol{\Theta}}\right], \end{aligned} \tag{10}$$

one can obtain

$$\begin{aligned} &\frac{\partial}{\partial\boldsymbol{\Theta}}\left[\frac{\partial \ln f(\tilde{\tau}_i|\boldsymbol{\Theta})}{\partial\boldsymbol{\Theta}}\right]^T \\ &= -\frac{1}{\sigma^2}\left[\frac{\partial\tau_i(\boldsymbol{\theta})}{\partial\boldsymbol{\Theta}} + \frac{\partial b_i(\boldsymbol{\theta}, \boldsymbol{v})}{\partial\boldsymbol{\Theta}}\right]\left[\frac{\partial\tau_i(\boldsymbol{\theta})}{\partial\boldsymbol{\Theta}} + \frac{\partial b_i(\boldsymbol{\theta}, \boldsymbol{v})}{\partial\boldsymbol{\Theta}}\right]^T \\ &\quad + \frac{\tilde{\tau}_i - \tau_i(\boldsymbol{\theta}) - b_i(\boldsymbol{\theta}, \boldsymbol{v})}{\sigma^2}\frac{\partial}{\partial\boldsymbol{\Theta}}\left[\frac{\partial\tau_i(\boldsymbol{\theta})}{\partial\boldsymbol{\Theta}} + \frac{\partial b_i(\boldsymbol{\theta}, \boldsymbol{v})}{\partial\boldsymbol{\Theta}}\right]^T. \end{aligned} \tag{11}$$

Since $\mathbb{E}\{\tilde{\tau}_i - \tau_i(\boldsymbol{\theta}) - b_i(\boldsymbol{\theta}, \boldsymbol{v})\} = 0$, $\boldsymbol{J}$ is recast as

$$\boldsymbol{J} = \frac{1}{\sigma^2}\sum_{i=1}^{N}\underbrace{\left[\frac{\partial\tau_i(\boldsymbol{\theta})}{\partial\boldsymbol{\Theta}} + \frac{\partial b_i(\boldsymbol{\theta}, \boldsymbol{v})}{\partial\boldsymbol{\Theta}}\right]}_{\triangleq\boldsymbol{e}_i}\left[\frac{\partial\tau_i(\boldsymbol{\theta})}{\partial\boldsymbol{\Theta}} + \frac{\partial b_i(\boldsymbol{\theta}, \boldsymbol{v})}{\partial\boldsymbol{\Theta}}\right]^T. \tag{12}$$

Obviously, $\boldsymbol{J}$ is spanned by N column vectors. As the true delay $\tau_i(\boldsymbol{\theta})$ does not depend on $\boldsymbol{v}$, the first term of $\boldsymbol{e}_i$ is specified as

$$\frac{\partial\tau_i(\boldsymbol{\theta})}{\partial\boldsymbol{\Theta}} = \begin{bmatrix} \frac{\partial\tau_i(\boldsymbol{\theta})}{\partial\boldsymbol{\theta}} \\ \frac{\partial\tau_i(\boldsymbol{\theta})}{\partial\boldsymbol{v}} \end{bmatrix} = \begin{bmatrix} \frac{\boldsymbol{h}_i}{c} \\ \boldsymbol{0} \end{bmatrix}, \tag{13}$$

where $\boldsymbol{h}_i \triangleq \frac{\boldsymbol{\theta} - \boldsymbol{s}_i}{||\boldsymbol{\theta} - \boldsymbol{s}_i||} + \frac{\boldsymbol{\theta} - \boldsymbol{u}}{||\boldsymbol{\theta} - \boldsymbol{u}||}$ for notational simplicity. Recalling (4), the second term can be expressed as

$$\frac{\partial b_i(\boldsymbol{\theta}, \boldsymbol{v})}{\partial\boldsymbol{\Theta}} = \begin{cases} \frac{\lambda c}{(c - 2\dot{r}_i(\boldsymbol{\theta}, \boldsymbol{v}))^2} \cdot \frac{\partial\dot{r}_i(\boldsymbol{\theta}, \boldsymbol{v})}{\partial\boldsymbol{\Theta}} & \text{HFM} \\ \frac{\lambda}{c} \cdot \frac{\partial\dot{r}_i(\boldsymbol{\theta}, \boldsymbol{v})}{\partial\boldsymbol{\Theta}} & \text{LFM}. \end{cases} \tag{14}$$

