ROBUST DESIGN OF BLOCK DIAGONALIZATION USING PERTURBATION ANALYSIS

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ABSTRACT

Block diagonalization (BD) is a low-complexity linear precoding technique for multi-user MIMO (MU-MIMO) downlink systems, which can provide a performance that is close to the MU-MIMO capacity. However, imperfect channel state information (CSI) will result in a degraded performance of the BD scheme. Thus, studying the performance of BD under imperfect CSI is crucial for a practical system design since the robustness of BD to real-world imperfections should be verified. In this paper we apply a first-order perturbation analysis of the SVD to derive analytic expressions of the signal to interference plus noise ratio (SINR) for each subchannel of each UT using BD in presence of imperfect CSI. To demonstrate the usefulness of these expressions, a robust BD technique via worst SINR maximization is developed. Numerical simulations show the accuracy and the usefulness of the derived analytical results.

Index Terms— Block diagonalization, MU-MIMO, perturbation analysis, convex optimization

1. INTRODUCTION AND THE STATE OF THE ART

Multi-user MIMO (MU-MIMO) downlink precoding is one of the major techniques to meet the demands of higher data rates in future wireless networks. The block diagonalization (BD) scheme is a linear precoding algorithm, which first nulls the multi-user interference (MUI) and then optimizes the performance of each user terminal (UT) separately [1]. BD is well recognized because it not only simplifies the system design but also has a close to optimum performance [2]. However, in practice perfect channel state information (CSI) is impossible to obtain due to channel estimation errors, quantization loss, high mobility, etc. BD with imperfect CSI will result in residual interference (including both MUI and co-channel interference (CCI)) and thus a degraded performance. As a result, the required quality of service of the UTs might not be met. This phenomenon is critical for wireless networks which use the BD technique. Thus, it is essential to study the sensitivity / analytic performance of BD under realistic conditions, i.e., imperfect CSI. Such a performance analysis not only illustrates whether BD is robust to imperfections but may also help us to adapt BD for real-world applications, i.e., to design a robust version of BD. The performance of BD in the presence of imperfect CSI has been studied in [3] and [4]. In [3] an upper bound of the rate loss due to limited feedback is derived, where long-term second-order statistics of the simulated Rayleigh fading channels [5] and random matrix theory [6] are used in the derivation. This upper bound is further applied in [4] to study the performance of BD using delayed CSI. Nevertheless, analytic expressions instead of an upper bound for BD using imperfect CSI have not been derived prior to our work.

In BD the SVD is used to project the channel of interest into the null space of the other UTs' channels and is also used to optimize the performance of each UT after the projection [1]. Thus, to analyze the effects of the perturbed subspaces for BD due to imperfect CSI, the perturbation analysis of the SVD in [7], [8], and [9] can be used. This perturbation analysis of SVD has been widely used to study the analytic performance of the subspace-based parameter estimation schemes, e.g., [7], [10], and [11]. However, it has not been applied in the performance analysis of MIMO techniques other than [12]. In [12] the analytic performance of a point-to-point MIMO system in the presence of imperfect CSI is studied, where the first-order SVD perturbation analysis in [9] is used since in contrast to [8] it also takes into account the contribution of the signal subspace to the perturbed subspaces. Therefore, we also use the perturbation analysis in [9] for our analysis.

In this paper, we derive closed-form SINR expressions of each subchannel of each UT with respect to an instantaneous channel realization when using imperfect CSI in BD based precoding techniques. The derivations in Section 3 are based on the first-order SVD perturbation analysis in [9]. As an example for the usefulness of these newly derived analytic performance of BD, we develop a robust BD technique via worst SINR maximization in Section 4. The simulation results in Section 5 demonstrate the accuracy and usefulness of the derived analytic results.

