JOINT TRANSMIT ARRAY INTERPOLATION AND TRANSMIT BEAMFORMING FOR SOURCE LOCALIZATION IN MIMO RADAR WITH ARBITRARY ARRAYS

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ABSTRACT

We consider a MIMO radar with arbitrary multi-dimensional array, and propose a method for transmit array interpolation that maps an arbitrary transmit array into an array with a certain desired structure. A properly designed interpolation matrix is used to jointly achieve transmit array interpolation and design transmit beamforming. The transmit array interpolation problem is cast as a convex optimization problem based on minmax criterion. Our designs enable to control the sidelobe levels of the transmit beampattern and enforce different transmit beams to have rotational invariance with respect to each other, a property that enables the use of computationally efficient direction finding techniques. It is shown that the rotational invariance can be achieved independently in both the elevation and the azimuth spatial domains, allowing for independent elevation and azimuth direction finding.

Index Terms— Arbitrary arrays, array interpolation, direction finding, MIMO radar, rotational invariance property.

1. INTRODUCTION

Multiple-input multiple-output (MIMO) radar has been recently the focus of intensive research [1]–[6]. It has been shown that MIMO radar with collocated transmit antennas suffers from the loss of coherent transmit processing gain as a result of omnidirectional transmission of orthogonal waveforms [6]. The concepts of phased-MIMO radar and transmit energy focussing have been developed to address the latter problem [6], [7]. Other transmit beamforming approaches have been also developed [8]–[13], but all of them address the case of one dimensional (1D) transmit array. Despite the great practical interest in two dimensional (2D) transmit arrays [14], the fact that the performance of MIMO radar with less number of waveforms than the number of transmit antennas and with transmit processing gain is better than the performance of MIMO radar with full waveform diversity and with no transmit beamforming gain [7] becomes more evident in

the case when the transmit array contains a large number of antennas, e.g., 2D transmit arrays.

In this paper, we consider a MIMO radar with arbitrary multi-dimensional arrays and develop a method for transmit array interpolation that maps an arbitrary transmit array into an array with a certain desired structure, e.g., a uniform rectangular array or an array with two perpendicular uniform linear arrays. A properly designed interpolation matrix is used to jointly achieve transmit array interpolation and design transmit beamforming. The transmit array interpolation problem is cast as a convex optimization problem based on the minmax criterion. Such formulation is flexible and enables applying constraints on the transmit power distribution across the array elements, controlling the sidelobe levels of the transmit beampattern, and enforcing different transmit beams to have rotational invariance with respect to each other, a property that enables efficient computationally cheap 2D direction finding at the receiver. The rotational invariance is achieved independently in both the elevation and the azimuth spatial domains, allowing for independent elevation and azimuth direction finding at the receiver using simple 1D techniques.

2. SIGNAL MODEL

Consider a mono-static MIMO radar with transmit and receive arrays of M and N elements, respectively. Both the transmit and receive arrays are assumed to be planar arrays with arbitrary geometries. In a Cartesian two-dimensional space, the transmit antennas are assumed to be located at $\mathbf{p}_m \triangleq [x_m \ y_m]^T$, $m = 1, \ldots, M$ where $(\cdot)^T$ stands for the transpose operator. The antenna locations are measured in wavelength. The $M \times 1$ steering vector of the transmit array is defined as

$$\mathbf{a}(\theta, \phi) = \left[e^{-j2\pi\boldsymbol{\mu}^T(\theta,\phi)\mathbf{p}_1}, \ldots, e^{-j2\pi\boldsymbol{\mu}^T(\theta,\phi)\mathbf{p}_M}\right]^T \quad (1)$$

where θ and ϕ denote the elevation and azimuth spatial angles, respectively, and $\boldsymbol{\mu}(\theta, \phi) = [\sin\theta\cos\phi \ \sin\theta\sin\phi]^T$ denotes the propagation vector.

