INTERFERENCE ALIGNMENT WITH DOUBLY LAYERED SIGNALING FOR CONSTANT SISO INTERFERENCE CHANNELS

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ABSTRACT

It has been conjectured by Høst-Madsen and Nosratinia that the K-user single-input single-output (SISO) complex Gaussian interference channels with constant channel coefficients have merely one degree of freedom (DoF) regardless of the number of users, i.e., K. Then, Cadambe and Jafar introduced the idea of interference alignment (IA) being able to achieve $K/2$ DoF in time-varying SISO interference channels. Moreover, their joint work with Wang settled the Høst-Madsen–Nosratinia conjecture in negative by using the idea of asymmetric complex signaling to achieve 1.2 DoF for K-user constant SISO interference channels. In this paper, a linear IA scheme for K-user constant SISO interference channels is proposed which could enable us to achieve $K/4$ DoF for almost all channel coefficients. This means that whenever $K \geq 5$, the proposed scheme could achieve at least 1.25 DoF. The main idea of the proposed method relies on the linear IA using symbol extension by Cadambe-Jafar which is not effective for constant channels. However, we show that along with signal rotation across every two consecutive time slots to artificially build a random time-varying channel out of a constant channel, the proposed method can be directly applied to constant channels to achieve $K/4$ DoF.

Index Terms— Constant SISO interference channels, doubly layered signaling, linear interference alignment.

1. INTRODUCTION

The study of degrees of freedom (DoF) of interference networks was pioneered by Høst-Madsen and Nosratinia who investigated that for the two user Gaussian interference channel, only one DoF can be achieved, even with cooperation between transmit nodes and/or cooperation between receive nodes [1]. They even showed that the DoF of the K-user Gaussian interference channel is less than or equal to $K/2$ and also conjectured that for the fully connected K-user constant single-input single-output (SISO) channel, the interference network has at most one DoF.

Later, Cadambe and Jafar showed that for the K-user SISO interference channels with time-varying or frequency-selective channel coefficients, $K/2$ DoF is achievable [2]. For constant channel coefficient the situation is somewhat different from time-varying channels. Although some examples have been discussed for constant channels that achieve more than one DoF, all these special cases span only a subset of measure zero [3, 4]. In other words, for almost all channel coefficient values, it was not known whether it is possible to achieve more than one DoF. The authors in [2] showed that for the 3-user multi-input multi-output (MIMO) interference channels with $M > 1$ antennas at each node, we can achieve exactly $3M/2$ DoF even with constant channel coefficients. However the situation for single-antenna nodes remained unsolved till Cadambe, Jafar and Wang showed that for the class of linear beamforming and interference alignment (IA) schemes, 1.2 DoF can be achieved on the complex Gaussian 3-user interference channel with constant channel coefficient, for almost all values of channel coefficients [5]. Recently, the idea of real interference alignment has been used in [6] which ensures that $\frac{MN}{M+N}K$ DoF is achievable for the K-user constant MIMO interference channels with M antennas at each receiver and N antennas at each transmitter. This means that for the K-user constant SISO interference channels $K/2$ DoF is achievable.

In this paper, a framework for K-user constant SISO interference channels is presented which could achieve $K/4$ DoF for almost all channel coefficients by using IA. In the proposed framework, we use random rotations at the transmit and receive sides to effectively fluctuate the constant channel. We also identified that it is necessary to transmit two versions of rotation for the same signals to design precoding matrices that are full rank using symbol-extension precoding. Since we are transmitting the same symbols twice, we achieve only $K/4$ DoF. The proposed scheme is applicable to both complex and real channel coefficients and noise components. Second and more importantly, the proposed interference alignment is linear and could achieve at least 1.25 DoF for $K \geq 5$. However, we also note that the proposed method may only enable us to

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achieve up to 0.75 DoF whenever $K \leq 3$.

The paper is organized as follows: In Section 2, we describe the general system model of channels based on symbol extensions. Section 3 reviews the standard IA in time-varying channels. In Section 4, we derive a useful signaling scheme, called doubly layered signaling, which enables us to convert a constant channel to a time-varying one. The system description and the implementation of doubly layered signaling in constant channels are presented in Section 5, and finally Section 6 contains conclusion.