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2. SYSTEM MODEL

We consider a MU-MIMO downlink system where a multi-antenna BS transmits data to K multi-antenna UTs. The BS has N transmit antennas. For notional simplicity, each UT has M receive antennas and we have $N \geq K \cdot M$. We assume perfect synchronization and frequency flat quasi-static block fading channel. The channel between the BS and the kth UT is denoted as $\boldsymbol{H}_k \in \mathbb{C}^{M \times N}$ ($k \in \{1, \cdots, K\}$). Define the SVD of $\tilde{\boldsymbol{H}}_k = \tilde{\boldsymbol{U}}_k \tilde{\boldsymbol{\Sigma}}_k \begin{bmatrix} \tilde{\boldsymbol{V}}_{\mathrm{s},k} & \tilde{\boldsymbol{V}}_{\mathrm{n},k} \end{bmatrix}^{\mathrm{H}}$, where $\tilde{\boldsymbol{H}}_k \in \mathbb{C}^{(K-1)M \times N}$ contains all channel matrices except for $\boldsymbol{H}_k$ [1]. Let the rank of $\tilde{\boldsymbol{H}}_k$ be $\tilde{r}_k$. Then the last $(N - \tilde{r}_k)$ right singular vectors $\tilde{\boldsymbol{V}}_{\mathrm{n},k}$ form an orthonormal basis for the null space of $\tilde{\boldsymbol{H}}_k$. Define the SVD of $\boldsymbol{H}_k \tilde{\boldsymbol{V}}_{\mathrm{n},k} = \bar{\boldsymbol{U}}_k \bar{\boldsymbol{\Sigma}}_k \begin{bmatrix} \bar{\boldsymbol{V}}_{\mathrm{s},k} & \bar{\boldsymbol{V}}_{\mathrm{n},k} \end{bmatrix}^{\mathrm{H}}$, where $\bar{\boldsymbol{V}}_{\mathrm{s},k} \in \mathbb{C}^{M \times \bar{r}_k}$ contains the dominant $\bar{r}_k$ right singular vectors which correspond to the $\bar{r}_k$ non-zero singular values. Let $\boldsymbol{H}_k$ be the channels known at the BS and $\boldsymbol{H}_k + \Delta \boldsymbol{H}_k$ denote the actual MIMO channels, where $\Delta \boldsymbol{H}_k$ is a zero-mean circularly symmetric complex perturbation with covariance matrix $\boldsymbol{C}_{\Delta \boldsymbol{H}_k} = \sigma_{\mathrm{p}}^2 \boldsymbol{I}_M$. After applying the BD based design at the BS, the transmitted signal vector is expressed as $\boldsymbol{x} = \sum_{k=1}^{K} \tilde{\boldsymbol{V}}_{\mathrm{n},k} \bar{\boldsymbol{V}}_{\mathrm{s},k} \boldsymbol{P}_k^{\frac{1}{2}} \boldsymbol{s}_k$ where the elements of the data vectors $\boldsymbol{s}_k \in \mathbb{C}^{\bar{r}_k}$ are i.i.d. with zero mean and unit variance [1]. The diagonal matrices $\boldsymbol{P}_k = \mathrm{diag}\{\boldsymbol{p}_k\} = \mathrm{diag}\{p_{k,1}, p_{k,2}, \cdots, p_{k,\bar{r}_k}\}$ allocate power onto each subchannels of each UT in the system and they are designed such that the transmit power constraint at the BS $\mathbb{E}\{\|\boldsymbol{x}\|^2\} = P_{\mathrm{T}}$ is fulfilled.

To design the decoding matrix, each UT estimates the effective channel $\hat{\boldsymbol{H}}_k = (\boldsymbol{H}_k + \Delta \boldsymbol{H}_k) \tilde{\boldsymbol{V}}_{\mathrm{n},k}$ using a channel estimation algorithm. If we assume $\hat{\boldsymbol{H}}_k$ is estimated without errors, the decoding matrix at the kth UT is chosen as $\hat{\boldsymbol{U}}_{\mathrm{s},k}^{\mathrm{H}}$, which contains the first $\bar{r}_k$ left singular vectors of $\hat{\boldsymbol{H}}_k$ and is defined in Section 3 (cf. equation (2)). Finally, the received signal at the kth UT is written as:

$$\boldsymbol{y}_k = \hat{\boldsymbol{U}}_{\mathrm{s},k}^{\mathrm{H}} \Bigg[\underbrace{(\boldsymbol{H}_k + \Delta \boldsymbol{H}_k) \tilde{\boldsymbol{V}}_{\mathrm{n},k} \bar{\boldsymbol{V}}_{\mathrm{s},k} \boldsymbol{P}_k^{\frac{1}{2}} \boldsymbol{s}_k}_{\text{perturbed signal}} + \underbrace{(\boldsymbol{H}_k + \Delta \boldsymbol{H}_k) \sum_{\substack{\ell=1 \\ \ell \neq k}}^{K} \tilde{\boldsymbol{V}}_{\mathrm{n},\ell} \bar{\boldsymbol{V}}_{\mathrm{s},\ell} \boldsymbol{P}_\ell^{\frac{1}{2}} \boldsymbol{s}_\ell}_{\text{residual MUI}} + \boldsymbol{n}_k \Bigg] \in \mathbb{C}^{\bar{r}_k} \quad (1)$$

where $\boldsymbol{n}_k$ denotes the zero-mean circularly symmetric Gaussian (ZMCSG) noise and $\mathbb{E}\{\boldsymbol{n}_k \boldsymbol{n}_k^{\mathrm{H}}\} = \sigma_{\mathrm{n}}^2 \boldsymbol{I}_M, \forall k$.

Given the system model with perturbation errors (1), the perturbation analysis in [9] derives explicitly the perturbed terms in the SVD of $\hat{\boldsymbol{H}}_k$, i.e., signal components and interference components in our application, in terms of a first-order expression of the perturbation error $\Delta \boldsymbol{H}_k$.

