ROBUST CODEBOOK-BASED DOWNLINK BEAMFORMING USING MIXED INTEGER CONIC PROGRAMMING

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ABSTRACT

This paper considers robust codebook-based downlink beamforming (i.e., single-layer precoding), where the beamformer of each user is chosen from a fixed beamformer codebook defined, e.g., in LTE and LTE-A. Admission control and power allocation are embedded in the precoding vector selection procedure. The objective is to maximize the system utility, defined as *the revenue gained from admitting users minus the cost for the transmitted power of the base station*. We adopt the quality-of-service constrained approach and the robustness against channel covariance estimation errors is realized with worst-case design. The robust codebook-based beamforming problem, which is a *bi-level mixed integer program*, is converted into a more tractable *mixed integer second-order cone program*. Techniques are proposed to customize the convex continuous relaxation based branch-and-cut algorithm to compute the optimal solutions. A low-complexity inflation procedure is also developed to compute the near-optimal solutions for practical applications. Numerical examples show that the gap between the average number of admitted users achieved by the fast inflation procedure and that of the optimal solutions is less than 11.6% for all considered simulation settings. Further, the inflation procedure yields optimal solutions in 88% of the Monte Carlo runs under specific parameter settings.

Index Terms— Codebook-based Precoding, Robustness, Mixed Integer Conic Programming, Fast Near-Optimal Algorithm

1. INTRODUCTION

Downlink transmit beamforming, in which multiple users are simultaneously served on the same time and frequency resource, can significantly enhance spectrum efficiency [1–22]. A common approach in the literature consists in the quality-of-service (QoS) constrained design, in which the total transmitted power of the base station (BS) is minimized under the individual received signal-to-interference-plus-noise-ratio (SINR) constraints (i.e., the QoS constraints) of the mobile stations (MSs). Downlink beamformer design requires the channel state information (CSI) of the MSs, in terms of either instantaneous channel vectors in slow-fading scenarios or channel covariance matrices in fast-fading scenarios, to be known at the BS [1–8]. However, it is difficult to obtain perfect CSI at the BS in practice due to limited resources available for pilot signaling and CSI feedback. To achieve robustness against CSI errors in the QoS-constrained beamformer design, worst-case robust beamforming has been proposed and intensively studied (see, e.g., [1, 9–16]).

The conventional [1–8] and the worst-case robust beamforming problems [1, 9–16] can easily become infeasible when the number of admitted MSs is large and/or the SINR targets of the admitted MSs are high. In this case, user admission control mechanisms need to be applied to schedule a subset of the MSs to be served by the BS in a given time-slot, as investigated, e.g., in [17–22]. To reduce the overhead for signaling the beamformers, codebook-based downlink beamforming is introduced in 4G wireless standards, e.g., LTE-A [23–25], and has been studied in prior works [26, 27]. In codebook-based beamforming [23–27], the beamformer of each MS is chosen from a predefined precoding vector codebook known to both the BS and the MSs. Since there are only finite precoding vectors (i.e., finite predefined transmit beamformers) available, in order to ensure feasibility, a suitable admission control mechanism is indispensable in the codebook-based beamforming problems [23–27]. However, to our best knowledge, robust codebook-based downlink beamforming with admission control and power allocation, has not yet been investigated in the literature due to the tremendous computational complexity that is involved.

We consider in this paper the robust codebook-based downlink beamforming (RCB) problem, with admission control and power allocation embedded in the precoding vector assignment procedure. The RCB problem is firstly stated as a *bi-level mixed integer program* (BL-MIP) [28]. Similar to [9–12], by transforming the *inner* optimization problems in the worst-case SINR constraints into independent convex semidefinite programs (SDPs) and applying the Lagrange duality theory [29] to the resulting inner SDPs, we convert the RCB problem into a more structured *mixed integer second-order cone program* (MI-SOCP) [30]. Based on the developed MI-SOCP formulation, we introduce several techniques to customize the convex continuous relaxation based branch-and-cut (BnC) algorithm [30–33] implemented, e.g., in the MI-SOCP solver CPLEX [31], to compute the optimal solutions of the RCB problem, which can be used as benchmarks to evaluate the performance of the proposed low-complexity algorithms that are suitable for practical applications. A fast inflation procedure is developed, which yields near-optimal solutions of the RCB problem with very low computational complexity. As an illustrative example, the RCB problem is configured to maximize the number of admitted users in the simulations. Numerical results show that the gap between the average number of admitted users achieved by the inflation procedure and that of the optimal solutions is less than 11.6% for all parameter settings, and under certain conditions the inflation procedure can yield optimal solutions in up to 88% of the Monte Carlo runs.