As $\dot{r}_i(\boldsymbol{\theta}, \boldsymbol{v})$ is a function of $\boldsymbol{\theta}$ and $\boldsymbol{v}$, $\frac{\partial\dot{r}_i(\boldsymbol{\theta}, \boldsymbol{v})}{\partial\boldsymbol{\Theta}}$ is obtained as

$$\frac{\partial\dot{r}_i(\boldsymbol{\theta}, \boldsymbol{v})}{\partial\boldsymbol{\Theta}} = \begin{bmatrix} \frac{\partial\dot{r}_i(\boldsymbol{\theta}, \boldsymbol{v})}{\partial\boldsymbol{\theta}} \\ \frac{\partial\dot{r}_i(\boldsymbol{\theta}, \boldsymbol{v})}{\partial\boldsymbol{v}} \end{bmatrix}, \tag{15}$$

where

$$\frac{\partial \dot{r}_i(\boldsymbol{\theta},\boldsymbol{v})}{\partial \boldsymbol{\theta}} = \frac{\partial}{\partial \boldsymbol{\theta}}\left[\frac{(\boldsymbol{\theta}-\boldsymbol{s}_i)^T\boldsymbol{v}}{||\boldsymbol{\theta}-\boldsymbol{s}_i||}\right] + \frac{\partial}{\partial \boldsymbol{\theta}}\left[\frac{(\boldsymbol{\theta}-\boldsymbol{u})^T\boldsymbol{v}}{||\boldsymbol{\theta}-\boldsymbol{u}||}\right]$$
$$= \frac{\boldsymbol{v}}{||\boldsymbol{\theta}-\boldsymbol{s}_i||} - \frac{(\boldsymbol{\theta}-\boldsymbol{s}_i)(\boldsymbol{\theta}-\boldsymbol{s}_i)^T\boldsymbol{v}}{||\boldsymbol{\theta}-\boldsymbol{s}_i||^3}$$
$$+ \frac{\boldsymbol{v}}{||\boldsymbol{\theta}-\boldsymbol{u}||} - \frac{(\boldsymbol{\theta}-\boldsymbol{u})(\boldsymbol{\theta}-\boldsymbol{u})^T\boldsymbol{v}}{||\boldsymbol{\theta}-\boldsymbol{u}||^3} \triangleq \boldsymbol{d}_i \qquad (16)$$

and

$$\frac{\partial \dot{r}_i(\boldsymbol{\theta},\boldsymbol{v})}{\partial \boldsymbol{v}} = \frac{\boldsymbol{\theta}-\boldsymbol{s}_i}{||\boldsymbol{\theta}-\boldsymbol{s}_i||} + \frac{\boldsymbol{\theta}-\boldsymbol{u}}{||\boldsymbol{\theta}-\boldsymbol{u}||} = \boldsymbol{h}_i. \qquad (17)$$

3.2. Analysis

The derivation of that CRLB implies that the FIM in (8) is invertible or nonsingular, and the property of $\boldsymbol{J}$ for this problem will be analyzed below. Substituting (13), (16), and (17) into (12), the FIM matrix for the LFM case can be written as

$$\boldsymbol{J} = \frac{1}{c^2\sigma^2}\sum_{i=1}^{N}\begin{bmatrix} \boldsymbol{h}_i + \lambda\boldsymbol{d}_i \\ \lambda\boldsymbol{h}_i \end{bmatrix}\begin{bmatrix} \boldsymbol{h}_i + \lambda\boldsymbol{d}_i \\ \lambda\boldsymbol{h}_i \end{bmatrix}^T. \qquad (18)$$