3. FIRST-ORDER PERTURBATION ANALYSIS

3.1. Derivation of the First Order Expressions

Following the perturbation analysis in [9], we compute the SVD of $\hat{\boldsymbol{H}}_k$ as

$$\hat{\boldsymbol{H}}_k = \hat{\boldsymbol{U}}_{\mathrm{s},k} \hat{\boldsymbol{\Sigma}}_{\mathrm{s},k} \hat{\boldsymbol{V}}_{\mathrm{s},k}^{\mathrm{H}} = (\bar{\boldsymbol{U}}_{\mathrm{s},k} + \Delta \bar{\boldsymbol{U}}_{\mathrm{s},k}) \cdot (\bar{\boldsymbol{\Sigma}}_{\mathrm{s},k} + \Delta \bar{\boldsymbol{\Sigma}}_{\mathrm{s},k}) (\bar{\boldsymbol{V}}_{\mathrm{s},k} + \Delta \bar{\boldsymbol{V}}_{\mathrm{s},k})^{\mathrm{H}} \quad (2)$$

Inserting (2) into the perturbed signal part of (1) and dropping higher-order terms (i.e., 2nd-order and higher), we obtain

$$\boldsymbol{y}_{\mathrm{S+CCI},k} \approx (\bar{\boldsymbol{\Sigma}}_{\mathrm{s},k} + \Delta \bar{\boldsymbol{\Sigma}}_{\mathrm{s},k} + \bar{\boldsymbol{\Sigma}}_{\mathrm{s},k} \Delta \bar{\boldsymbol{V}}_{\mathrm{s},k}^{\mathrm{H}} \bar{\boldsymbol{V}}_{\mathrm{s},k}) \boldsymbol{P}_k^{\frac{1}{2}} \boldsymbol{s}_k \quad (3)$$

where $\bar{\boldsymbol{\Sigma}}_{\mathrm{s},k} = \mathrm{diag}\{\bar{\sigma}_{k,1}, \bar{\sigma}_{k,2}, \cdots, \bar{\sigma}_{k,\bar{r}_k}\}$. If the perturbation $\Delta \boldsymbol{H}_k$ is assumed to be small enough, then we can apply a similar derivation as in [9]. Afterwards, the matrices $\Delta \bar{\boldsymbol{\Sigma}}_{\mathrm{s},k}$ and $\Delta \bar{\boldsymbol{V}}_{\mathrm{s},k}$ are obtained in terms of the perturbation $\Delta \boldsymbol{H}_k$ as

$$\Delta \bar{\boldsymbol{\Sigma}}_{\mathrm{s},k} = \mathrm{diag}\{\Delta \bar{\sigma}_{k,1}, \Delta \bar{\sigma}_{k,2}, \cdots, \Delta \bar{\sigma}_{k,\bar{r}_k}\} \quad (4)$$

where

$$\Delta \bar{\sigma}_{k,i} \approx \frac{1}{2} (\bar{\boldsymbol{u}}_{k,i}^{\mathrm{H}} \Delta \boldsymbol{H}_k \tilde{\boldsymbol{V}}_{\mathrm{n},k} \bar{\boldsymbol{v}}_{k,i} + \bar{\boldsymbol{v}}_{k,i}^{\mathrm{H}} \tilde{\boldsymbol{V}}_{\mathrm{n},k}^{\mathrm{H}} \Delta \boldsymbol{H}_k^{\mathrm{H}} \bar{\boldsymbol{u}}_{k,i}),$$

$$\Delta \bar{\boldsymbol{V}}_{\mathrm{s},k} \approx \bar{\boldsymbol{V}}_{\mathrm{s},k} \bar{\boldsymbol{Q}}_k + \bar{\boldsymbol{V}}_{\mathrm{n},k} \bar{\boldsymbol{V}}_{\mathrm{n},k}^{\mathrm{H}} \tilde{\boldsymbol{V}}_{\mathrm{n},k}^{\mathrm{H}} \Delta \boldsymbol{H}_k^{\mathrm{H}} \bar{\boldsymbol{U}}_{\mathrm{s},k} \bar{\boldsymbol{\Sigma}}_{\mathrm{s},k}^{-1}, \quad (5)$$

and

$$\bar{\boldsymbol{Q}}_k = \bar{\boldsymbol{D}}_k \odot (\bar{\boldsymbol{V}}_{\mathrm{s},k}^{\mathrm{H}} \tilde{\boldsymbol{V}}_{\mathrm{n},k}^{\mathrm{H}} \Delta \boldsymbol{H}_k^{\mathrm{H}} \bar{\boldsymbol{U}}_{\mathrm{s},k} \bar{\boldsymbol{\Sigma}}_{\mathrm{s},k} + \bar{\boldsymbol{\Sigma}}_{\mathrm{s},k} \bar{\boldsymbol{U}}_{\mathrm{s},k}^{\mathrm{H}} \Delta \boldsymbol{H}_k \tilde{\boldsymbol{V}}_{\mathrm{n},k} \bar{\boldsymbol{V}}_{\mathrm{s},k})$$

