THE SEPARATING FUNCTION ESTIMATION TEST AND THE UMPI TEST FOR TARGET DETECTION PROBLEM USING OPTIMAL SIGNAL DESIGN IN MIMO RADAR

A. Ghobadzadeh†, M. R. Taban†, A. Tadaion†, S. Gazor**

* Department of Electrical and Computer Engineering
Queen's University, Ontario, Canada
† Department of Electrical and Computer Engineering
Yazd University, Yazd, Iran

ABSTRACT

In this paper, we study the MIMO signal detection problem using widely separated antennas in Gaussian interference. The interference is assumed to be colored with unknown $N \times N$ covariance matrix. We derive the Uniformly Most Powerful Invariant (UMPI) test for this detection problem as the upper performance bound for invariant tests. Also the Separating Function Estimation Test (SFET) is derived for this problem using the Signal and Scatter to Interference Ratio (SIR). Then, based on the eigenvalues expansion of SIR, we propose a set of orthogonal signals in transmitters which maximizes the detection probability of UMPI and SFET. Simulation results show that 1) the performance of our proposed detector is close to the UMPI bound and 2) the performance of the optimal invariant bound improves when the transmitters use the proposed set of signals instead of the orthogonal complex exponential signals.

Index Terms— MIMO signal detection, Gaussian interference, UMPI, eigenvalues expansion.

1. INTRODUCTION

Multiple-Input Multiple-Output (MIMO) signal processing has various applications involving radar, wireless communication, and cognitive radio [1, 2]. MIMO radar is a type of multi-static radars, that employs multiple transmit antennas with different waveforms and jointly processes signals for detection of target and estimation of its parameters [2]. In general, the MIMO radar systems are classified into the Widely Separated Antennas (WSA) and Co-Located Antennas (CLA) [2, 3]. In WSA MIMO radar, the antennas are spatially separated at the transmitters and the receivers, such that the target signals received at the receivers are independent [2–4]. The focus of this paper is on the WSA MIMO radar target detection and signal design.

According to the Neyman-Pearson criterion, the optimal test provides the maximum probability of detection P_{d} while the false alarm detection P_{fa} is limited by a given value. In many signal detection problems with unknown parameters, this test may not exist [5]. Hence, in such problems, proposing a suboptimal test is very important. In this paper, we attempt to find the Uniformly Most Powerful Invariant (UMPI) test and Separating Function Estimation Test (SFET) which are asymptotically optimal. The UMPI test is the optimal test between invariant tests [7]. The UMPI test is given by constructing the likelihood ratio of the maximal invariant statistic [5]. Unfortunately, the UMPI test cannot often be derived in many practical problems; instead it can provide an upper performance bound for the invariant tests such as the Generalized Likelihood Ratio Test (GLRT) and SFET (for example see [5, 6]).

In [4], a model for the received signals in the WSA MIMO radar, based on the scattering effects of the electromagnetic signals, is proposed. A moving target detection problem in Gaussian interference with unknown variance is investigated in [2]. The authors of [8] develop a GLRT for moving target detection in a nonhomogeneous clutter. It is shown that this GLRT is also a constant false alarm rate (CFAR) detector. In [9], based on the expectation maximization algorithm, an estimate of correlationamong the data from different transmitter and receiver pairs is derived. Then the authors use these estimators to develop a test for target detection. In [10] two detectors based on the Rao and Wald criterions are derived for target detection problem in clutter with known covariance matrix. The authors of [11] present a detector based on time reversal method, and design transmitted signal to improve the performance of the test. In [12] the authors have proposed a GLRT for target detection with unknown covariance matrix and channel coefficients to improve the detection probability of the GLRT ; a method for sampling of received signal is also presented.

In this paper, we derive the UMPI test for target detection problem in clutter with unknown covariance matrix using WSA MIMO radar. It is shown that the UMPI test depends on the scatter and signal to interference ratio (SIR). Hence, for a given SIR, this test provides the Most Powerful Invariant (MPI) bound. Since in this problem the UMPI test does not exist, we utilize the SFET using SIR as a suboptimal invariant test. Based on the eigenvalues expansion of SIR, a set of signals is proposed such that maximizes the MPI bound.