Clearly, if $(\lambda\boldsymbol{d}_i)$ goes to zero,

$$\lim_{\lambda\boldsymbol{d}_i\to 0}\boldsymbol{J} = \begin{bmatrix} \sum_{i=1}^{N}\boldsymbol{h}_i\boldsymbol{h}_i^T & \lambda\sum_{i=1}^{N}\boldsymbol{h}_i\boldsymbol{h}_i^T \\ \lambda\sum_{i=1}^{N}\boldsymbol{h}_i\boldsymbol{h}_i^T & \lambda^2\sum_{i=1}^{N}\boldsymbol{h}_i\boldsymbol{h}_i^T \end{bmatrix} \qquad (19)$$

becomes singular. In a real situation, λ is less than unity, the target is generally not especially proximate to the sensors, and its velocity is low. These imply that the elements of $\boldsymbol{d}_i$ are much smaller than those in $\boldsymbol{h}_i$, and hence $\boldsymbol{J}$ is ill-conditioned. The argument for the HFM case is similar, as $\dot{r}_i(\boldsymbol{\theta},\boldsymbol{v}) \ll c$, which results in $\frac{\lambda c}{(c-2\dot{r}_i(\boldsymbol{\theta},\boldsymbol{v}))^2} \approx \frac{\lambda}{c}$ [9].

An exact analysis of the condition number (CN) of $\boldsymbol{J}$ is challenging, and we will give some numerical results instead. An underwater sonar system is visualized, and its two-dimensional configuration is shown in Fig. 2 (a). The system parameters are: $c = 1500$ m/s, $T = 100$ ms, $f_1 = 11$ kHz, $f_2 = 9$ kHz. The CNs of FIMs for different target locations and velocities are shown in Figs. 2 (b) and (c), where a specific value at point (x_0, y_0) stands for the corresponding CN if the target is located there. The minimum CNs for (b) and (c) are 1.21×10^4 and 1.05×10^5, respectively. From those figures, we can see that *1)* the CN of the FIMs are very high (even though the sensor geometry is very good here), and information matrices are ill-conditioned. *2)* The CN will increase with the decrease of velocity. These observations indicate that one may not achieve satisfactory estimation of both target-location and -velocity simultaneously.

Here is an intuitive explanation. Based on (4), only the bias part $b_i(\boldsymbol{\theta},\boldsymbol{v})$ contains the velocity information. However, the amount of information is, with reasonable parameter settings, far too wee to guarantee a reliable velocity estimation, and this causes the FIM to be ill-conditioned. In addition, the estimate of velocity with model (4) may also degrade the estimation of target location. In [10], the authors show that there is no unbiased estimator with finite variance if the FIM is singular, and CRLB fails to provide any valuable information. Some approaches have been suggested to modify the CRLB under such circumstances: adding a prior distribution on the parameters or adding constraints to the parameter space. However, those methods may not work here as the target is usually hostile, and extra information is generally unavailable.

In brief, then, localization with Doppler biased TOAs is an ill-conditioned problem.

4. QUASI MAXIMUM LIKELIHOOD ESTIMATION

4.1. Misspecified Estimator

Localization with Doppler biased TOAs is ill-conditioned, and an unbiased estimator with finite variance does not exist. In order to find a *feasible* location estimator, and we are interested in a misspecified estimator in this section. The idea is simple: one may intentionally or unintentionally neglect the TOA bias $b_i(\boldsymbol{\theta},\boldsymbol{v})$ in (4), and reformulate a misspecified likelihood function as

$$f_q(\tilde{\tau}_i|\boldsymbol{\theta}) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{\left(\tilde{\tau}_i - \tau_i(\boldsymbol{\theta})\right)^2}{2\sigma^2}\right), \qquad (20)$$

which yields a misspecified ML estimator[1]

$$\hat{\boldsymbol{\theta}}_q = \arg\max_{\boldsymbol{\theta}} f_q(Z|\boldsymbol{\theta}) = \arg\max_{\boldsymbol{\theta}} \ln f_q(Z|\boldsymbol{\theta})$$
$$= \arg\min_{\boldsymbol{\theta}} \sum_{i=1}^{N}\left[\tilde{\tau}_i - \tau_i(\boldsymbol{\theta})\right]^2. \qquad (21)$$

Statistically, the misspecified ML is termed a Quasi maximum likelihood (QML) [12], and it is similar to the classic bias-free localization formula. As opposed to the classic ML, the QML relies on a misspecified or approximate rather than exact signal model.