with

$$\bar{\boldsymbol{D}}_{k,i,j} = \begin{cases} 1/(\bar{\sigma}_{k,j}^2 - \bar{\sigma}_{k,i}^2) & i \neq j \\ 0 & i = j \end{cases}$$

and $\odot$ denotes the Hadamard (element-wise) product. Inserting (5) into (3) and using the fact that $\bar{\boldsymbol{V}}_{\mathrm{n},k}^{\mathrm{H}} \bar{\boldsymbol{V}}_{\mathrm{s},k} = \boldsymbol{0}$, equation (3) is further simplified as

$$\boldsymbol{y}_{\mathrm{S+CCI},k} \approx (\bar{\boldsymbol{\Sigma}}_{\mathrm{s},k} + \Delta \bar{\boldsymbol{\Sigma}}_{\mathrm{s},k} + \bar{\boldsymbol{\Sigma}}_{\mathrm{s},k} \bar{\boldsymbol{Q}}_{,k}^{\mathrm{H}}) \boldsymbol{P}_k^{\frac{1}{2}} \boldsymbol{s}_k \quad (6)$$

Similarly, the term of the residual MUI in (1) is expanded as

$$\begin{aligned} \boldsymbol{y}_{\mathrm{MUI},k} &\approx \hat{\boldsymbol{U}}_{\mathrm{s},k}^{\mathrm{H}} (\boldsymbol{H}_k + \Delta \boldsymbol{H}_k) \sum_{\substack{\ell=1 \\ \ell \neq k}}^{K} \tilde{\boldsymbol{V}}_{\mathrm{n},\ell} \bar{\boldsymbol{V}}_{\mathrm{s},\ell} \boldsymbol{P}_\ell^{\frac{1}{2}} \boldsymbol{s}_\ell \\ &= (\bar{\boldsymbol{U}}_{\mathrm{s},k}^{\mathrm{H}} + \Delta \bar{\boldsymbol{U}}_{\mathrm{s},k}^{\mathrm{H}}) \Delta \boldsymbol{H}_k \sum_{\substack{\ell=1 \\ \ell \neq k}}^{K} \tilde{\boldsymbol{V}}_{\mathrm{n},\ell} \bar{\boldsymbol{V}}_{\mathrm{s},\ell} \boldsymbol{P}_\ell^{\frac{1}{2}} \boldsymbol{s}_\ell \\ &\approx \bar{\boldsymbol{U}}_{\mathrm{s},k}^{\mathrm{H}} \Delta \boldsymbol{H}_k \sum_{\substack{\ell=1 \\ \ell \neq k}}^{K} \tilde{\boldsymbol{V}}_{\mathrm{n},\ell} \bar{\boldsymbol{V}}_{\mathrm{s},\ell} \boldsymbol{P}_\ell^{\frac{1}{2}} \boldsymbol{s}_\ell \end{aligned} \quad (7)$$

where the fact $\boldsymbol{H}_k \tilde{\boldsymbol{V}}_{\mathrm{n},\ell} = \boldsymbol{0}, \forall \ell$ is used in the derivation.

3.2. Derivation of the Perturbed SINR

Given the first-order expression of (6) and (7), we derive the effective SINR of each stream at each UT. Take the ith stream of the kth UT as an example, we start with (6) which contains the desired signal as well as the CCI caused by the perturbation.

In equation (6) only the first and second term contribute to the signal power since the third term has only zeros on the main diagonal. The signal power is derived as:

$$\begin{aligned}\mathbb{E}\{|y_{\mathrm{S},k,i}|^2\} &= \mathbb{E}\{|\bar{\Sigma}_{k,i,i}\sqrt{p_{k,i}}s_{k,i} + \Delta\bar{\Sigma}_{k,i,i}\sqrt{p_{k,i}}s_{k,i}|^2\} \\ &= (\bar{\sigma}_{k,i}^2 + \mathbb{E}\{|\Delta\bar{\sigma}_{k,i}|^2\})p_{k,i} \\ &= (\bar{\sigma}_{k,i}^2 + \frac{1}{4}\mathbb{E}\{2|\bar{\boldsymbol{u}}_{k,i}^{\mathrm{H}}\Delta\boldsymbol{H}_k\tilde{\boldsymbol{V}}_{\mathrm{n},k}\bar{\boldsymbol{v}}_{k,i}|^2\})p_{k,i} \\ &= (\bar{\sigma}_{k,i}^2 + \frac{1}{2}((\tilde{\boldsymbol{V}}_{\mathrm{n},k}\bar{\boldsymbol{v}}_{k,i})^{\mathrm{T}} \otimes \bar{\boldsymbol{u}}_{k,i}^{\mathrm{H}}) \\ &\quad \cdot \boldsymbol{C}_{\Delta\boldsymbol{H}_k}((\tilde{\boldsymbol{V}}_{\mathrm{n},k}\bar{\boldsymbol{v}}_{k,i})^* \otimes \bar{\boldsymbol{u}}_{k,i}))p_{k,i} \\ &= (\bar{\sigma}_{k,i}^2 + \frac{1}{2}\sigma_{\mathrm{p}}^2)p_{k,i} \end{aligned} \tag{8}$$

where the properties that $\mathrm{vec}\{\boldsymbol{AXB}\} = (\boldsymbol{B}^{\mathrm{T}} \otimes \boldsymbol{A})\mathrm{vec}\{\boldsymbol{X}\}$ and $\mathbb{E}\{\mathrm{vec}\{\Delta\boldsymbol{H}_k\}\mathrm{vec}\{\Delta\boldsymbol{H}_k\}^{\mathrm{H}}\} = \boldsymbol{C}_{\Delta\boldsymbol{H}_k}$ are used.