2. SYSTEM MODEL

Consider the K-user SISO interference channel consisting of $2K$ nodes, K of which are denoted as transmitters while the other K are receivers. Each single-antenna transmitter is paired with a single-antenna receiver in a one-to-one mapping. Let define the set of all users by $\mathcal{K} = \{1, \ldots, K\}$. We denote the channel from k-th transmit node to j-th receive node during time slot t by $h_{jk}^{[t]}$. The channel output of receiver j over the t-th time slot is described as follows:

$$y_j^{[t]} = \sum_{k=1}^{K} h_{jk}^{[t]} x_k^{[t]} + z_j^{[t]} \tag{1}$$

where $x_k^{[t]}$ is the input signal of the k-th transmitter at time slot t, and $z_j^{[t]}$ is independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian noise at the j-th receiver at time slot t, which has mean zero and variance σ_z^2, i.e., $z_j^{[t]} \sim \mathcal{CN}\left(0, \sigma_z^2\right)$.

The usefulness of symbol extensions for constant channel coefficients has been investigated in [7] for MIMO X channel. Here the idea of symbol extension is used to develop the proposed method. The supersymbol is defined as the T symbols transmitted over T time slots. We call this the T symbol extension of the channel. For a symbol extension of length T, the effective $T \times T$ channel from k-th transmit node to the j-th receive node can be shown by

$$\mathbf{H}_{jk} = \mathrm{diag}\left(h_{jk}^{[T(t-1)+1]}, \ldots, h_{jk}^{[Tt]}\right) \tag{2}$$

where "diag" represents the diagonal matrix. In the case of constant channels, we have $\mathbf{H}_{jk} = h_{jk} \cdot \mathbf{I}_{T\times T}$.

In the extended channel, the signal vector at the j-th receive node can be expressed as

$$\mathbf{Y}_j^{[t]} = \sum_{k=1}^{K} \mathbf{H}_{jk} \mathbf{X}_k^{[t]} + \mathbf{Z}_j^{[t]} \tag{3}$$

where $\mathbf{X}_k^{[t]}$ is a T-vector representing the T symbol extension of the transmitted symbol $x_k^{[t]}$. i.e.,

$$\mathbf{X}_k^{[t]} \triangleq \left(x_k^{[T(t-1)+1]}, \ldots, x_k^{[Tt]}\right)^{\top} \tag{4}$$

where the superscript $(\cdot)^{\top}$ denotes the transpose of a vector or matrix. Similarly $\mathbf{Y}_j^{[t]}$ and $\mathbf{Z}_j^{[t]}$ represents T symbol extension of $y_j^{[t]}$ and $z_j^{[t]}$, respectively. Further, $\mathbf{X}_k^{[t]}$ can be shown as

$$\mathbf{X}_k^{[t]} = \mathbf{V}_k \mathbf{s}_k^{[t]} \tag{5}$$

where $\mathbf{V}_k \in \mathbb{C}^{T\times d_k}$ is the precoding matrix of user k, d_k is the number of independent streams into which the message of transmitter k is encoded, and $\mathbf{s}_k^{[t]} \in \mathbb{C}^{d_k \times 1}$ is the intended data vector for receiver k, thus $\left(\frac{d_1}{T}, \ldots, \frac{d_K}{T}\right)$ can be defined as the set of achievable DoF tuple, and therefore the total number of DoF of the network is equal to $\frac{d_1 + \cdots + d_K}{T}$.

3. IA FOR TIME-VARYING CHANNELS

In this section, the IA conditions for time-varying channels are presented. In this case, all channel coefficients $h_{jk}^{[t]}$ can be modeled by i.i.d. complex Gaussian random variables with zero mean and unit variance.

