RECONFIGURABLE ADAPTIVE LINEAR ARRAY SIGNAL PROCESSING IN GNSS APPLICATIONS

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ABSTRACT

The configuration of an antenna array plays a fundamental role in the ability of array signal processing to mitigate interference. We propose in this paper a novel reconfigurable adaptive linear array scheme to overcome the drawbacks of traditional array processing. We employ the effective carrier to noise density ratio (C/N_0), which is a reliable measure of the performance in the Global Navigation Satellite Systems (GNSS) applications, expressing its dependence on the spatial separation through the Spatial Correlation Coefficient (SCC), with a lower SCC giving better interference mitigation performance. We then formulate the problem of determining the optimal orientation of a linear array in terms of the minimization of SCC. Simulation results show that the proposed method is effective in reducing the SCC and improving the effective C/N_0. Finally, we propose a practical implementation where the array orientation is chosen from a number of present orientations.

***Index Terms*—** DOA, GNSS, Reconfigurable adaptive linear array, SCC, STAP

1. INTRODUCTION

Global Navigation Satellite Systems (GNSS) are vulnerable to Radio Frequency Interference (RFI) due to the limited transmitting power from the satellite. Consequently, adaptive antenna arrays have been proposed to boost the received signal and suppress the RFI. As the array configuration plays a fundamental role in the array signal processing performance, it is an important design problem [1, 2, 3]. However, most of the work in the literature focuses on adaptive beamforming and filtering techniques with the array configuration taken to be fixed [4, 5, 6]. Under certain scenarios, the number of available Degree of Freedom (DOF) may be substantially reduced and the performance of adaptive array severely degraded by the fixed array architecture. This problem is exacerbated in the case of a linear array where the array exhibits a cone of ambiguity [7]. The linear array cannot distinguish the desired signal and interference if they are in the same ambiguity cone. This situation can only be rectified if the orientation of the linear array is changed.

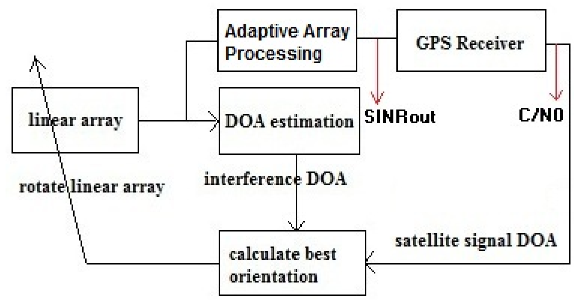

Fig. 1. Block diagram of reconfigurable adaptive linear array

In this paper, we propose a novel reconfigurable linear array method to adaptively adjust the array orientation in order to overcome the drawbacks of existing adaptive array processing algorithms. The block diagram of this method is shown in Fig. 1. The linear array is placed along an initial direction and beamforming is implemented under this array orientation. The GPS receiver decodes the navigation information which gives the accurate Direction of Arrival (DOA) of the satellite signal. The DOA of interference, on the other hand, is estimated prior to the array processing step. The optimal array orientation is then derived from the DOAs of the satellite signal and interference. Subsequently array processing algorithm is applied based on this optimal array configuration to enhance the receiver's performance. Therefore, the distinguishing aspect of the proposed technique is in adopting a strategy of adaptively reconfiguring both the antenna array configuration as well as setting the weights of the adaptive filter in order to maximize the interference suppression.

The paper is organised as follows. In Section II the mathematical theory is formulated and the relationship between the SCC and effective C/N_0 is described as well. Section III presents an effective method to estimate the DOA of the interference. The technique for determining the optimal array orientation is also given in this section. In Section IV a set of representative numerical results are reported and discussed.

Finally, some conclusions are drawn in Section V.

2. MATHEMATICAL FORMULATION

The effective C/N_0 is an important indicator of the achievable performance of GNSS receivers [8]. The C/N_0 is intimately related to the Spectral and Spatial Separation Coefficient (SSSC), [9], which takes into account the spatial dimension of the antenna array in order to quantify its effect on the desired signal. The spatial separation between interference and the desired signal, on the other hand, is characterized with a single parameter called the Spatial Correlation Coefficient (SCC) [1]. The definition of these parameters and the relationship between them are mathematically formulated in what follows.