Let $\mathbf{C} = [\mathbf{c}_1, \ldots, \mathbf{c}_{\tilde{M}}]$ be the $M \times \tilde{M}$ interpolation matrix, where $\tilde{M} \leq M$ is the number of elements in the desired interpolated array and $\mathbf{c}_m$ is the mth column of $\mathbf{C}$. The relationship between the actual and the interpolated transmit arrays is given by

$$\mathbf{C}^H \mathbf{a}(\theta, \phi) \simeq \tilde{\mathbf{a}}(\theta, \phi) \qquad \theta \in \Theta, \; \phi \in \Phi \qquad (2)$$

where $\tilde{\mathbf{a}}(\theta, \phi)$ is the $\tilde{M} \times 1$ steering vector associated with the desired array, Θ and Φ are predefined sectors in the elevation and azimuth domains, respectively, and $(\cdot)^H$ stands for the Hermitian transpose. Let $\mathbf{s}(t) = [s_1(t), \ldots, s_{\tilde{M}}(t)]$ be the $\tilde{M} \times 1$ vector of predesigned independent waveforms which satisfy the orthogonality condition $\int_T \mathbf{s}(t)\mathbf{s}^H(t) = \mathbf{I}_{\tilde{M}}$ where T is the radar pulse duration and $\mathbf{I}_{\tilde{M}}$ is the identity matrix of size $\tilde{M}$. Each of the orthogonal waveforms is radiated via one element of the interpolated array. Therefore, the signal radiated towards a hypothetical spatial location (θ, ϕ) is given by

$$s(t, \theta, \phi) = \tilde{\mathbf{a}}^T(\theta, \phi)\mathbf{s}(t) = \sum_{i=1}^{\tilde{M}} \left(\mathbf{c}_i^H \mathbf{a}(\theta, \phi) \right) s_i(t). \quad (3)$$

It can be observed from (3) that the radiation pattern of the power of the ith orthogonal waveform $s_i(t)$ is given by $|\mathbf{c}_i^H \mathbf{a}(\theta, \phi)|^2$. Therefore, the vector $\mathbf{c}_i$ can be used to achieve a desired transit beampattern. In other words, the interpolation matrix $\mathbf{C}$ can be properly designed to jointly achieve transmit array interpolation and transmit beamforming.

Assuming that L targets are present in the far-field of the array, the $N \times 1$ receive array observation vector can be written as

$$\mathbf{x}(t, \tau) = \sum_{l=1}^{L} \beta_l(\tau) \left(\tilde{\mathbf{a}}^T(\theta_l, \phi_l)\mathbf{s}(t) \right) \mathbf{b}(\theta_l, \phi_l) + \mathbf{z}(t, \tau) \quad (4)$$

where t and τ are the fast and slow time indexes respectively, $\mathbf{b}(\theta, \phi)$ is the $N \times 1$ steering vector of the receive array, $\beta_l(\tau)$ is the reflection coefficient associated with the lth target with variance σ_β^2, and $\mathbf{z}(t, \tau)$ is the $N \times 1$ vector of zero-mean white Gaussian noise. We assume that the reflection coefficients obey the Swerling II target model, i.e., they remain constant within the whole duration of the radar pulse but change from pulse to pulse. The receive array observation vector $\mathbf{x}(t, \tau)$ is matched-filtered to each of the orthogonal basis waveforms $\mathbf{s}_i(t), i = 1, \ldots, \tilde{M}$, producing the $N \times 1$ virtual data vectors

$$\begin{aligned}
\mathbf{y}_i(\tau) &= \int_T \mathbf{x}(t, \tau)\mathbf{s}_i^*(t)dt \\
&= \sum_{l=1}^{L} \beta_l(\tau) \left(\mathbf{c}_i^H \mathbf{a}(\theta_l, \phi_l) \right) \mathbf{b}(\theta_l, \phi_l) + \mathbf{z}_i(\tau) \quad (5)
\end{aligned}$$

where $\mathbf{z}_i(\tau) \triangleq \int_T \mathbf{z}(t, \tau)\mathbf{s}_i^*(t)dt$ is the $N \times 1$ noise term whose covariance is $\sigma_z^2 \mathbf{I}_N$. Note that $\mathbf{z}_i(\tau)$ and $\mathbf{z}_{i'}(\tau)$ $(i \neq i')$

are independent due to the orthogonality between $\mathbf{s}_i(t)$ and $\mathbf{s}_{i'}(t)$.