To derive the CCI power, we define $\boldsymbol{e}_i$ as the ith column of an identity matrix and $\boldsymbol{a}_{k,i}^{\mathrm{H}} = \boldsymbol{e}_i^{\mathrm{H}}\bar{\boldsymbol{Q}}_k^{\mathrm{H}}$. Then we find

$$\begin{aligned}\mathbb{E}\{|y_{\mathrm{CCI},k,i}|^2\} &= \mathbb{E}\{|(\bar{\boldsymbol{\Sigma}}_{\mathrm{s},k}\bar{\boldsymbol{Q}}_k^{\mathrm{H}}\boldsymbol{P}_k^{\frac{1}{2}}\boldsymbol{s}_k)_i|^2\} \\ &= \bar{\sigma}_{k,i}^2\mathbb{E}\{|\boldsymbol{e}_i^{\mathrm{H}}\bar{\boldsymbol{Q}}_k^{\mathrm{H}}\boldsymbol{P}_k^{\frac{1}{2}}\boldsymbol{s}_k|^2\} \\ &= \bar{\sigma}_{k,i}^2\mathbb{E}\{|\boldsymbol{a}_{k,i}^{\mathrm{H}}\boldsymbol{P}_k^{\frac{1}{2}}\boldsymbol{s}_k|^2\} \\ &= \bar{\sigma}_{k,i}^2\mathbb{E}\{|\mathrm{vec}\{\boldsymbol{a}_{k,i}^{\mathrm{H}}\}^{\mathrm{T}}\mathrm{vec}\{\boldsymbol{P}_k^{\frac{1}{2}}\boldsymbol{s}_k\}|^2\} \\ &= \bar{\sigma}_{k,i}^2\mathbb{E}\{\mathrm{Tr}\{\mathrm{vec}\{\boldsymbol{a}_{k,i}^{\mathrm{H}}\}^* \\ &\quad \cdot \mathrm{vec}\{\boldsymbol{a}_{k,i}^{\mathrm{H}}\}^{\mathrm{T}}\}\}\boldsymbol{P}_k \end{aligned} \tag{9}$$

where the property $\mathrm{Tr}\{\boldsymbol{AB}\} = \mathrm{Tr}\{\boldsymbol{BA}\}$ is applied. Note that the expectation operates on both $\boldsymbol{s}_k$ and $\Delta\boldsymbol{H}_k$. Now we compute $\mathbb{E}\{\mathrm{Tr}\{\mathrm{vec}\{\boldsymbol{a}_{k,i}^{\mathrm{H}}\}^*\mathrm{vec}\{\boldsymbol{a}_{k,i}^{\mathrm{H}}\}^{\mathrm{T}}\}\}$.

Noticing that $\boldsymbol{e}_i(\boldsymbol{A} \odot \boldsymbol{B}) = (\boldsymbol{e}_i\boldsymbol{A}) \odot (\boldsymbol{e}_i\boldsymbol{B})$, we further expand $\boldsymbol{a}_{k,i}^{\mathrm{H}}$ as

$$\begin{aligned}\boldsymbol{a}_{k,i}^{\mathrm{H}} = (\bar{\boldsymbol{d}}_{k,i}^{\mathrm{H}} \odot (&\bar{\boldsymbol{v}}_{\mathrm{s},k,i}^{\mathrm{H}}\tilde{\boldsymbol{V}}_{\mathrm{n},k}^{\mathrm{H}}\Delta\boldsymbol{H}_k^{\mathrm{H}}\bar{\boldsymbol{U}}_{\mathrm{s},k}\bar{\boldsymbol{\Sigma}}_{\mathrm{s},k} \\ &+ \bar{\boldsymbol{\sigma}}_{\mathrm{s},k,i}^{\mathrm{H}}\bar{\boldsymbol{U}}_{\mathrm{s},k}^{\mathrm{H}}\Delta\boldsymbol{H}_k\tilde{\boldsymbol{V}}_{\mathrm{n},k}\bar{\boldsymbol{V}}_{\mathrm{s},k})) \end{aligned} \tag{10}$$