Assuming the desired signal and interference are coming from (θ_s, ϕ_s) and (θ_j, ϕ_j) respectively, then the **u**-space DOA parameters are

$$\tilde{\mathbf{s}} = [u_{sx} \quad u_{sy}]^T, \qquad \tilde{\mathbf{j}} = [u_{jx} \quad u_{jy}]^T. \tag{1}$$

where $u_{ix} = \cos\theta_i \cos\phi_i$ and $u_{iy} = \cos\theta_i \sin\phi_i$, for $i = s, j$. The superscripts ^{T} and ^{H} denote the transpose and conjugate transpose respectively. The steering vectors of the desired signal and interference are

$$\mathbf{v}_s = e^{jk_0\mathbf{p}\tilde{\mathbf{s}}}, \qquad \mathbf{v}_j = e^{jk_0\mathbf{p}\tilde{\mathbf{j}}}. \tag{2}$$

with **p** being the $N \times 2$ matrix whose entries are the positions of antenna elements. Let the linear array be at azimuth angle φ. Then the n_{th} row of **p** is

$$\mathbf{p}_n = [nd\cos\varphi \quad nd\sin\varphi] = nd\mathbf{v}_o. \tag{3}$$

with the array orientation vector to be

$$\mathbf{v}_o = [cos\varphi \quad sin\varphi] \tag{4}$$

Here, the inter-element space d is set to be half the wavelength. Assuming the noise and interference to be uncorrelated, their covariance matrix is given by

$$\mathbf{R}_n = \sigma^2\mathbf{I} + P_j\mathbf{v}_j\mathbf{v}_j^H, \tag{5}$$

where σ^2 is the thermal noise power, P_j that of the interference. The optimal array weight vector based on maximum signal to interference ratio is then, [10],

$$\mathbf{w}_{opt} = \gamma\mathbf{R}_n^{-1}\mathbf{v}_s, \tag{6}$$

where γ is an arbitrary constant. At the output of the adaptive array filter, the signal to interference plus noise ratio becomes

$$SINR_{out} = P_s\mathbf{v}_s^H\mathbf{R}_n^{-1}\mathbf{v}_s. \tag{7}$$

where P_s denotes the signal power. Applying the Sherman-Morrison-Woodbury identity to Eq.(5) and assuming the thermal noise power is much smaller than the interference, Eq. (6) can be written as

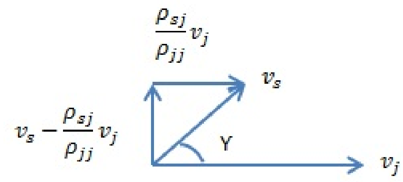

Fig. 2. Relationship between SCC and adaptive array weight vector

$$\mathbf{w}_{opt} \approx k(\mathbf{v}_s - \frac{\rho_{sj}}{\rho_{jj}}\mathbf{v}_j) = k(\mathbf{v}_s - \alpha_{sj}\mathbf{v}_j), \tag{8}$$

where $k = \gamma/\sigma^2$, $\rho_{sj} = \mathbf{v}_j^H\mathbf{v}_s$, $\rho_{ss} = \mathbf{v}_s^H\mathbf{v}_s$ and $\rho_{jj} = \mathbf{v}_j^H\mathbf{v}_j$. We can see that when the interference is much stronger than the thermal noise, then the adaptive array weight vector becomes orthogonal to the interference from Fig. 2 and thus the adaptive array processor approximates a null-steering algorithm. Now setting, without loss of generality, $\|\mathbf{v}_j\| = \|\mathbf{v}_s\| = \sqrt{N}$ (N is the number of antennas) we define

$$\alpha_{sj} = \frac{\rho_{js}}{\sqrt{\rho_{jj}}\sqrt{\rho_{ss}}} = \frac{\rho_{js}}{\|\mathbf{v}_j\|\|\mathbf{v}_s\|} = \frac{\rho_{js}}{N}. \tag{9}$$