4.2. Asymptotical Performance Analysis

The objective function of (21) is non-convex, and an analytical solution of $\hat{\boldsymbol{\theta}}_q$ is unavailable. An accurate performance analysis of a QML is rather challenging. In this part, its asymptotical performance will be examined. Let the Kullback-Leibler divergence between the true probability density function (pdf) and the misspecified pdf

$$\mathcal{D}\left(f(Z|\boldsymbol{\Theta}_t)||f_q(Z|\boldsymbol{\theta})\right) = \int_Z f(Z|\boldsymbol{\Theta}_t)\ln\frac{f(Z|\boldsymbol{\Theta}_t)}{f_q(Z|\boldsymbol{\theta})}dZ \qquad (22)$$

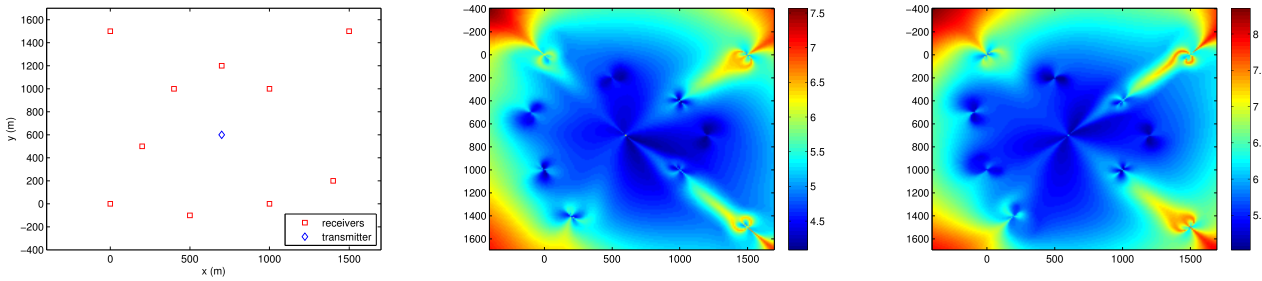
have a unique minimum $\boldsymbol{\theta}^*$, where $\boldsymbol{\Theta}_t = [\boldsymbol{\theta}_t^T, \boldsymbol{v}_t^T]^T$ denotes the parameters of truth, and then we have that

- $\hat{\boldsymbol{\theta}}_q$ exists, and $\hat{\boldsymbol{\theta}}_q$ converges to $\boldsymbol{\theta}^*$ almost surely if either the number of sample Z [12] or the signal-to-noise ratio (SNR) of measurements [2] goes to infinity.

In other words, $\boldsymbol{\theta}^*$ is the asymptotic mean of $\hat{\boldsymbol{\theta}}_q$, and it offers an insight in misspecified estimator design. Since the $\tilde{\tau}_i$'s are independent, $\mathcal{D}\left(f(Z|\boldsymbol{\Theta}_t)||f_q(Z|\boldsymbol{\theta})\right)$ is recast as

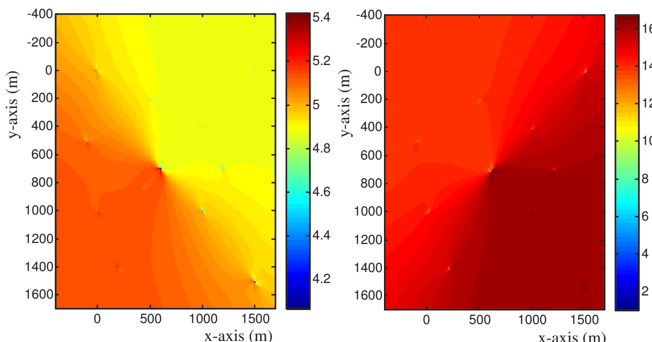
$$\mathcal{D}\left(f(Z|\boldsymbol{\Theta}_t)||f_q(Z|\boldsymbol{\theta})\right) = \sum_{i=1}^{N}\mathcal{D}\left(f(\tilde{\tau}_i|\boldsymbol{\Theta}_t)||f_q(\tilde{\tau}_i|\boldsymbol{\theta})\right) \qquad (23)$$
$$= \sum_{i=1}^{N}\int_{\tilde{\tau}_i} f(\tilde{\tau}_i|\boldsymbol{\Theta}_t)\ln\frac{f(\tilde{\tau}_i|\boldsymbol{\Theta}_t)}{f_q(\tilde{\tau}_i|\boldsymbol{\theta})}d\tilde{\tau}_i.$$