It can be observed from (5) that the amplitude of signal component associated with the lth target in $\mathbf{y}_i(\tau)$, $i = 1, \ldots, \tilde{M}$ is amplified by factor $\mathbf{c}_i^H \mathbf{a}(\theta_l, \phi_l)$. In the following section, we propose a method for designing the interpolation matrix $\mathbf{C}$ and show how to jointly achieve transmit array interpolation and transmit beamforming. We also show how to enforce the rotational invariance property at the transmit side of the MIMO radar.

3. TRANSMIT ARRAY INTERPOLATION

Let $\theta_k \in \Theta, k = 1, \ldots, K_\theta$ be the angular grid chosen (uniform or nonuniform) which properly approximates the desired elevation sector Θ by a finite number K_θ of directions. Similarly, let $\phi_k \in \Phi, k = 1, \ldots, K_\phi$ be the angular grid chosen (uniform or nonuniform) which properly approximates the desired azimuth sector Φ by a finite number K_ϕ of directions. The interpolation matrix $\mathbf{C}$ can be computed as the least squares (LS) solution to

$$\mathbf{C}^H \mathbf{A} = \tilde{\mathbf{A}} \qquad (6)$$

where the $M \times K_\theta K_\phi$ and the $\tilde{M} \times K_\theta K_\phi$ matrices $\mathbf{A}$ and $\tilde{\mathbf{A}}$ are, respectively, defined as follows

$$\mathbf{A} = [\mathbf{a}(\theta_1, \phi_1), \ldots, \mathbf{a}(\theta_{K_\Theta}, \phi_1), \ldots, \mathbf{a}(\theta_{K_\Theta}, \phi_{K_\phi})] \quad (7)$$

$$\tilde{\mathbf{A}} = [\tilde{\mathbf{a}}(\theta_1, \phi_1), \ldots, \tilde{\mathbf{a}}(\theta_{K_\Theta}, \phi_1), \ldots, \tilde{\mathbf{a}}(\theta_{K_\Theta}, \phi_{K_\phi})]. \quad (8)$$

Given that $K_\theta K_\phi \geq M$, the LS solution to (6) can be given as [15]

$$\mathbf{C} = \left(\mathbf{A} \mathbf{A}^H \right)^{-1} \mathbf{A} \tilde{\mathbf{A}}^H. \qquad (9)$$

Unfortunately, the LS solution (9) does not enable controlling the sidelobe levels of the transmit beampattern. In fact, the sidelobe levels in this case can be higher than the in-sector levels. This may result in wasting most of the transmit power in the out-of-sector areas which can lead to severe degradation in the MIMO radar performance.

To incorporate robustness against sidelobe levels, we propose to use the minmax criterion to minimize the difference between the interpolated array steering vector and the desired one while keeping the sidelobe level bounded by some constant. Therefore, the interpolation matrix design problem can be formulated as the following optimization problem

$$\min_{\mathbf{C}} \max_{\theta_k, \phi_{k'}} \left\| \mathbf{C}^H \mathbf{a}(\theta_k, \phi_{k'}) - \tilde{\mathbf{a}}(\theta_k, \phi_{k'}) \right\| \qquad (10)$$

$$\theta_k \in \Theta, \; k = 1, \ldots, K_\theta, \quad \phi_{k'} \in \Phi, \; k' = 1, \ldots, K_\phi$$

$$\text{subject to } \left\| \mathbf{C}^H \mathbf{a}(\theta_n, \phi_{n'}) \right\| \leq \gamma, \qquad (11)$$

$$\theta_n \in \bar{\Theta}, \; n = 1, \ldots, N_\theta, \quad \phi_{n'} \in \bar{\Phi}, \; n' = 1, \ldots, N_\phi$$

where $\bar{\Theta}$ and $\bar{\Phi}$ are the out-of-sector regions in the elevation and azimuth domains, respectively, $\theta_n \in \bar{\Theta}, \; n = 1, \ldots, N_\theta$

and $\phi_{n'} \in \bar{\Phi}$, $n' = 1, \ldots, N_\phi$ are angular grids used to approximate $\bar{\Theta}$ and $\bar{\Phi}$, respectively, and γ is a positive number of user choice used to upper-bound the sidelobe level. The optimization problem (10)–(11) is convex and can be efficiently solved using interior-point methods. Choosing $\gamma \geq \gamma_{min}$, the problem (10)–(11) is guaranteed to have a feasible and unique solution (for discussions on how to find γ_{min}, see [16], [17]). Note that the interpolation achieved by (10)–(11) is performed only in a certain spatial sector. The accuracy of such approximation depends on the size of sector.