Then the $SINR_{out}$ can be written as

$$SINR_{out} \approx \frac{NP_s}{\sigma^2}(1 - |\alpha_{sj}|^2), \tag{10}$$

We call α_{sj} the spatial correlation coefficient (SCC). Its absolute value is bounded between zero and one and can be interpreted as $\cos\gamma$ as seen from Fig. 2. The smaller the SCC is, the more separable in space the desired signal and interference are. Now substituting the array weight vector Eq. (6) into the effective C/N_0 [9] and using the SCC expression Eq. (9) yields

$$(\frac{C}{N_0})_{eff} = \frac{NP_s^d}{G_nN_0}(1 - |\alpha_{sj}|^2). \tag{11}$$

where P_s^d is the satellite signal power after code de-spreading, N_0 is the white noise power density and G_n is the noise processing gain. Eq. (11) reveals that the effective C/N_0 does not depend on the Doppler separation any more because the adaptive antenna array cancels the interference completely (Recall that the array weight vector essentially implements a null-steering algorithm when the interference is much stronger than white noise). In summary, the effective C/N_0 is a function of both the number of antennas, N and the SCC parameter. When the number of antennas N is fixed, the effective C/N_0 can be improved by changing the array configuration (here rotating the linear array) to reduce the SCC value under the specific scenario.

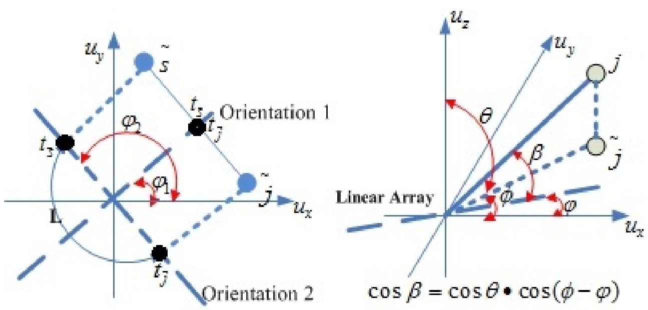

Fig. 3. Orientation calculated with projection method

3. OPTIMAL ORIENTATION OF LINEAR ARRAY

In order to calculate the optimal orientation that gives the minimum SCC value, the DOAs of the desired signal and interference must be known or estimated. Although several algorithms [11] have been proposed to solve this problem, we employ an effective discrete Fourier transform (DFT)-based technique due to its simplicity, robustness, and excellent performance, [12]. We then present the method of calculating the optimal array orientation according to the estimated DOAs of the desired signal and interference.

The estimation of the DOA of a signal incident on the array can be solved as a frequency estimation problem, [7]. A linear array can only resolve one direction which is in the plane containing both the signal and the array. As the DOA problem is two-dimensional, we use two 1-Dimensional spatial DFT measurements $u_1 = \cos\beta_1$ and $u_2 = \cos\beta_2$ obtained from two orthogonal orientations of the array, φ_1 and $\varphi_2 = \varphi_1 + \frac{\pi}{2}$. Here $\beta_{1,2}$ is the angle between the linear array and the estimated signal and the relationship among β, array orientation φ, the estimated signal's azimuth angle ϕ and elevation angle θ is $\cos\beta = \cos\theta\cos(\phi - \varphi)$ as shown in Fig. 3. The Interpolation on Fourier Coefficients method of [12] is then used to get fine estimates of the DOA parameters (here $u_1 = u_x$ and $u_2 = u_y$ by choosing $\varphi_1 = 0$).

Having derived the DOAs of the signal and interference, we can now find the optimal array orientation as follows. The **u**-space DOA parameters in Eq. (1) are also the projections of two point sources on the x-y plane shown in Fig. 3. Substituting Eq. (1), Eq. (2), Eq. (3) into Eq. (9), the SCC can be expressed as

$$\alpha_{sj} = \frac{1}{N}\sum_{n=-(N-1)/2}^{(N-1)/2} e^{jk_0dn[cos\varphi(u_{sx}-u_{jx})+sin\varphi(u_{sy}-u_{jy})]}. \tag{12}$$

Next we project $\tilde{\mathbf{s}}$ and $\tilde{\mathbf{j}}$ onto the line along the array orientation which we denote by $\mathbf{t}_{\tilde{\mathbf{s}}}$ and $\mathbf{t}_{\tilde{\mathbf{j}}}$ shown in Fig. 3.