[1] As the estimator here intentionally employs a signal generation model different from what is true, it is often known in the statistics literature as a *misspecified* estimator [12].



(a) system configuration (b) $\log_{10}(\text{CN})$ (c) $\log_{10}(\text{CN})$

Fig. 2. An illustration of sensor configuration and CNs for FIM with different velocity parameters: (a) the sensor configuration, which includes a single transmitter and ten receivers, (b) CNs with $\boldsymbol{v} = [30\cos(\pi/4), 30\sin(\pi/4)]^T$ m/s, and (c) CNs with $\boldsymbol{v} = [10\cos(-\pi/4), 10\sin(-\pi/4)]^T$ m/s.



(a) (b)

Fig. 3. The distance between $\boldsymbol{\theta}^*$ and $\boldsymbol{\theta}_t$, say $||\boldsymbol{\theta}^* - \boldsymbol{\theta}_t||$, for different target parameters: (a) $\boldsymbol{v} = [10\cos(-\pi/4), 10\sin(-\pi/4)]^T$ m/s, and (b) $\boldsymbol{v} = [30\cos(\pi/4), 30\sin(\pi/4)]^T$ m/s.

Recall (6) and (20), and hence we obtain

$$\begin{aligned}&\mathcal{D}\left(f(\tilde{\tau}_i|\boldsymbol{\Theta}_t)||f_q(\tilde{\tau}_i|\boldsymbol{\theta})\right)\\ &= \int_{\tilde{\tau}_i} f(\tilde{\tau}_i|\boldsymbol{\Theta}_t)\frac{[\tilde{\tau}_i - \tau_i(\boldsymbol{\theta})]^2 - [\tilde{\tau}_i - \tau_i(\boldsymbol{\theta}_t) - b_i(\boldsymbol{\theta}_t, \boldsymbol{v}_t)]^2}{2\sigma^2} d\tilde{\tau}_i\\ &= \frac{[\tau_i(\boldsymbol{\theta}) - \tau_i(\boldsymbol{\theta}_t) - b_i(\boldsymbol{\theta}_t, \boldsymbol{v}_t)]^2}{2\sigma^2}. \qquad (24)\end{aligned}$$

Thus, the minimum can be reached by

$$\boldsymbol{\theta}^* = \arg\min_{\boldsymbol{\theta}} \sum_{i=1}^{N} \left[\tau_i(\boldsymbol{\theta}) - \tau_i(\boldsymbol{\theta}_t) - b_i(\boldsymbol{\theta}_t, \boldsymbol{v}_t)\right]^2. \qquad (25)$$

Interestingly, $\boldsymbol{\theta}^*$ does not depend on σ^2. In addition, if $b_i(\boldsymbol{\theta}_t, \boldsymbol{v}_t) = 0$ for all i, one can obtain that $\boldsymbol{\theta}^* = \boldsymbol{\theta}_t$. This is physically reasonable, as the 'misspecified' model becomes exact under this condition, which can happen if the target is stationary: $\boldsymbol{v} = \mathbf{0}$.