Let $N = (K-1)(K-2) - 1$. In [2], it has been shown that $\left(\frac{d_1}{T}, \ldots, \frac{d_k}{T}\right)$ lies in the DoF region of the K-user interference channel for any $n \in \mathbb{N}$ where

$$\begin{array}{l} d_1 = (n+1)^N \\ d_k = n^N, \quad \forall k \in \mathcal{K} \backslash \{1\} \end{array} \tag{6}$$

and $T = (n+1)^N + n^N$ is the symbol extension of the channel. In other words, $\left(d_1 = (n+1)^N, d_2 = n^N, \ldots, d_K = n^N\right)$ lies in the DoF region of the $T = (n+1)^N + n^N$ symbol extension of the original channel. In this case, the precoding matrices can be defined as [2]

$$\mathbf{V}_1 = \left\{ \left(\prod_{\substack{j,k \in \mathcal{K}\backslash\{1\} \\ j\neq k, (j,k)\neq(2,3)}} \mathbf{T}_{j,k}^{m} \right) \cdot \mathbf{1}_T \,\middle|\, \forall m \in [0, 1, \ldots, n] \right\} \tag{7}$$

$$\mathbf{V}_3 = \left\{ \mathbf{H}_{23}^{-1} \mathbf{H}_{21} \left(\prod_{j,k \in \mathcal{K}\backslash\{1\}, j\neq k, (j,k)\neq(2,3)} \mathbf{T}_{j,k}^{m} \right) \cdot \mathbf{1}_T \,\middle|\, \forall m \in [0, 1, \ldots, n-1] \right\} \tag{8}$$

$$\mathbf{V}_k = \mathbf{H}_{1k}^{-1} \mathbf{H}_{13} \mathbf{V}_3 \quad \forall k \in \mathcal{K} \backslash \{1, 3\} \tag{9}$$

where $\{\cdot\}$, in equations (7) and (8), denotes the set of $(n+1)^N$ column vectors for $\mathbf{V}_1$, and n^N column vectors for $\mathbf{V}_3$, $\mathbf{T}_{jk}$ $(j, k \in \mathcal{K}\backslash\{1\}, j \neq k, (j,k) \neq (2,3))$ is equal to

$$\mathbf{T}_{jk} = \mathbf{H}_{j1}^{-1} \mathbf{H}_{jk} \mathbf{H}_{1k}^{-1} \mathbf{H}_{13} \mathbf{H}_{23}^{-1} \mathbf{H}_{21} \tag{10}$$

and $\mathbf{1}_T$ is the all one vector of size T. To further proceed, the following facts are considered:

Fact 1 *If all diagonal elements of even one of $\mathbf{T}_{jk}$ would be the same, i.e., $\mathbf{T}_{jk}$ be a scaled identity matrix, then precoding matrices $\mathbf{V}_k$, $k \in \mathcal{K}$ are rank deficient* [1] *[8].*

Fact 2 *If the diagonal elements of each matrix $\mathbf{T}_{jk}$ are drawn i.i.d. from a continuous distribution, then all $\mathbf{V}_k,\ k \in \mathcal{K}$ are full column rank almost surely [2].*

Remark 1 *For constant interference channels, each time-extended channel matrix $\mathbf{H}_{jk}$ becomes a scaled identity matrix, and this causes each $\mathbf{T}_{jk}$ in* (10) *to be turned into a scaled identity matrix too. Therefore, based on* **Fact 1**, *for constant interference channels, the precoding matrices become rank deficient, and thus the proposed method of Cadambe–Jafar is not effective for these types of channels. However, in this paper and in the following section, an idea of doubly layered signaling is presented which enables us to build a virtual time-varying channel out of a constant channel, and consequently we can use Cadambe–Jafar scheme to approach $K/4$ DoF.*

4. DOUBLY LAYERED SIGNALING FOR CONSTANT CHANNELS

One method to make the constant channel time-varying is to fluctuate the coding at all nodes with an arbitrarily gain. Let $\alpha_k^{[\tau]}$ denote the gain of the k-th transmit node at a specific time slot τ, and $\beta_j^{[\tau]}$ represent the gain of the j-th receive node at the same time slot, with this assumption that all $\alpha_k^{[\tau]}$ and $\beta_j^{[\tau]}$ are drawn i.i.d. from a continuous distribution. Moreover we assume that all nodes have a knowledge about all values of $\alpha_k^{[\tau]}$ and $\beta_j^{[\tau]}$.