2. RELATION TO PRIOR WORK

This paper addresses the robust codebook-based downlink beamforming problem, together with admission control and power allocation, using a novel MI-SOCP framework. Prior works on conventional and codebook-based beamforming [2–16, 26, 27] did not consider user admission control, and the existing contributions on joint beamforming and admission control [17–22] did not consider robust design or codebook-based beamforming.

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3. SYSTEM MODEL

Consider the downlink of a cellular network with one BS equipped with M transmit antennas, and K single-antenna MSs. We denote $\mathbf{h}_k \in \mathbb{C}^{M\times 1}$ and $\mathbf{w}_k \in \mathbb{C}^{M\times 1}$ as the frequency-flat channel vector and the beamforming weight vector, respectively, of the kth MS, $\forall k \in \mathcal{K} \triangleq \{1, 2, \cdots, K\}$. The received signal $y_k \in \mathbb{C}$ at the kth MS can be written as

$$y_k = \mathbf{h}_k^H \mathbf{w}_k x_k + \sum_{j=1, j\neq k}^{K} \mathbf{h}_k^H \mathbf{w}_j x_j + z_k, \forall k \in \mathcal{K} \quad (1)$$

with $x_k \in \mathbb{C}$ denoting the normalized data symbol, i.e., $\mathrm{E}\left\{|x_k|^2\right\} = 1$, of the kth MS, and $z_k \in \mathbb{C}$ representing the additive Gaussian noise at the kth MS, with zero mean and variance $\sigma_k^2, \forall k \in \mathcal{K}$.

In this paper, we consider codebook-based beamforming, as defined in the wireless standards, e.g., LTE-A [23–25], where the beamforming direction $\mathbf{w}_k/\|\mathbf{w}_k\|_2$ is chosen as one of the fixed precoding vectors in the beamformer codebook $\mathcal{B}$ that consists of L precoding vectors, i.e., $\mathcal{B} \triangleq \{\mathbf{v}_1,\ \mathbf{v}_2,\ \cdots,\ \mathbf{v}_L\}$, with $\mathbf{v}_l \in \mathbb{C}^{M\times 1}$ and $\|\mathbf{v}_l\|_2 = 1$, $\forall l \in \mathcal{L} \triangleq \{1, 2, \cdots, L\}$. To model the precoding vector assignment procedure, we introduce the *binary* variable $b_{k,l} \in \{0, 1\}$ to indicate with $b_{k,l} = 1$ that the lth precoding vector $\mathbf{v}_l$ is assigned to the kth MS, and $b_{k,l} = 0$ otherwise. We further introduce the variable $p_{k,l} \geq 0$ to model the power allocated to the lth precoding vector for the kth MS, $\forall k \in \mathcal{K}, \forall l \in \mathcal{L}$. Since at most one precoding vector shall be assigned to a MS in single-layer precoding, we have

$$\sum_{l=1}^{L} b_{k,l} \leq 1, \forall k \in \mathcal{K} \quad (2)$$

$$0 \leq p_{k,l} \leq b_{k,l} P^{(\mathrm{MAX})}, \forall k \in \mathcal{K}, \forall l \in \mathcal{L} \quad (3)$$

$$\sum_{k=1}^{K}\sum_{l=1}^{L} p_{k,l} \leq P^{(\mathrm{MAX})} \quad (4)$$

$$\mathbf{w}_k = \sum_{l=1}^{L} \sqrt{p_{k,l}}\mathbf{v}_l, \forall k \in \mathcal{K} \quad (5)$$

where the constant $P^{(\mathrm{MAX})}$ denotes the transmission power budget of the BS, and Eq. (4) represents the per-BS sum-power constraint. Eq. (3) implements the so-called big-M method [32, 33] to ensure that when $b_{k,l} = 0$, we have $p_{k,l} = 0$. Note that if $\sum_{l=1} b_{k,l} = 0$, i.e., if no precoding vector is assigned to the kth MS, the kth MS is not admitted. Hence, admission control is embedded in the precoding vector assignment procedure via the multiple-choice constraints (2). Assuming that the data symbols of the MSs are mutually independent and independent from the noise, the average received SINR at the kth MS, denoted by SINR_k, under single-user detection can then be expressed as (see, e.g., [1–3, 9–13])