where $\bar{\boldsymbol{d}}_{k,i}$, $\bar{\boldsymbol{v}}_{\mathrm{s},k,i}$, and $\bar{\boldsymbol{\sigma}}_{\mathrm{s},k,i}$ are the ith columns of $\bar{\boldsymbol{D}}_k$, $\bar{\boldsymbol{V}}_{\mathrm{s},k}$, and $\bar{\boldsymbol{\Sigma}}_{\mathrm{s},k}$, respectively. Utilizing $\mathrm{vec}\{\boldsymbol{A} \odot \boldsymbol{B}\} = \mathrm{diag}\{\mathrm{vec}\{\boldsymbol{A}\}\}\mathrm{vec}\{\boldsymbol{B}\}$, we get

$$\begin{aligned}\mathrm{vec}\{\boldsymbol{a}_{k,i}^{\mathrm{H}}\} = \mathrm{diag}\{\mathrm{vec}\{\bar{\boldsymbol{d}}_{k,i}^{\mathrm{H}}\}\}\mathrm{vec}\{&\bar{\boldsymbol{v}}_{\mathrm{s},k,i}^{\mathrm{H}}\tilde{\boldsymbol{V}}_{\mathrm{n},k}^{\mathrm{H}}\Delta\boldsymbol{H}_k^{\mathrm{H}} \\ &\cdot \bar{\boldsymbol{U}}_{\mathrm{s},k}\bar{\boldsymbol{\Sigma}}_{\mathrm{s},k} + \bar{\boldsymbol{\sigma}}_{\mathrm{s},k,i}^{\mathrm{H}}\bar{\boldsymbol{U}}_{\mathrm{s},k}^{\mathrm{H}}\Delta\boldsymbol{H}_k\tilde{\boldsymbol{V}}_{\mathrm{n},k}\bar{\boldsymbol{V}}_{\mathrm{s},k}\}\end{aligned}$$

Again applying $\mathrm{vec}\{\boldsymbol{AXB}\} = (\boldsymbol{B}^{\mathrm{T}} \otimes \boldsymbol{A})\mathrm{vec}\{\boldsymbol{X}\}$, we obtain

$$\begin{aligned}\mathbb{E}\{\mathrm{vec}\{\boldsymbol{a}_{k,i}^{\mathrm{H}}\}^*\mathrm{vec}\{\boldsymbol{a}_{k,i}^{\mathrm{H}}\}^{\mathrm{T}}\} &= \sigma_{\mathrm{p}}^2\mathrm{diag}\{\mathrm{vec}\{\bar{\boldsymbol{d}}_{k,i}^{\mathrm{H}}\}\}^2 \\ &\quad \cdot (\bar{\boldsymbol{\Sigma}}_{\mathrm{s},k}^2 + \bar{\sigma}_{k,i}^2\boldsymbol{I}_M)\end{aligned} \tag{11}$$

Inserting (11) into (9) and after some algebraic manipulations, the co-channel interference power is expressed as

$$\mathbb{E}\{|y_{\mathrm{CCI},k,i}|^2\} = \bar{\sigma}_{k,i}^2\sigma_{\mathrm{p}}^2\sum_{\substack{j=1\\j\neq i}}^{\bar{r}_k}\frac{(\bar{\sigma}_{k,j}^2 + \bar{\sigma}_{k,i}^2)p_{k,j}}{(\bar{\sigma}_{k,j}^2 - \bar{\sigma}_{k,i}^2)^2} \tag{12}$$

Similarly, the residual MUI power is computed as:

$$\begin{aligned}\mathbb{E}\{|y_{\mathrm{MUI},k,i}|^2\} &= \mathbb{E}\{|\bar{\boldsymbol{u}}_{\mathrm{s},k,i}^{\mathrm{H}}\Delta\boldsymbol{H}_k\sum_{\substack{\ell=1\\\ell\neq k}}^{K}\tilde{\boldsymbol{V}}_{\mathrm{n},\ell}\bar{\boldsymbol{V}}_{\mathrm{s},\ell}\boldsymbol{P}_\ell^{\frac{1}{2}}\boldsymbol{s}_\ell|^2\} \\ &= \sum_{\substack{\ell=1\\\ell\neq k}}^{K}\mathbb{E}\{|\bar{\boldsymbol{u}}_{\mathrm{s},k,i}^{\mathrm{H}}\Delta\boldsymbol{H}_k\tilde{\boldsymbol{V}}_{\mathrm{n},\ell}\bar{\boldsymbol{V}}_{\mathrm{s},\ell}\boldsymbol{P}_\ell^{\frac{1}{2}}\boldsymbol{s}_\ell|^2\} \\ &= \sigma_{\mathrm{p}}^2\sum_{\substack{\ell=1\\\ell\neq k}}^{K}\sum_{\bar{i}=1}^{\bar{r}_\ell}p_{\ell,\bar{i}} \end{aligned} \tag{13}$$