We consider the duration of the symbol extension to be twice as that of the time-varying scenario described in Section 3, i.e., $2T$ symbol extension of the original channel. We categorize the time slots as odd time slots (beginning from $2T(t-1)+1$ with steps of length two) and even time slots (beginning from $2T(t-1)+2$ with steps of length two). In this case, we can add the two consecutive time slots (including one odd-indexed time slot with one even-indexed time slot) of the doubly extended channel to from one virtual status of the network. *Note that during the two consecutive time slots of this doubly extended channel, we send the same information*. So we still have T virtual statuses in total. Therefore the artificially built time-varying channel coefficients (related to one virtual status of the network) can be described by

$$\begin{aligned}\widetilde{h}_{jk}^{[\tau]} &= \beta_j^{[2\tau-1]} h_{jk} \alpha_k^{[2\tau-1]} + \beta_j^{[2\tau]} h_{jk} \alpha_k^{[2\tau]} \\ &= h_{jk}\left(\beta_j^{[2\tau-1]}\alpha_k^{[2\tau-1]} + \beta_j^{[2\tau]}\alpha_k^{[2\tau]}\right) \\ &\forall \quad T(t-1)+1 \le \tau \le Tt\end{aligned} \tag{11}$$

where h_{jk} is the constant channel coefficient and consequently $\mathbf{H}_{jk}$ in (2) can now be written as

$$\begin{aligned}\widetilde{\mathbf{H}}_{jk} = h_{jk}\cdot \mathrm{diag}\Big(&\beta_j^{[2T(t-1)+1]}\alpha_k^{[2T(t-1)+1]} + \beta_j^{[2T(t-1)+2]} \\ &\times \alpha_k^{[2T(t-1)+2]}, \ldots, \beta_j^{[2Tt-1]}\alpha_k^{[2Tt-1]} + \beta_j^{[2Tt]}\alpha_k^{[2Tt]}\Big)\end{aligned} \tag{12}$$

[1]Matrix $\mathbf{V} \in \mathbb{C}^{p\times q}$ is *rank deficient*, if $\mathrm{rank}(\mathbf{V}) < \min(p,q)$.

Let define

$$\begin{aligned}\mathbf{A}_k^o &= \mathrm{diag}\left(\alpha_k^{[2T(t-1)+1]}, \ldots, \alpha_k^{[2Tt-1]}\right) \\ \mathbf{A}_k^e &= \mathrm{diag}\left(\alpha_k^{[2T(t-1)+2]}, \ldots, \alpha_k^{[2Tt]}\right) \\ \mathbf{B}_j^o &= \mathrm{diag}\left(\beta_j^{[2T(t-1)+1]}, \ldots, \beta_j^{[2Tt-1]}\right) \\ \mathbf{B}_j^e &= \mathrm{diag}\left(\beta_j^{[2T(t-1)+2]}, \ldots, \beta_j^{[2Tt]}\right)\end{aligned} \tag{13}$$

then (12) can be expressed as

$$\widetilde{\mathbf{H}}_{jk} = h_{jk}\left(\mathbf{B}_j^o\mathbf{A}_k^o + \mathbf{B}_j^e\mathbf{A}_k^e\right) \tag{14}$$

and $\mathbf{T}_{jk}$ in (10) can be written as

$$\begin{aligned}\mathbf{T}_{jk} &= h_{j1}^{-1}h_{jk}h_{1k}^{-1}h_{13}h_{23}^{-1}h_{21}\left(\mathbf{B}_j^o\mathbf{A}_1^o + \mathbf{B}_j^e\mathbf{A}_1^e\right)^{-1} \\ &\times \left(\mathbf{B}_j^o\mathbf{A}_k^o + \mathbf{B}_j^e\mathbf{A}_k^e\right)\left(\mathbf{B}_1^o\mathbf{A}_k^o + \mathbf{B}_1^e\mathbf{A}_k^e\right)^{-1}\left(\mathbf{B}_1^o\mathbf{A}_3^o + \mathbf{B}_1^e\mathbf{A}_3^e\right) \\ &\times \left(\mathbf{B}_2^o\mathbf{A}_3^o + \mathbf{B}_2^e\mathbf{A}_3^e\right)^{-1}\left(\mathbf{B}_2^o\mathbf{A}_1^o + \mathbf{B}_2^e\mathbf{A}_1^e\right)\end{aligned} \tag{15}$$