$$\begin{aligned}\mathrm{SINR}_k &\triangleq \frac{\mathbf{w}_k^H \mathbf{R}_k \mathbf{w}_k}{\sum_{j=1,j\neq k} \mathbf{w}_j^H \mathbf{R}_k \mathbf{w}_j + \sigma_k^2} \\ &= \frac{\sum_{l=1}^{L} p_{k,l} \mathrm{Tr}\{\mathbf{R}_k \mathbf{V}_l\}}{\sum_{j=1,j\neq k}^{K}\sum_{l=1}^{L} p_{j,l}\mathrm{Tr}\{\mathbf{R}_k\mathbf{V}_l\} + \sigma_k^2}, \forall k \in \mathcal{K} \quad (6)\end{aligned}$$

with the matrix $\mathbf{R}_k \in \mathbb{C}^{M\times M}$ representing the channel covariance matrix (CCM) of the kth MS, the operation $\mathrm{Tr}\{\cdot\}$ denoting the trace of a matrix, the constant matrix $\mathbf{V}_l \in \mathbb{C}^{M\times M}$ defined as

$$\mathbf{V}_l \triangleq \mathbf{v}_l\mathbf{v}_l^H \succeq \mathbf{0}, \forall l \in \mathcal{L} \quad (7)$$

and to obtain Eq. (6), we have used the fact that

$$\mathbf{w}_j^H \mathbf{R}_k \mathbf{w}_j = \sum_{l=1}^{L} p_{j,l} \mathrm{Tr}\{\mathbf{R}_k\mathbf{V}_l\}, \forall j, k \in \mathcal{K} \quad (8)$$

which hold because of Eqs. (2), (3), (5), and (7).

Due to limited channel training and/or feedback resources [1, 2, 9–16], the *true* CCM $\mathbf{R}_k$ is usually not available at the BS, and only the estimated CCM of the kth MS, denoted by $\widehat{\mathbf{R}}_k \in \mathbb{C}^{M\times M}$, is known to the BS. In practical systems, the estimated CCM $\widehat{\mathbf{R}}_k$ is generally different from the true CCM $\mathbf{R}_k$. Following the approach of [1, 2, 9–16], we model in this paper the estimated CCM $\widehat{\mathbf{R}}_k$ as

$$\widehat{\mathbf{R}}_k = \mathbf{R}_k + \mathbf{\Delta}_k, \forall k \in \mathcal{K} \quad (9)$$

where the matrix $\mathbf{\Delta}_k \in \mathbb{C}^{M\times M}$ denotes the estimation error in the estimated channel covariance matrix $\widehat{\mathbf{R}}_k$. We know from practical considerations that the matrices $\mathbf{R}_k$ and $\widehat{\mathbf{R}}_k$ are positive semidefinite, i.e., $\mathbf{R}_k \succeq \mathbf{0}$ and $\widehat{\mathbf{R}}_k \succeq \mathbf{0}$, and the mismatch matrix $\mathbf{\Delta}_k$ is Hermitian. It is commonly assumed in the literature that the Frobenius norm of the error matrix $\mathbf{\Delta}_k$ is upper bounded by a known constant $\varepsilon_k \geq 0$ (see, e.g., [1, 2, 9–16]), i.e.,

$$\|\mathbf{\Delta}_k\|_F \leq \varepsilon_k, \forall k \in \mathcal{K}. \quad (10)$$

We remark that the framework proposed in the following sections can also accommodate other robust design approaches with different channel uncertainty models.

4. PROBLEM FORMULATIONS

4.1. Problem Statement

In this paper, we consider the problem of precoding vector assignment for the K MSs to maximize the system utility function $f(\{b_{k,l}\}, \{p_{k,l}\})$, which is defined as

$$f(\{b_{k,l}\}, \{p_{k,l}\}) \triangleq \sum_{k=1}^{K}\beta_k\sum_{l=1}^{L} b_{k,l} - \rho\sum_{k=1}^{K}\sum_{l=1}^{L} p_{k,l} \quad (11)$$

where the constant $\beta_k > 0$ denotes the revenue that is gained from admitting the kth MS, and the constant $\rho > 0$ represents the expense-per-watt for the total transmitted power of the BS. Similarly as in the QoS-constrained design [1–22], if the kth MS is admitted, then the received SINR of the kth MS must exceed a prescribed threshold $\Gamma_k^{(\mathrm{MIN})}$ to guarantee the QoS that the kth MS is subscribed to. To achieve robustness against the channel covariance estimation errors $\{\mathbf{\Delta}_k, \forall k \in \mathcal{K}\}$, we adopt the worst-case design approach [9–16]. Specifically, we define the following SINR constraints for the K MSs:

$$\left(\min_{\mathbf{\Delta}_k\in\mathcal{E}_k} \mathrm{SINR}_k\right) \geq \Gamma_k^{(\mathrm{MIN})}\sum_{l=1}^{L} b_{k,l}, \forall k \in \mathcal{K} \quad (12a)$$

$$\mathcal{E}_k \triangleq \left\{\mathbf{\Delta}_k | \mathbf{R}_k = \widehat{\mathbf{R}}_k - \mathbf{\Delta}_k \succeq 0, \text{ and } \|\mathbf{\Delta}_k\|_F^2 \leq \varepsilon_k^2\right\} \quad (12b)$$

where the expression of SINR_k is given in Eq. (6).

With the worst-case SINR constraints in Eq. (12), the RCB problem of interest can be stated as

$$\Psi^{(\mathrm{bmi})} \triangleq \max_{\{b_{k,l}, p_{k,l}\}} f(\{b_{k,l}\}, \{p_{k,l}\}) \quad (13a)$$

$$\text{s.t.} \quad (2) – (4), (12a), \text{and } (12b) \quad (13b)$$

$$b_{k,l} \in \{0, 1\}, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}. \quad (13c)$$

The RCB problem (13) contains the *inner* optimization problems in the SINR constraints (12) and the *outer* optimization problem (13). Hence, the RCB problem (13) represents a BL-MIP [28], which is generally intractable due to the inner optimization step (12) and the integer constraints (13c) [28]. To facilitate the development of efficient numerical algorithms, we derive in the next subsection an equivalent MI-SOCP formulation of the RCB problem (13).

4.2. A MI-SOCP Formulation of the RCB Problem

The main difficulty of problem (13) stems from the inner optimization problems (12), i.e., the worst-case SINR constraints, and the integer constraints (13c). We develop here more tractable equivalent reformulations of the SINR constraints (13c) and a MI-SOCP formulation of the RCB problem (13). Note that the constraints

$$\mathrm{SINR}_k \geq \Gamma_k^{(\mathrm{MIN})} \sum_{l=1}^{L} b_{k,l}, \forall k \in \mathcal{K} \tag{14}$$

with SINR_k given in Eq. (6), are equivalent to

$$\Big(\sum_{j=1, j\neq k}^{K} \sum_{l=1}^{L} p_{j,l} \mathrm{Tr}\big\{(\widehat{\mathbf{R}}_k - \mathbf{\Delta}_k)\mathbf{V}_l\big\} + \sigma_k^2 \Big) \Gamma_k^{(\mathrm{MIN})} \sum_{l=1}^{L} b_{k,l} \leq \sum_{l=1}^{L} p_{k,l} \mathrm{Tr}\big\{(\widehat{\mathbf{R}}_k - \mathbf{\Delta}_k)\mathbf{V}_l\big\}, \forall k \in \mathcal{K}. \tag{15}$$

The constraints (15) are difficult to handle as they involve the products of the continuous variables $\{p_{k,l}, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}\}$ and binary variables $\{b_{k,l}, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}\}$ in the left hand side of (15). Define respectively the matrix $\mathbf{A}_k \in \mathbb{C}^{M\times M}$ and the constant U_k as

$$\mathbf{A}_k \triangleq \frac{1}{\Gamma_k^{(\mathrm{MIN})}} \sum_{l=1}^{L} p_{k,l}\mathbf{V}_l - \sum_{j=1, j\neq k} \sum_{l=1}^{L} p_{j,l}\mathbf{V}_l, \forall k \in \mathcal{K} \tag{16}$$

$$U_k \triangleq \Big(\max_{l\in\mathcal{L}} \mathrm{Tr}\big\{\widehat{\mathbf{R}}_k \mathbf{V}_l\big\} + \varepsilon_k \Big) P^{(\mathrm{MAX})} + \sigma_k^2, \forall k \in \mathcal{K}. \tag{17}$$

Then, we can adopt the big-M method [32, 33] to rewrite the constraints in Eq. (15) as

$$\mathrm{Tr}\big\{(\widehat{\mathbf{R}}_k - \mathbf{\Delta}_k)\mathbf{A}_k\big\} \geq \sigma_k^2 + \Big(\sum_{l=1}^{L} b_{k,l} - 1 \Big) U_k, \forall k \in \mathcal{K}. \tag{18}$$