Given (8), (12), and (13), the analytic expression of the SINR of the ith subchannel of the kth UT becomes

$$\begin{aligned}\mathrm{SINR}_{k,i} &= \frac{\mathbb{E}\{|y_{\mathrm{S},k,i}|^2\}}{\mathbb{E}\{|y_{\mathrm{CCI},k,i}|^2\} + \mathbb{E}\{|y_{\mathrm{MUI},k,i}|^2\} + \sigma_{\mathrm{n}}^2} \\ &= \frac{(\bar{\sigma}_{k,i}^2 + \frac{1}{2}\sigma_{\mathrm{p}}^2)p_{k,i}}{\bar{\sigma}_{k,i}^2\sigma_{\mathrm{p}}^2\sum_{\substack{j=1\\j\neq i}}^{\bar{r}_k}\frac{(\bar{\sigma}_{k,j}^2 + \bar{\sigma}_{k,i}^2)p_{k,j}}{(\bar{\sigma}_{k,j}^2 - \bar{\sigma}_{k,i}^2)^2} + \sigma_{\mathrm{p}}^2\sum_{\substack{\ell=1\\\ell\neq k}}^{K}\sum_{\bar{i}=1}^{\bar{r}_\ell}p_{\ell,\bar{i}} + \sigma_{\mathrm{n}}^2}\end{aligned}$$

4. WORST SINR MAXIMIZATION

In this section, we demonstrate a possible application of the derived expressions. More specifically, we develop a power allocation scheme which maximizes the minimum per-stream SINR after applying BD. Such a design criterion is common for a MIMO system, e.g., [13]. Define $\tilde{\boldsymbol{p}}_k = [\boldsymbol{p}_k^{\mathrm{T}}, \boldsymbol{0}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}_+^M$ and then $\boldsymbol{p} = [\tilde{\boldsymbol{p}}_1^{\mathrm{T}}, \cdots, \tilde{\boldsymbol{p}}_K^{\mathrm{T}}, \boldsymbol{0}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}_+^N$. Mathematically, we solve the following problem:

$$\begin{aligned}\max_{\boldsymbol{p}}\min_{\forall k,i} \quad & \mathrm{SINR}_{k,i} \\ \text{s.t.} \quad & \boldsymbol{1}^{\mathrm{T}}\boldsymbol{p} \leq P_{\mathrm{T}}\end{aligned} \tag{14}$$

or equivalently

$$\begin{aligned}\max_{\boldsymbol{p},t} \quad & t \\ \text{s.t.} \quad & \boldsymbol{1}^{\mathrm{T}}\boldsymbol{p} \leq P_{\mathrm{T}}, \\ & (t\boldsymbol{b}_{k,i}^{\mathrm{T}} - \boldsymbol{a}_{k,i}^{\mathrm{T}})\boldsymbol{p} \leq -t\sigma_{\mathrm{n}}^2,\ \forall k,i\end{aligned} \tag{15}$$

where the mth elements ($m \in \{1, \cdots, N\}$) $a_{k,i,m}$ of $\boldsymbol{a}_{k,i} \in \mathbb{R}_+^N$ and $b_{k,i,m}$ of $\boldsymbol{b}_{k,i} \in \mathbb{R}_+^N$ are defined as

$$a_{k,i,m} = \begin{cases}\bar{\sigma}_{k,i}^2 + \frac{1}{2}\sigma_{\mathrm{p}}^2 & m = i + (k-1)\cdot M \\ 0 & \text{otherwise}\end{cases} \tag{16}$$

and

$$b_{k,i,m} = \begin{cases} \bar{\sigma}_{k,i}^2 \sigma_{\mathrm{p}}^2 \frac{(\bar{\sigma}_{k,j}^2+\bar{\sigma}_{k,i}^2)}{(\bar{\sigma}_{k,j}^2-\bar{\sigma}_{k,i}^2)^2} & m = j + (k-1)\cdot M \\ \sigma_{\mathrm{p}}^2 & m = \tilde{i} + (\ell-1)\cdot M \\ 0 & \text{otherwise,} \end{cases} \quad (17)$$

respectively. Moreover, the index $\{i,j\} \in [1,\cdots,\bar{r}_k]$ and we have $i \neq j$. The index $\ell \in [1,\cdots,K]$ ($\ell \neq k$) and $\tilde{i} \in [1,\cdots,\bar{r}_\ell]$.

In case of the non-robust design, i.e., the perturbation is not considered in the SINR ($\sigma_{\mathrm{p}}^2 = 0$), problem (15) degrades into the following linear programming problem which can be solved using the interior-point algorithm in [14]

$$\begin{aligned} \max_{\boldsymbol{p},t} \quad & t \\ \text{s.t.} \quad & \mathbf{1}^{\mathrm{T}}\boldsymbol{p} \leq P_{\mathrm{T}}, \\ & \boldsymbol{a}_{k,i}^{\mathrm{T}}\boldsymbol{p} - t\sigma_{\mathrm{n}}^2 \geq 0, \; \forall k,i \end{aligned} \quad (18)$$

In case of the robust design, i.e., $\sigma_{\mathrm{p}}^2 \neq 0$, we notice that for a fixed t, problem (15) is still a linear programming problem which can also be solved using the interior-point algorithm in [14]. Thereby, it is straightforward to apply the bisection search method where a linear programming problem is solved at each step. If we further define $t_{\mathrm{lb}} = 0$ and $t_{\mathrm{ub}} = \max_{\forall k,i}((\boldsymbol{a}_{k,i}^{\mathrm{T}}\boldsymbol{e}_i)P_{\mathrm{T}}/\sigma_{\mathrm{n}}^2)$, i.e., the total transmit power is allocated to a stream which has the best channel condition, a suitable search interval for the proposed bisection search algorithm is given by $[t_{\mathrm{lb}}, t_{\mathrm{ub}}]$.