5. NUMERICAL RESULTS

The first example shows distances between $\boldsymbol{\theta}^*$ and $\boldsymbol{\theta}_t$ for different target parameters and in two dimensions. The system configuration and parameters are the same as the example in Section 3. The numerical result is in Fig. 3, where a specific value at point (x_0, y_0) stands for the distance if the target is located there. From those figures, we see that *1)* the difference between $\boldsymbol{\theta}^*$ and $\boldsymbol{\theta}_t$ depends on target location; and, *2)* the distance becomes large if target velocity increases. Intuitively, $\boldsymbol{\theta}^*$ is the closest point for the two different model spaces. As the bias $b_i(\boldsymbol{\theta}, \boldsymbol{v})$ is a function of target location, the space for the exact model is target location dependent, and hence so is $\boldsymbol{\theta}^*$. Thus, observation *1)* is physically reasonable. If the system parameters such as frequency span and pulse width are fixed, the bias relies on $\boldsymbol{v}$ as well as target location $\boldsymbol{\theta}$. A larger $\boldsymbol{v}$ will introduce more bias, which means the model divergence becomes larger, and hence there will be an increase on $||\boldsymbol{\theta}^* - \boldsymbol{\theta}_t||$. As a consequence, observation *2)* is intuitively correct.

6. CONCLUSIONS

Many real radar and sonar systems utilize DTWs to extract waveform propagation delays to deliver a Cartesian measurement reflective of the target. A DTW enables an active system to be capable of a certain robustness in detection, however, at the cost of introducing Doppler dependent TOA (or range) bias. The TOA bias is a function of unknown target location and velocity vectors, and thus the parameter space has been enlarged twice with respect to the traditional TOA based localization. In this paper, we show that the FIM of joint location and velocity estimation is unfortunately execrable, and this means that while in principle joint estimation of position and velocity is possible, the numerical issues are at present insurmountable. However, an estimator based on a misspecified model has been suggested, and it provides acceptable location estimates.

7. REFERENCES

[1] Y. T. Chan, H. Y. C. Hang, and P. C. Ching, "Exact and approximate maximum likelihood localization algorithms," *IEEE Trans. Veh. Technol.*, vol. 55, no. 1, pp. 10–16, Jan. 2006.

[2] Q. Ding and S. Kay, "Maximum likelihood estimator under a misspecified model with high signal-to-noise ratio," *IEEE Trans. Signal Process.*, vol. 59, no. 8, pp. 4012–4016, Aug. 2011.

[3] R. J. Fitzgerald, "Effects of range-Doppler coupling on chirp radar tracking accuracy," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 10, no. 4, pp. 528–532, Jul. 1974.

[4] K. C. Ho, "Bias reduction for an explicit solution of source localization using TDOA," *IEEE Trans. Signal Process.*, vol. 60, no. 5, pp. 2101–2114, May 2012.

[5] X. Sheng and Y.-H. Hu, "Maximum likelihood multiple-source localization using acoustic energy measurements with wireless sensor networks," *IEEE Trans. Signal Process.*, vol. 53, no. 1, pp. 44–53, Jan. 2005.

[6] X. Song, P. Willett, and S. Zhou, "Target localization with NLOS circularly reflected AoAs," in *Proc. of Intl. Conf. on Acoustics, Speech and Signal Process.*, Prague, Czech, May 2011.

[7] ——, "On Fisher information reduction for range-only localization with imperfect detection," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 48, no. 4, pp. 3694–3702, Oct. 2012.

[8] ——, "Posterior Cramér-Rao bounds for Doppler biased distributed tracking," *J. Adv. Inf. Fusion*, vol. 7, no. 1, pp. 16–27, Jun. 2012.

[9] ——, "Range bias modeling for hyperbolic frequency modulated waveforms in target tracking," *IEEE J. Ocean. Eng.*, vol. 37, no. 4, pp. 670–679, Oct. 2012.

[10] P. Stoica and T. L. Marzetta, "Parameter estimation problems with singular information matrices," *IEEE Trans. Signal Process.*, vol. 49, no. 1, pp. 87–90, Jan. 2001.

[11] H. Van Trees, *Detection, Estimation, and Modulation Theory*, 1st ed. New York: John Wiley & Sons, Inc., 1968.

[12] H. White, "Maximum likelihood estimation of misspecified models," *Econometrica*, vol. 50, no. 1, pp. 1–25, Jan. 1982.