Note that Eq. (18) consists of only *linear* terms of the optimization variables $\{b_{k,l}, p_{k,l}, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}\}$. Further, comparing Eq. (18) to Eq. (15), when $\sum_{l=1}^{L} b_{k,l} = 1$, Eqs. (15) and (18) become identical. On the other hand, when $\sum_{l=1}^{L} b_{k,l} = 0$, we have $\sum_{l=1}^{L} p_{k,l} = 0$ due to Eq. (3) and thus the constraint corresponding to the kth MS in Eq. (15) is automatically satisfied. Since the constant U_k satisfies the property that

$$U_k \geq \max_{\mathbf{\Delta}_k \in \mathcal{E}_k} \sum_{j=1, j\neq k}^{K} \sum_{l=1}^{L} p_{j,l} \mathrm{Tr}\big\{(\widehat{\mathbf{R}}_k - \mathbf{\Delta}_k)\mathbf{V}_l\big\} + \sigma_k^2 \tag{19}$$

when $\sum_{l=1}^{L} b_{k,l} = 0$, the constraint corresponding to the kth MS in Eq. (18) is also automatically satisfied due to the big-M constant U_k. As a result, under the multiple-choice constraints (2) and the binary constraints (13c), Eqs. (15) and (18) are equivalent.

With Eq. (18), we can rewrite the SINR constraints (12) as

$$\Big(\min_{\mathbf{\Delta}_k \in \mathcal{E}_k} \mathrm{Tr}\big\{(\widehat{\mathbf{R}}_k - \mathbf{\Delta}_k)\mathbf{A}_k\big\} \Big) \geq \sigma_k^2 + \big(\sum_{l=1}^{L} b_{k,l} - 1 \big) U_k, \forall k \in \mathcal{K}. \tag{20}$$

Similar to [9–12], we treat the inner optimization problem corresponding to the kth MS in the left hand side of Eq. (20) as an independent *convex* semidefinite program (SDP) given by

$$\Phi_k \triangleq \min_{\mathbf{\Delta}_k} \ \mathrm{Tr}\big\{(\widehat{\mathbf{R}}_k - \mathbf{\Delta}_k)\mathbf{A}_k\big\}\big) \tag{21a}$$

$$\text{s.t.} \quad \widehat{\mathbf{R}}_k - \mathbf{\Delta}_{\boldsymbol{k}} \succeq 0 \tag{21b}$$

$$\|\mathbf{\Delta}_k\|_F^2 \leq \varepsilon_k^2. \tag{21c}$$

Note that the SDP (21) is *strictly* feasible, e.g., $\mathbf{\Delta}_{\boldsymbol{k}} = \mathbf{0}$ is a feasible solution. As a result, we can apply Lagrange duality theory [29] to problem (21). Further, following a similar procedure as in [9, 10], it can be proved that, after maximizing over the Lagrange multiplier associated with the constraint (21c), the dual problem associated with the convex SDP (21) can finally be expressed as

$$\Phi_k = \max_{\mathbf{Q}_k \succeq \mathbf{0}} -\varepsilon_k \|\mathbf{A}_k - \mathbf{Q}_k\|_F + \mathrm{Tr}\big\{\widehat{\mathbf{R}}_k(\mathbf{A}_k - \mathbf{Q}_k)\big\} \tag{22}$$

where the matrix $\mathbf{Q}_k \succeq \mathbf{0}$ represents the Lagrange multiplier corresponding to the constraint (21b). With the dual problem (22) of the SDP (21), we obtain the following equivalent formulations of the worst-case SINR constraints in Eq. (20):

$$\Big(\max_{\mathbf{Q}_k \succeq \mathbf{0}} -\varepsilon_k \|\mathbf{A}_k - \mathbf{Q}_k\|_F + \mathrm{Tr}\big\{\widehat{\mathbf{R}}_k(\mathbf{A}_k - \mathbf{Q}_k)\big\} \Big) \geq \sigma_k^2 + \big(\sum_{l=1}^{L} b_{k,l} - 1 \big) U_k, \forall k \in \mathcal{K}. \tag{23}$$