$$\mathbf{t}_{\tilde{\mathbf{s}}} = \frac{\mathbf{v}_o^H\mathbf{v}_o}{\mathbf{v}_o\mathbf{v}_o^H}\tilde{\mathbf{s}}, \qquad \mathbf{t}_{\tilde{\mathbf{j}}} = \frac{\mathbf{v}_o^H\mathbf{v}_o}{\mathbf{v}_o\mathbf{v}_o^H}\tilde{\mathbf{j}}. \tag{13}$$

Now, the distance between $\mathbf{t}_{\tilde{\mathbf{s}}}$ and $\mathbf{t}_{\tilde{\mathbf{j}}}$ is

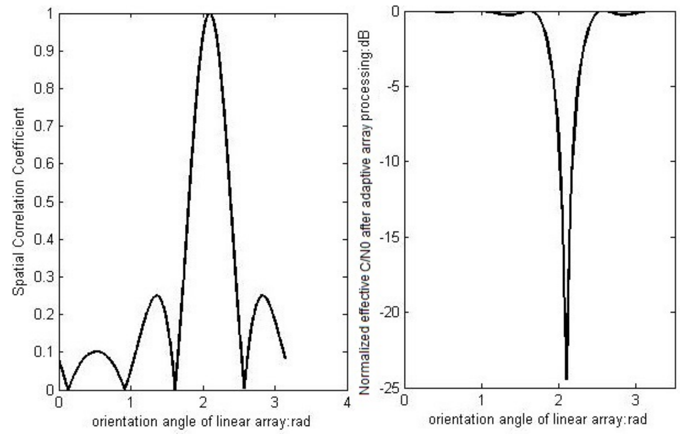

Fig. 4. Normalized effective C/N_0 and SCC versus different orientation of linear array

$$L = \| \mathbf{t}_{\tilde{\mathbf{s}}} - \mathbf{t}_{\tilde{\mathbf{j}}} \|_2 = |\cos\varphi(u_{sx}-u_{jx}) + \sin\varphi(u_{sy}-u_{jy})|. \tag{14}$$

Substituting Eq. (14) into Eq. (12) yields

$$\alpha_{sj} = \frac{1}{N}\sum_{n=-(N-1)/2}^{(N-1)/2} e^{jk_0dnL} = \frac{1}{N}\frac{\sin(\frac{N}{2}k_0dL)}{\sin(\frac{1}{2}k_0dL)}. \tag{15}$$

We can see that there is sinc function relationship between the value of SCC and L. So the optimal solution should satisfy

$$L = \frac{2m}{N}, \tag{16}$$

where m is an integer which is not divisible by N and $L \leq \|\tilde{\mathbf{s}} - \tilde{\mathbf{j}}\|$. When a solution exists, we can find an orientation that makes the SCC become zero. Otherwise, The orientation which is parallel to the line passing through $\tilde{\mathbf{s}}$ and $\tilde{\mathbf{j}}$ (such as Orientation 2 in Fig. 3) is the optimal solution. It is clear that the worst performance occurs when $L = 0$ (such as Orientation 1 in Fig. 3), which means that $\mathbf{t}_{\tilde{\mathbf{s}}}$ and $\mathbf{t}_{\tilde{\mathbf{j}}}$ are the same point and the SCC has the maximum value one.

The final optimal orientation of a linear array should be rotated from the orientation 2 in Fig. 3 by an angle δ such that

$$\cos\delta = \frac{L}{\|\tilde{\mathbf{s}} - \tilde{\mathbf{j}}\|} = \frac{2m/N}{\sqrt{(u_{sx}-u_{jx})^2 + (u_{sy}-u_{jy})^2}}. \tag{17}$$

Then the optimal orientations of a linear array which can give the smallest SCC are

$$\varphi = \varphi_o \pm \delta(mod\,\pi), \quad \text{with } \tan\varphi_o = \frac{u_{sy}-u_{jy}}{u_{sx}-u_{jx}}. \tag{18}$$

4. SIMULATION RESULTS

We use a linear array with 5 half-wavelength spaced antenna elements to simulate the proposed strategy. In the