Alternatively, it is possible to minimize the worst-case out-of-sector sidelobe level while upper-bounding the norm of the difference between the interpolated array steering vector and the desired one. This can be formulated as the following optimization problem

$$\min_{\mathbf{C}} \max_{\theta_n, \phi_{n'}} \left\| \mathbf{C}^H \mathbf{a}(\theta_n, \phi_{n'}) \right\| \tag{12}$$

$$\theta_n \in \bar{\Theta}, \ n = 1, \ldots, N_\theta, \quad \phi_{n'} \in \bar{\Phi}, \ n' = 1, \ldots, N_\phi$$

$$\text{subject to} \left\| \mathbf{C}^H \mathbf{a}(\theta_k, \phi_{k'}) - \tilde{\mathbf{a}}(\theta_k, \phi_{k'}) \right\| \leq \Delta \tag{13}$$

$$\theta_k \in \Theta, \ k = 1, \ldots, K_\theta, \quad \phi_{k'} \in \Phi, \ k' = 1, \ldots, K_\phi$$

where Δ is a positive number of user choice used to control the deviation of the interpolated array from the desired one.

ESPRIT-based DOA Estimation: The interpolation matrix design formulations given in (10)–(11) and (12)–(13) can be used to achieve any desired planar array geometry. Here, we choose the desired array to be two perpendicular linear subarrays of two elements each; one located along the x-axis while the other is located along the y-axis. The desired locations of the elements of the first subarray are $[\tilde{x}_1, \ 0]^T$ and $[\tilde{x}_2, \ 0]^T$ while the desired locations of the elements of the second subarray are $[0, \ \tilde{y}_1]^T$ and $[0, \ \tilde{y}_2]^T$, where $\tilde{x}_1, \tilde{x}_2, \tilde{y}_1$, and $\tilde{y}_2$ are measured in wavelength. We also choose $\tilde{\mathbf{a}}(\theta, \phi)$ to take the following format

$$\tilde{\mathbf{a}}(\theta, \phi) = \begin{bmatrix} e^{-j2\pi\tilde{x}_1 \sin \theta} \\ e^{-j2\pi\tilde{x}_2 \sin \theta} \\ e^{-j2\pi\tilde{y}_1 \sin \phi} \\ e^{-j2\pi\tilde{y}_2 \sin \phi} \end{bmatrix}, \quad \theta \in \Theta, \ \phi \in \Phi. \tag{14}$$

It is worth noting that the array response of the subarray located along the x-axis is chosen to be dependant on the elevation angle θ and independent on the azimuth angle ϕ. Moreover, the array response of the subarray located along the y-axis is chosen to be dependant on the the azimuth angle ϕ and independent on elevation angle θ. This specific choice enables us to apply ESPRIT at the receive side to the data associated with the first subarray to estimate the elevation directions of the targets. Also, it enables applying the ESPRIT to the data associated with the second subarray to estimate the azimuth directions. In other words, it enables estimating the elevation and azimuth angles independently using simple search-free direction finding techniques.

Solving (10)–(11) or (12)–(13) for $\mathbf{C}$, we obtain

$$\mathbf{C}^H \mathbf{a}(\theta, \phi) \approx \tilde{\mathbf{a}}(\theta, \phi), \quad \theta \in \Theta, \ \phi \in \Phi. \tag{15}$$