We can then follow a similar argument as in [9, 10] with modifications required for accommodating the integer variables to prove that, without loss of optimality of the RCB problem (13), we can choose $\mathbf{Q}_k = \mathbf{0}$, $\forall k \in \mathcal{K}$, in Eq. (23). As a result, the RCB problem (13) can be equivalently rewritten as the following MI-SOCP:

$$\Psi^{(\mathrm{bmi})} = \max_{\{b_{k,l}, p_{k,l}\}} \ f(\{b_{k,l}\}, \{p_{k,l}\}) \tag{24a}$$

$$\text{s.t.} \quad (2) – (4) \tag{24b}$$

$$-\varepsilon_k \|\mathrm{vec}\{\mathbf{A}_k\}\|_2 + \mathrm{Tr}\big\{\widehat{\mathbf{R}}_k \mathbf{A}_k\big\} \geq \sigma_k^2 + \Big(\sum_{l=1}^{L} b_{k,l} - 1 \Big) U_k, \forall k \in \mathcal{K} \tag{24c}$$

$$b_{k,l} \in \{0, 1\}, \forall k \in \mathcal{K}, \forall l \in \mathcal{L} \tag{24d}$$

where the vectorizing operation $\mathrm{vec}\{\cdot\}$ stacks the columns of a matrix into a column vector. We have thus converted the RCB problem in the form of the BL-MIP (13) into the MI-SOCP formulation (24) which admits convex continuous relaxations and can be solved using the convex continuous relaxation based BnC method [30–33].

5. OPTIMAL AND NEAR-OPTIMAL ALGORITHMS

5.1. Optimal Solutions via the BnC Method

Due to advancement of parallel computing, the convex continuous relaxation based BnC method [30–33] is commonly used to solve MI-SOCPs, such as the formulated RCB problem (24). The upper bounds and/or optimal solutions computed by the BnC procedure can be used as benchmarks to evaluate the performance of fast near-optimal algorithms when the optimal solutions cannot be reached in reasonable time in practice. The BnC algorithm relies on solving the continuous relaxation of (24), given by the following SOCP:

$$\Psi^{(\mathrm{bmc})} \triangleq \max_{\{b_{k,l}, p_{k,l}\}} \ f(\{b_{k,l}\}, \{p_{k,l}\}) \tag{25a}$$

$$\text{s.t.} \quad (2) – (4), \text{ and } (24c) \tag{25b}$$

$$0 \leq b_{k,l} \leq 1, \forall k \in \mathcal{K}, \forall l \in \mathcal{L} \tag{25c}$$

where the variables $\{b_{k,l}, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}\}$ originally constrained in the discrete set $\{0, 1\}$ as in Eq. (24d) are relaxed to be continuous variables in the closed interval $[0, 1]$ as in Eq. (25c). The non-integer valued variables in $\{b_{k,l}, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}\}$ are gradually branched and set to binary values in the BnC procedure [30–33].

Based on the structure of the RCB problem (24), strategies can be introduced to customize the BnC algorithm implemented, e.g., in the MI-SOCP solver CPLEX [31], to reduce the computational efforts. For instance, the following necessary condition

$$\Big(\min_{\mathbf{\Delta}_{\boldsymbol{k}} \in \mathcal{E}_k} \mathrm{Tr}\{(\widehat{\mathbf{R}}_k - \mathbf{\Delta}_{\boldsymbol{k}})\mathbf{V}_l\} P^{(\mathrm{MAX})} \Big) \geq \sigma_k^2 \Gamma_k^{(\mathrm{MIN})} \tag{26}$$

can easily be tested. In case the necessary condition (27) is *not* satisfied, i.e., if it is *infeasible* to assign the lth precoding vector $\mathbf{v}_l$ to the kth MS, we fix $b_{k,l} = 0$ in the RCB problem (24). We can also introduce problem-specific cuts [30–33], i.e., the constraints which are redundant to problem (24), but reduce the size of the feasible set of the continuous relaxation (25). For instance, the following cuts can be added to the RCB problem (24):

$$p_{k,l}\mathrm{Tr}\{\widehat{\mathbf{R}}_k\mathbf{V}_l\} \geq b_{k,l}\sigma_k^2\Gamma_k^{(\mathrm{MIN})}, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}. \quad (27)$$

More importantly, we can customize the branching priorities [30–33] of the relaxed binary variables according to, e.g., the term $P^{(\mathrm{MAX})}\mathrm{Tr}\{\widehat{\mathbf{R}}_k\mathbf{V}_l\}$ representing the maximum signal power received at the kth MS with the lth precoding vector, $\forall k \in \mathcal{K}, \forall l \in \mathcal{L}$. That is, a larger term $P^{(\mathrm{MAX})}\mathrm{Tr}\{\widehat{\mathbf{R}}_k\mathbf{V}_l\}$ corresponds to a larger branching priority of the variable $b_{k,l}$, and the non-integer valued relaxed binary variable with the largest branching priority is branched in each branching step in the BnC procedure [30–33]. *Further customizing strategies for the RCB problem* (24) *are proposed in [34] and are omitted here due to limited space.*