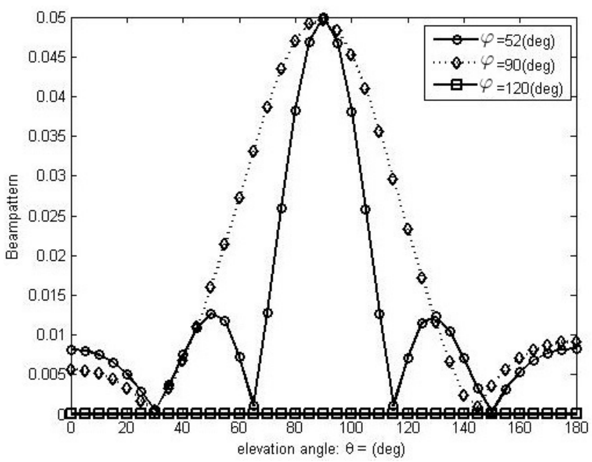

Fig. 5. Beampattern versus different orientations of linear array

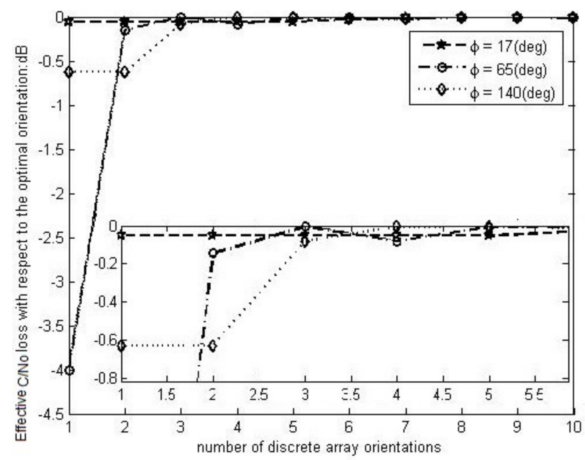

Fig. 6. Effective C/N_0 loss versus the number of orientations

simulations, we set the signal-to-noise ratio to $-20dB$ and interference-to-noise ratio to $10dB$. Let the satellite signal and interference have the same azimuth angle $\phi_s = \phi_j = 30°$, but different elevation angles $\theta_s = 90°$ and $\theta_j = 30°$ respectively. Furthermore we assume that, in the ideal case, the linear array can be rotated around its central element continuously from $0°$ to $180°$. We get four optimal orientations $\varphi = 7°, 52°, 92°, 147°$ using Eq. (18). In these cases, the satellite signal and interference are orthogonal with each other, and the smallest SCC (zero) and highest C/N_0 result. When the array has an orientation along the direction $\varphi = 120°$, we get the maximum SCC and minimum C/N_0. These results are confirmed by the simulation results shown in Fig. 4.

We can clearly see that the effective C/N_0 has an inverse relationship with the SCC in agreement with Eq. (11). The corresponding beampatterns of different orientations using the optimal array weight vector Eq. (6) are shown in Fig. 5. We can conclude that the linear array loses its function completely when it is rotated along the $\varphi = 120°$ direction and gives the best beampattern when it is placed along $\varphi = 52°$ direction.

An important fact to note is that the C/N_0 curve is nearly flat when SCC is within the range between 0.1 and 0.5, which occurs over a wide range of angles. This implies that finely positioning the array is not necessary. Therefore instead of continuously rotating the array, a more practical strategy is to discretely position the array by switching various antennas in and out of the receiver front end to choose the most suitable orientation under the specific scenario. Looking at such a strategy here, let there be M different orientations at equal angle intervals (each interval being $(180/M)°$). The SINR loss of the discrete case with respect to the optimal orientation is shown in Fig. 6 against M. We can see that $M = 4$ uniformly distributed orientations are enough to guarantee SINR loss is less than $0.1dB$. Therefore, using switching antennas in and out of the receiver front end to choose the most suitable orientation is an acceptable and more practical substitute for the continuous method.

5. CONCLUSION

In this paper, the effect of different orientations of linear array on the performance of adaptive array processing is investigated and a new reconfigurable adaptive linear array method is proposed to overcome the existing weaknesses of conventional array processing technique based on the fixed array structure. Adaptively changing the orientation of the linear array can separate the desired signal and interference in space as much as possible and thus the effective C/N_0 can be improved. Simulation results also show that using switching method to reconfigure the linear array to the suitable orientation discretely is a good substitute to the continuous one.