5. SIMULATION RESULTS

In this section, the accuracy of the derived expressions and the performance of the proposed robust design are evaluated via Monte-Carlo simulations. For this purpose, we consider a system with 2 UTs, where each UT has 2 antennas and the BS has 4 antennas. Two uncorrelated 2×4 channels $\boldsymbol{H}_k$ are selected such that $\mathrm{vec}\{\boldsymbol{H}_k\} \sim \mathbb{CN}\{\mathbf{0}, \boldsymbol{I}_8\}$. The total transmit power P_{T} is fixed to unity and a noise variance of -30 dB with respect to P_{T} is assumed. All the simulation results are obtained by averaging over 1000 perturbation realizations of $\mathrm{vec}\{\Delta\boldsymbol{H}_k\} \sim \mathbb{CN}(\mathbf{0}, \sigma_{\mathrm{p}}^2\boldsymbol{I}_8)$.

In Fig. 1 we compare the per-stream SINR values generated through a Monte Carlo method (denoted as "Empirical") with the analytic first-order SINR approximations using the SVD perturbation analysis in [9] (denoted as "Analytic"). The transmit power is uniformly allocated to all the subchannels. Obviously, the derived first-order expressions provide SINR estimates with good accuracy and thus are suitable for further applications, e.g., a worst SINR maximization.

Fig. 2 compares the achievable minimum per-stream SINR with the robust design (denoted as "Robust Design") and without the robust design (denoted as "Non-Robust Design"). Compared to the case with perfect CSI, a space for further improvements still exists as the perturbation power increases. Nevertheless, compared to the case without the robust design, a significant gain in terms of the achievable minimum SINR is already obtained by using the robust design especially when the perturbation power is high.

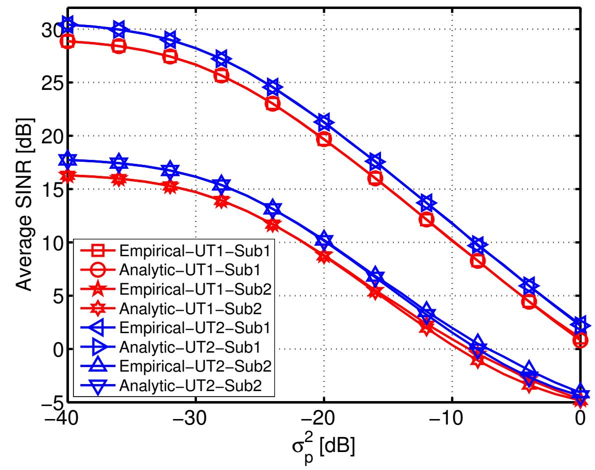

Fig. 1. A comparison of the first-order per-stream SINR approximation with the Monte Carlo simulation results as a function of the perturbation power σ_{p}^2.

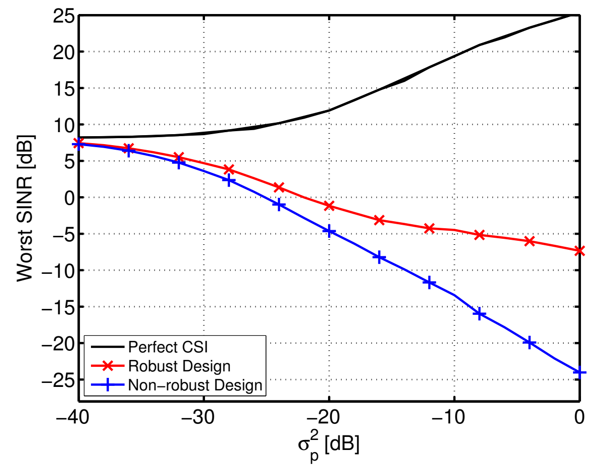

Fig. 2. A comparison of the minimum per-stream SINR achieved with and without a robust design as a function of the perturbation power σ_{p}^2.

6. CONCLUSION

Given imperfect CSI in a MU-MIMO downlink system using the BD based precoding technique, we have derived a first-order per-stream average SINR approximation for each UT. The accuracy of the derived expressions is verified via Monte-Carlo simulations. Furthermore, a robust design via worst SINR maximization is proposed based on the derived SINR expressions to demonstrate the possible application of these newly derived analytic results. Numerical results show that a substantial gain is obtained via the proposed robust design.

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