5.2. A Fast Near-Optimal Algorithm

To facilitate practical applications, we propose here a fast inflation procedure [21] to compute near-optimal solutions of the RCB problem (24). The integer feasible solutions found by the inflation procedure can also be used to initialize the BnC algorithm to reduce the efforts in computing tight upper bounds and/or optimal solutions. The inflation procedure starts with $\{b_{k,l}^{(0)} = 0, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}\}$, and the objective value $\Psi^{(0)} = -\rho P^{(\mathrm{MAX})}$. In the nth iteration ($1 \leq n \leq K$), the best candidate among the *zero-valued* variables in $\{b_{k,l}^{(\mathrm{n}-1)}, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}\}$ is chosen and set to *one*. To determine the best candidate in the nth iteration, an enumerating process is introduced. Define $\widetilde{b}_{k,l}^{(\mathrm{n})} \triangleq b_{k,l}^{(\mathrm{n}-1)}, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}$. In the (j, m)th enumeration ($1 \leq j \leq K, 1 \leq m \leq L$), if $\widetilde{b}_{j,m}^{(\mathrm{n})} = 0$, set $\widetilde{b}_{j,m}^{(\mathrm{n})} = 1$, and the following convex SOCP

$$\widetilde{\Psi}_{j,m}^{(\mathrm{n})} \triangleq \max_{\{p_{k,l}\}} \sum_{k=1}^{K}\beta_k\sum_{l=1}^{L}\overline{\overline{b}}_{k,l}^{(\mathrm{n})} - \rho\sum_{k=1}^{K}\sum_{l=1}^{L}p_{k,l} \quad (28a)$$

$$\text{s.t.} \quad (4) \quad (28b)$$

$$-\varepsilon_k\|\mathrm{vec}\{\mathbf{A}_k\}\|_2 + \mathrm{Tr}\{\widehat{\mathbf{R}}_k\mathbf{A}_k\} \geq \sigma_k^2, \quad \text{if } \widetilde{b}_{k,l}^{(\mathrm{n})} = 1, \forall k \in \mathcal{K}, \forall l \in \mathcal{L} \quad (28c)$$

is solved, with $\overline{\overline{b}}_{k,l}^{(\mathrm{n})} \triangleq \max\{\widetilde{b}_{k,l}^{(\mathrm{n})}, 0\}, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}$, and then set $\widetilde{b}_{j,m}^{(\mathrm{n})} = -1$ to prevent infinite cycles. If problem (28) is infeasible, set $b_{j,m}^{(\mathrm{n}-1)} = \widetilde{b}_{j,m}^{(\mathrm{n})}$. The best candidate in the nth iteration is the zero-valued variable in $\{b_{k,l}^{(\mathrm{n}-1)}, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}\}$ that corresponds to the largest entry in $\{\widetilde{\Psi}_{k,l}^{(\mathrm{n})}, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}\}$. The proposed fast inflation procedure is summarized in Alg. 1.

Init.: $b_{k,l}^{(0)} = 0, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}$, and $\Psi^{(0)} = -\rho P^{(\mathrm{MAX})}$
for $n = 1$ to K **do**
  If $\{b_{k,l}^{(\mathrm{n}-1)} \neq 0, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}\}$, stop and return the results of the $(n-1)$th iteration. Otherwise, define $\widetilde{b}_{k,l}^{(\mathrm{n})} = b_{k,l}^{(\mathrm{n}-1)}, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}$.
  for $j = 1$ to K, and $m = 1$ to L **do**
    Initialize $\widetilde{\Psi}_{j,m}^{(\mathrm{n})} = 0$. If $\widetilde{b}_{j,m}^{(\mathrm{n})} = 0$, set $\widetilde{b}_{j,m}^{(\mathrm{n})} = 1$, solve the SOCP (28), and then set $\widetilde{b}_{j,m}^{(\mathrm{n})} = -1$.
    If the SOCP (28) is infeasible, set $b_{j,m}^{(\mathrm{n}-1)} = \widetilde{b}_{j,m}^{(\mathrm{n})}$.
  end
  Compute $(k^\star, l^\star) = \underset{k\in\mathcal{K}, l\in\mathcal{L}}{\mathrm{argmax}}\ \widetilde{\Psi}_{k,l}^{(\mathrm{n})}$. If $\widetilde{\Psi}_{k^\star,l^\star}^{(\mathrm{n})} < \Psi^{(n-1)}$, stop and return the results of the $(n-1)$th iteration.
  Otherwise, set $\Psi^{(\mathrm{n})} = \widetilde{\Psi}_{k^\star,l^\star}^{(\mathrm{n})}$; $b_{k,l}^{(\mathrm{n})} = b_{k,l}^{(\mathrm{n}-1)}, \forall k \in \mathcal{K}$, $\forall l \in \mathcal{L}$; set $b_{k^\star,l}^{(\mathrm{n})} = -1, \forall l \in \mathcal{L}$; and set $b_{k^\star,l^\star}^{(\mathrm{n})} = 1$.
end