6. RELATION TO PRIOR WORK

In this work we proposed a novel reconfigurable adaptive linear array method which combines array processing and adaptively changing the array orientation together. Most of the prior work either focused on developing adaptive beamforming and filtering technique [4, 5, 6] to suppress the interference under the predetermined array configuration or does not utilise adaptive array algorithm [13, 14, 15] to suppress interference adaptively. Therefore the distinguishing aspect of this proposal is in adopting a strategy of adaptively reconfiguring both the antenna array configuration as well as setting the weights of the adaptive filter in order to maximize the interference suppression.

7. REFERENCES

[1] H.C. Lin, "Spatial correlations in adaptive arrays," *Antennas and Propagation, IEEE Transactions on*, vol. 30, no. 2, pp. 212–223, 1982.

[2] K. Yang, N. Zhang, and T. Ohira, "Array configuration design for space-time adaptive processing systems," in *Sensor Array and Multichannel Signal Processing Workshop Proceedings, 2002*. IEEE, 2002, pp. 145–148.

[3] K. Yang, Y. Zhang, and Y. Mizuguchi, "Space-time adaptive processing based on unequally spaced antenna arrays," in *Vehicular Technology Conference Proceedings, 2000. VTC 2000-Spring Tokyo. 2000 IEEE 51st*. IEEE, 2000, vol. 2, pp. 1220–1224.

[4] R.L. Fante and JJ Vaccaro, "Wideband cancellation of interference in a gps receive array," *Aerospace and Electronic Systems, IEEE Transactions on*, vol. 36, no. 2, pp. 549–564, 2000.

[5] M. Li, A.G. Dempster, A.T. Balaei, C. Rizos, and F. Wang, "Switchable beam steering/null steering algorithm for cw interference mitigation in gps c/a code receivers," *Aerospace and Electronic Systems, IEEE Transactions on*, vol. 47, no. 3, pp. 1564–1579, 2011.

[6] JR Guerci, JS Goldstein, and IS Reed, "Optimal and adaptive reduced-rank stap," *Aerospace and Electronic Systems, IEEE Transactions on*, vol. 36, no. 2, pp. 647–663, 2000.

[7] H. Van Trees, *Optimum array processing*, vol. 7, Wiley Online Library, 2002.

[8] J.W. Betz, "Effect of partial-band interference on receiver estimation of c/n0: Theory," Tech. Rep., DTIC Document, 2001.

[9] A.T. Balaei and E. Aboutanios, "Characterization of interference effects in multiple antenna gnss receivers," in *Image and Signal Processing (CISP), 2010 3rd International Congress on*. IEEE, 2010, vol. 8, pp. 3930–3934.

[10] R. Klemm, *Principles of space-time adaptive processing*, Number 159. Inspec/Iee, 2002.

[11] F. Liu, J. Wang, C. Sun, and R. Du, "Spatial differencing method for doa estimation under the coexistence of both uncorrelated and coherent signals," *Antennas and Propagation, IEEE Transactions on*, , no. 99, pp. 1–1, 2011.

[12] E. Aboutanios and B. Mulgrew, "Iterative frequency estimation by interpolation on fourier coefficients," *Signal Processing, IEEE Transactions on*, vol. 53, no. 4, pp. 1237–1242, 2005.

[13] S.E. Nai, W. Ser, Z.L. Yu, and H. Chen, "Beampattern synthesis for linear and planar arrays with antenna selection by convex optimization," *Antennas and Propagation, IEEE Transactions on*, vol. 58, no. 12, pp. 3923–3930, 2010.

[14] Y. Liu, Z. Nie, and Q.H. Liu, "Reducing the number of elements in a linear antenna array by the matrix pencil method," *Antennas and Propagation, IEEE Transactions on*, vol. 56, no. 9, pp. 2955–2962, 2008.

[15] L. Cen, W. Ser, Z.L. Yu, S. Rahardja, and W. Cen, "Linear sparse array synthesis with minimum number of sensors," *Antennas and Propagation, IEEE Transactions on*, vol. 58, no. 3, pp. 720–726, 2010.