Now in this case, the only combination of j and k that makes $\mathbf{T}_{jk}$ a scaled identity matrix (or more precisely, an exact identity matrix) is $(j,k) = (2,3)$, but since in the construction of all matrices $\mathbf{V}_k$, it has been assumed that $(j,k) \neq (2,3)$, although $\mathbf{T}_{jk}$ is still a diagonal matrix, it is not a scaled identity matrix with probability one, as its diagonal elements are distinct (due to being drawn i.i.d. from a continuous distribution); thus based on **Fact 2**, all precoding matrices $\mathbf{V}_k$ that are built based on $\mathbf{T}_{jk}$ in (15) are all full column rank.

Remark 2 *Note that if we only use $\mathbf{A}_k \in \mathbb{C}^{T\times T}$ and $\mathbf{B}_j \in \mathbb{C}^{T\times T}$, i.e., we do not have to double the time extension and then add up every two consecutive time slots, the equivalent channel matrix would be $\widetilde{\mathbf{H}}_{jk} = \mathbf{B}_j\mathbf{A}_k$ instead of the one in* (14)*, and in this case, it is easy to verify that $\mathbf{T}_{jk}$ in* (10) *becomes, once again, a scaled identity matrix.*

Remark 3 *In the doubly extended channel, if receive fluctuation gains β_j or transmit fluctuation gains α_k become equal to one, i.e., $\mathbf{A}_k^o = \mathbf{A}_k^e = \mathbf{I}$ or $\mathbf{B}_j^o = \mathbf{B}_j^e = \mathbf{I}$, it is also easy to show that $\mathbf{T}_{jk}$ in* (15) *becomes a scaled identity matrix.*

5. IA WITH DOUBLY LAYERED SIGNALING

In Section 4, it has been shown that with deploying a fluctuation-based coding at each node and using the idea of doubly layered signaling across two consecutive time slots of the doubly extended channel, it is possible to build matrices $\mathbf{T}_{jk}$ such that they are not scaled versions of the identity matrix anymore, and this leads to full column rank precoding matrices $\mathbf{V}_k$. In this section, one possible system description for this doubly extended channel is provided.

Since we assume that we are sending the same information during the two consecutive time slots of the doubly extended channel, for a virtual time slot $T(t-1)+1 \le \tau \le Tt$ related to the two consecutive time slots $2\tau-1$ and 2τ, the received signals of receiver j are equal to

$$\widehat{y}_j^{[2\tau-1]} = \sum_{k=1}^{K} h_{jk}\beta_j^{[2\tau-1]}\alpha_k^{[2\tau-1]}\widehat{x}_k^{[2\tau-1]} + \beta_j^{[2\tau-1]}\widehat{z}_j^{[2\tau-1]} \tag{16}$$

$$\widehat{y}_j^{[2\tau]} = \sum_{k=1}^{K} h_{jk}\beta_j^{[2\tau]}\alpha_k^{[2\tau]}\widehat{x}_k^{[2\tau-1]} + \beta_j^{[2\tau]}\widehat{z}_j^{[2\tau]} \tag{17}$$