Alg. 1: The proposed fast inflation procedure

6. NUMERICAL RESULTS

In our simulations, we consider a downlink system with $M = 4$ and $P^{(\mathrm{MAX})} = 15$ dB at the BS, and $K = 16$ MSs. The beamformer codebook with $L = 16$ precoding vectors defined in [23] is used. The SINR requirements of the K MSs are chosen to be identical, and the noise power is normalized to one. The (m, l)th entry of the *normalized* true CCM $\mathbf{R}_k$ is modeled as (see, e.g., [2, 3, 9–11])

$$[\mathbf{R}_k]_{m,l} = \exp\left(-\left(\pi(m-l)\sigma_\theta\cos\theta_k\right)^2/2\right) \times \exp\left(\sqrt{-1}\pi(m-l)\sin\theta_k\right), \forall m, l = 1, 2, \cdots M, \forall k \in \mathcal{K} \quad (29)$$

where $\sigma_\theta = 2^{\circ}$ denotes the spread angle, and θ_k represents the *random* angular direction of the kth MS. The estimation error matrix $\mathbf{\Delta}_k$ is uniformly generated in a sphere centered at zero and with a radius of $\varepsilon_k = 0.1$ [9–11]. The MI-SOCP solver CPLEX [31] is applied to the RCB problem (24) in the simulations for reference.

As an illustrative example, we choose $\beta_k = 1$, $\forall k \in \mathcal{K}$, and $\rho = 0$, i.e., the RCB problem (24) is configured to maximize the total number of admitted MSs. Fig. 1 displays the average number of admitted MSs versus the SINR requirement $\Gamma_k^{(\mathrm{MIN})}$, with the simulation results averaged over 500 Monte Carlo runs. The optimal solutions are obtained from CPLEX with an average runtime of 40 seconds, and the remaining curves (i.e., the feasible solutions) were obtained with the same average runtime of 16 seconds. We observe from Fig. 1 that the average number of admitted MSs achieved by the proposed Alg. 1 is very close to the global optimum, and the relative gap in the objective function value at optimum is less than 11.6% for all considered values of $\Gamma_k^{(\mathrm{MIN})}$.

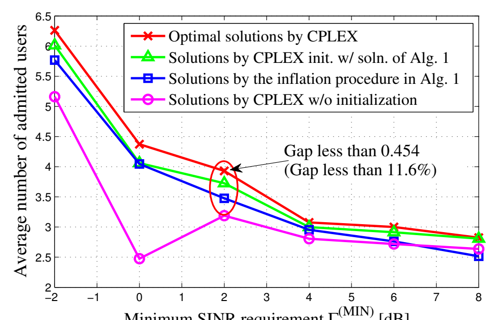

Fig. 1: Average number of admitted MSs vs. $\Gamma_k^{(\mathrm{MIN})}$.

More interestingly, the proposed inflation procedure yields optimal solutions of the RCB problem (24) in more than 54% of the Monte Carlo runs for all chosen values of $\Gamma_k^{(\mathrm{MIN})}$, as shown in Tab. 1. Particularly, the proposed Alg. 1 generates optimal solutions in up to 88% of the Monte Carlo runs with $\Gamma_k^{(\mathrm{MIN})} = 4$ dB.

Tab. 1: Percent. of optimal soln. achieved by the proposed Alg. 1

$\Gamma_k^{(\mathrm{MIN})}$ [dB]	-2	0	2	4	6	8
Percentage	54%	67%	55%	88%	76%	69%

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