Substituting (14) and (15) in (5), we obtain

$$\mathbf{y}_1(\tau) \approx \sum_{l=1}^{L} \beta_l(\tau) e^{-j2\pi\tilde{x}_1 \sin \theta_l} \mathbf{b}(\theta_l, \phi_l) + \mathbf{z}_1(\tau) \tag{16}$$

$$\mathbf{y}_2(\tau) \approx \sum_{l=1}^{L} \beta_l(\tau) e^{-j2\pi\tilde{x}_2 \sin \theta_l} \mathbf{b}(\theta_l, \phi_l) + \mathbf{z}_2(\tau) \tag{17}$$

$$\mathbf{y}_3(\tau) \approx \sum_{l=1}^{L} \beta_l(\tau) e^{-j2\pi\tilde{y}_1 \sin \phi_l} \mathbf{b}(\theta_l, \phi_l) + \mathbf{z}_3(\tau) \tag{18}$$

$$\mathbf{y}_4(\tau) \approx \sum_{l=1}^{L} \beta_l(\tau) e^{-j2\pi\tilde{y}_2 \sin \phi_l} \mathbf{b}(\theta_l, \phi_l) + \mathbf{z}_4(\tau) \tag{19}$$

Inspecting (16) and (17), it can be observed that $\mathbf{y}_1(\tau)$ and $\mathbf{y}_2(\tau)$ are related to each other through rotational invariance. The rotational invariance associated with the lth target is given by $\psi_l = 2\pi \sin \theta_l(\tilde{x}_2 - \tilde{x}_1)$. Therefore, the ESPRIT algorithm can be used to estimate the phase rotations ψ_l, $l = 1, \ldots, L$ from $\mathbf{y}_1(\tau)$ and $\mathbf{y}_2(\tau)$. Then, the elevation angles θ_l, $l = 1, \ldots, L$ can be obtained from ψ_l, $l = 1, \ldots, L$. Similarly, it can be observed from (18) and (19) that $\mathbf{y}_3(\tau)$ and $\mathbf{y}_4(\tau)$ are related to each other through rotational invariance. The rotational invariance associated with the lth target is given by $\varphi_l = 2\pi \sin \theta_l(\tilde{y}_2 - \tilde{y}_1)$. Therefore, the ESPRIT algorithm can be used to estimate the phase rotation associated with each target. Then, the azimuth angles ϕ_l, $l = 1, \ldots, L$ can be obtained from the estimated phase rotations φ_l, $l = 1, \ldots, L$.

It is worth noting that for the case $L > 1$ an extra step is need to match the estimated elevation and azimuth angles to each other. One simple way to achieve that is to use ESPRIT to obtain the rotational invariance between $\mathbf{y}_1(\tau)$ and $\mathbf{y}_3(\tau)$ and to use it to match the elevation and azimuth estimates.

4. SIMULATION RESULTS

In our simulations, we assume a transmit array of 64 elements and a receive array of 16 elements. The desired sector is defined by $\Theta = [30°, \ 40°]$ and $\Phi = [95°, \ 105°]$. We allow for a transition zone of width $10°$ at each side of the mainlobe in the elevation domain and $20°$ at each side of the mainlobe in the azimuth domain. The remaining areas of the elevation and azimuth domains are assumed to be a stopband region. The desired interpolated array is assumed to consist of 4 elements as given in (14) with $\tilde{x}_1 = \tilde{y}_1 = \lambda/2$ and $\tilde{x}_2 = \tilde{y}_2 = \lambda$, where λ is the wavelength.

In the first example we assume that the transmit array is a non-uniform rectangular array of size 8×8 where the x-position of each row and the y-position of each column is chosen randomly from the set $[0 \ 4\lambda]$. The interpolation matrix $\mathbf{C}$ is designed using (12)–(13) where $\Delta = 0.1$ is used. To solve the problem (12)–(13) we used CVX, a package for specifying and solving convex programs [18]. The normalized overall beampattern is shown in Fig. 1. It can be observed
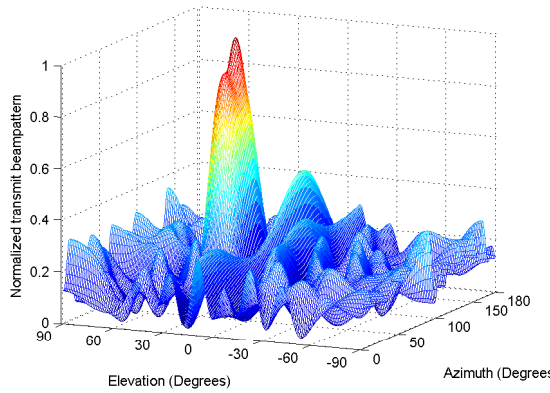
Fig. 1. Normalized transmit beampattern using an 8×8 non-uniform actual transmit array.