Therefore the received signal of the j-th receiver related to the $2T$ symbol extensions can be described as follows:

$$\widehat{\mathbf{Y}}_j^{[t]} = \sum_{k=1}^{K} \widehat{\mathbf{H}}_{jk}^{[t]}\widehat{\mathbf{X}}_k^{[t]} + \left(\mathbf{B}_j^o \otimes \mathbf{K}^o + \mathbf{B}_j^e \otimes \mathbf{K}^e\right)\widehat{\mathbf{Z}}_j^{[t]} \tag{18}$$

such that

$$\widehat{\mathbf{X}}_k^{[t]} = \mathbf{X}_k^{[t]} \otimes \mathbf{1}_2 = \left(\mathbf{V}_k \otimes \mathbf{1}_2\right)\mathbf{s}_k \tag{19}$$

$$\widehat{\mathbf{H}}_{jk}^{[t]} = h_{jk}\left(\left(\mathbf{B}_j^o\mathbf{A}_k^o \otimes \mathbf{K}^o\right) + \left(\mathbf{B}_j^e\mathbf{A}_k^e \otimes \mathbf{K}^e\right)\right) \tag{20}$$

where $\otimes$ represents the Kronecker product, $\mathbf{1}_2$ denotes the 2×1 column vector of all one, $\widehat{\mathbf{Y}}_j^{[t]}$ and $\widehat{\mathbf{Z}}_j^{[t]}$ designate the $2T \times 1$ vectors of the received signal and additive white Gaussian noise, respectively, where each element of $\widehat{\mathbf{Z}}_j^{[t]}$ has mean zero and variance σ_z^2, i.e., $\widehat{\mathbf{Z}}_j^{[t]} \sim \mathcal{CN}\left(\mathbf{0}, \sigma_z^2\mathbf{I}\right)$; also $\mathbf{X}_k^{[t]}$ is defined in (4), and

$$\mathbf{K}^o = \operatorname{diag}(1,0) \qquad \mathbf{K}^e = \operatorname{diag}(0,1) \tag{21}$$

Note that the fluctuation gains α_k and β_j may be drawn from any continuous distribution, but for simplifying the system description, we assume that β_j are i.i.d Gaussian random variables with mean zero and variance $\frac{1}{2}$, i.e., $\beta_j \sim \mathcal{CN}\left(0, \frac{1}{2}\right)$, since this assures that the final noise (resulted from adding up every two consecutive received signals) would be a white noise and its variance remains constant.

With respect to equations (16) and (17), the j-th receiver adds every two consecutive time slots (which contain the same information) as follows:

$$\begin{aligned}\widehat{y}_j^{[2\tau-1]} + \widehat{y}_j^{[2\tau]} &= \sum_{k=1}^{K} h_{jk}\left(\beta_j^{[2\tau-1]}\alpha_k^{[2\tau-1]} + \beta_j^{[2\tau]}\alpha_k^{[2\tau]}\right) \\ &\times \quad \widehat{x}_k^{[2\tau-1]} + \beta_j^{[2\tau-1]}\widehat{z}_j^{[2\tau-1]} + \beta_j^{[2\tau]}\widehat{z}_j^{[2\tau]}\end{aligned} \tag{22}$$

thus receiver j considers the received signal associated with the two consecutive time slots (or one virtual status of the network) as

$$\widetilde{y}_j^{[\tau]} = \widetilde{h}_{jk}^{[\tau]}x_k^{[\tau]} + \widetilde{z}_j^{[\tau]} \tag{23}$$

where $\widetilde{y}_j^{[\tau]} = \widehat{y}_j^{[2\tau-1]} + \widehat{y}_j^{[2\tau]}$, $x_k^{[\tau]} = \widehat{x}_k^{[2\tau-1]}$, and $\widetilde{z}_j^{[\tau]} = \beta_j^{[2\tau-1]}\widehat{z}_j^{[2\tau-1]} + \beta_j^{[2\tau]}\widehat{z}_j^{[2\tau]}$. Note that (23) is similar to (1), but the time-varying channel coefficients $h_{jk}^{[t]}$ have been replaced by the artificially built time-varying channel coefficients $\widetilde{h}_{jk}^{[\tau]}$ in (11). Also note that since the noise components of $\widehat{\mathbf{Z}}_j^{[t]}$ and also β_j are i.i.d Gaussian random variables, the incurred noise $\widetilde{z}_j^{[\tau]}$ is still white; plus, since we assumed that $\beta_j \sim \mathcal{CN}\left(0, \frac{1}{2}\right)$, the variance of $\widetilde{z}_j^{[\tau]}$ in (23) remains unchanged, i.e., $\widetilde{z}_j^{[\tau]} \sim \mathcal{CN}\left(0, \sigma_z^2\right)$. From the matricial notation perspective, we can write (23) as