Fig. 2. Left: Phase rotation between first and second elements of the interpolated arrays. Right: Phase rotation between third and forth elements of the interpolated arrays

from the figure that the transmit power is concentrated in the desired sector. The left side of Fig. 2 shows the phase rotation between the first and second elements of the interpolated array while the right side of the same figure shows the rotational invariance between the third and fourth elements of the interpolated array. It can be seen from the figure that the phase rotation between the first and second elements varies versus the elevation angle and remains constant versus the azimuth angle. It can also be seen that the phase rotation between the third and forth elements of the interpolated array varies versus the azimuth and remains constant versus the elevation angle.

In the second example, we assume that two targets in the far-field are located at $[33°, 98°]$ and $[37°, 101°]$, respectively. We use the interpolation matrix $\mathbf{C}$ obtained from the first example to radiate four orthogonal waveforms. The total transmit energy is fixed to M. The arbitrary geometry of the receive array is chosen by selecting the x- and y-components of the locations of all elements randomly from the set $[0\ 2\lambda]$. The noise term is chosen to be white-Gaussian with unit variance. The ESPRIT algorithm is used to estimate the elevation and azimuth angles of the targets. The root mean-square error (RMSE) of the estimated angles versus the signal-to-noise ratio (SNR) is shown in Fig. 3. The Cramer-Rao bound (CRB) is computed numerically and used as a benchmark for comparison of the estimation as shown in Fig. 3. It can be observed from that figure that the proposed method offers excellent DOA estimation performance at medium and high SNR regions. The RMSE saturates at low SNR regions because it is limited by the width of the desired sector.

5. CONCLUSIONS

The problem of MIMO radar with arbitrary multi-dimensional arrays is considered. A method for transmit array interpolation that maps the arbitrary transmit array into an array with
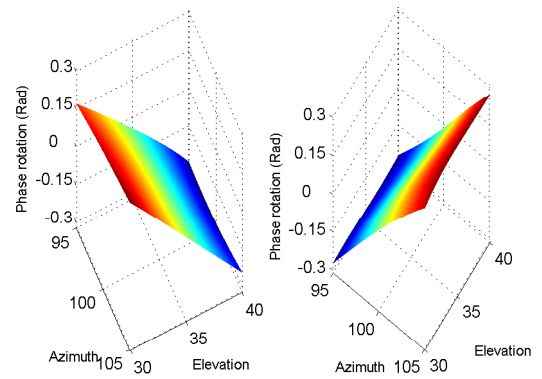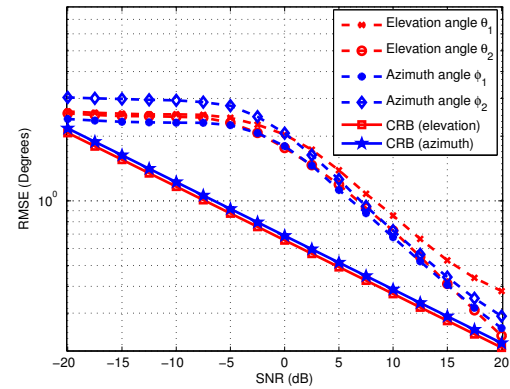

Fig. 3. RMSE versus SNR.

a certain desired structure has been proposed. A properly designed interpolation matrix is used to jointly achieve transmit array interpolation and transmit beamforming. The transmit array interpolation problem has been cast as an optimization problem that can be solved using the minmax criterion. It enables controlling the sidelobe levels of the transmit beampattern, and enforcing different transmit beams to have rotational invariance with respect to each other, a property that enables the use of computationally efficient direction finding techniques. Moreover, it has been shown that the rotational invariance can be achieved independently in both the elevation and the azimuth spatial domains, allowing for independent elevation and azimuth direction finding using simple 1D DOA estimation techniques. It has been shown that the formulated optimization problem is convex and can be solved efficiently using interior point optimization methods.

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