$$\widetilde{\mathbf{Y}}_j^{[t]} = \sum_{k=1}^{K} \widetilde{\mathbf{H}}_{jk}\mathbf{X}_k^{[t]} + \widetilde{\mathbf{Z}}_j^{[t]} \tag{24}$$

where $\mathbf{X}_k^{[t]}$ and $\widetilde{\mathbf{H}}_{jk}$ have been defined in (4) and (14), respectively, and we have $\widetilde{\mathbf{Z}}_j^{[t]} \sim \mathcal{CN}\left(\mathbf{0}, \sigma_z^2\mathbf{I}\right)$.

Note that (24) resembles (3) which is related to time-varying channels, therefore the precoding matrices $\mathbf{V}_k$ can be built via replacing $\mathbf{H}_{jk}$ by the artificially built time-varying channel matrices $\widetilde{\mathbf{H}}_{jk}$ defined in (14), thus based on discussions in Section 4, all $\mathbf{V}_k$ are full column rank and consequently satisfy the interference alignment conditions. Finally, the actual precoders can be built via $\widehat{\mathbf{V}}_k = \mathbf{V}_k \otimes \mathbf{1}_2$. The following theorem is the result of this paper:

Theorem 1 *Interference alignment with doubly layered signaling could achieve $K/4$ DoF for K-user constant SISO interference channels which implies the achievability of at least 1.25 DoF when $K \geq 5$.*

Proof 1 *It has been shown that $\left((n+1)^N, n^N, \ldots, n^N\right)$ lies in the degrees of freedom region of the $(n+1)^N + n^N$ symbol extension of the time varying channel, i.e., $\left(\frac{(n+1)^N}{(n+1)^N+n^N}, \frac{n^N}{(n+1)^N+n^N}, \ldots, \frac{n^N}{(n+1)^N+n^N}\right)$ is the achievable set of DoF tuple [2]. Since the virtual time varying channel is built out of a constant channel by doubling the symbol extensions and then sending the same information during the two consecutive time slots of the doubly extended channel, thus $\left(\frac{(n+1)^N}{2((n+1)^N+n^N)}, \frac{n^N}{2((n+1)^N+n^N)}, \ldots, \frac{n^N}{2((n+1)^N+n^N)}\right)$ is the achievable set of DoF tuple for the proposed scheme, and the total number of DoF of the network is equal to $D = \dfrac{(n+1)^N + (K-1)\,n^N}{2\left((n+1)^N + n^N\right)}$, therefore $\lim_{n\to\infty} D = K/4$.* ■

Note that for $K = 5$, we have $N = 11$. If we set $n = 82$, then $\mathcal{D} \approx 1.2001$, which is more than 1.2 DoF achieved in [5]. In this case, the actual size of channel matrices is: $2((n+1)^N + n^N) \times 2((n+1)^N + n^N) \approx \left(4.83 \times 10^{21}\right) \times \left(4.83 \times 10^{21}\right)$, which prohibits us from showing numerical results.

6. CONCLUSION

In this paper, we proposed a framework for K-user constant SISO interference channels by exploiting symbol rotations and Cadambe–Jafar symbol-extension interference alignment scheme to settle the Høst-Madsen–Nosratinia conjecture in negative. The main idea of the proposed scheme is to build a virtual time-varying channel out of a constant channel and then to use the Cadambe–Jafar symbol-extension linear interference alignment scheme to achieve the DoF greater than 1. However, as we have to retransmit the signals twice to achieve full-rank precoding matrices, we can only achieve $K/4$ DoF. This means that whenever $K \geq 5$, the proposed method could achieve at least 1.25 DoF.

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