Consider a MIMO radar with K transmit and L receive antennas. It is shown that the received signal at the l^{th} receiver from a target located at the i^{th} cell, is

$$r_l^i(n) = \sqrt{\frac{E_s}{K}} \sum_{k=1}^{K} h_{lk}^i s_k(nT_s) + w_l^i(nT_s),$$

where $n = 0, \cdots, N-1$, $w_l^i(nT_s)$ is Gaussian interference term, E_s is the total energy of transmit signals, $s_k(nT_s)$ denotes the transmitted signal from the k^{th} transmitter, and h_{lk}^i denotes the channel coefficients. The samples of the interference are zero mean Gaussian with unknown covariance matrix Σ_{ww} [4]. It is shown that h_{lk}^i's are identically independent distribution (i.i.d.) Gaussian random variables which depend on the field scattering [4].

Assuming $\mathbf{r}_l^i \triangleq [r_l^i(0), \cdots, r_l^i(N-1)]^T$ and $\mathbf{s}_k \triangleq [s_k(0), \cdots, s_k(N-1)]^T$, the observation $\mathbf{r}_l^i$ is a zero mean, Gaussian random vector with covariance matrix $\Sigma_l^i = \sigma^2 \mathbf{R}_{ss} + \Sigma_{ww}$, where Σ_{ww} is the interference covariance matrix, σ^2 is variance of h_{lk}^i, $\mathbf{R}_{ss} = \sum_{k=1}^{K} \mathbf{s}_k \mathbf{s}_k^H$, where the superscript ^{H} denotes the Hermitian operator. Hence, we can rewrite the MIMO target detection problem using C cells by the following hypothesis test:

$$\begin{cases} \mathcal{H}_0 : \mathbf{r}_l^i \sim \mathcal{N}(\mathbf{0}, \Sigma_{ww}), \\ \mathcal{H}_1 : \begin{cases} \mathbf{r}_l^i \sim \mathcal{N}(\mathbf{0}, \sigma^2 \mathbf{R}_{ss} + \Sigma_{ww}), i = c, \\ \mathbf{r}_l^i \sim \mathcal{N}(\mathbf{0}, \Sigma_{ww}), i \neq c, \end{cases} \end{cases}, \quad (1)$$

where $l = 1, \cdots, L, i = 1, \cdots, C$, and c is the number of cell under test (CUT).

The rest of the paper is organized as follows. In Section 2 and 3, we derive the UMPI test and the SFET respectively. Section 4 develops a set of transmitted signals such that it maximizes the MPI bound. Section 5 concludes the paper.

2. UMPI TEST

The decision problem remains unchanged by the set of transformations $G_{qu} = \{g_{qu} : g_{qu}(\mathbf{r}_l^i) = \mathbf{Q}\mathbf{r}_l^i\}$ and $G_s = \{g_s : g_s(\mathbf{r}_l^i) = c\mathbf{r}_l^i, c \neq 0\}$, where the matrix $\mathbf{Q}$ is a $N \times N$ quasi-unitary matrix with respect to $\mathbf{R}_{ss}$. This matrix must satisfy $\mathbf{Q}\mathbf{R}_{ss}\mathbf{Q}^H = \mathbf{R}_{ss}$. Since G_{qu} and G_s are two linear transformation groups, the distribution of their each element is Gaussian and also the induced parameter transformation groups maintain the parameters space under each hypothesis as follows. Note that the induced group of transformations remained the unknown parameters space under each hypothesis. The induced parameter transformation groups under each hypothesis are written as follows

$$\begin{aligned} \overline{G}_{qu}|_{\mathcal{H}_1} &= \{\overline{g}_{qu}|\overline{g}_{qu,1}([\sigma^2, \Sigma_{ww}]) = [\sigma^2, \mathbf{Q}\Sigma_{ww}\mathbf{Q}^H]\}, \\ \overline{G}_{qu}|_{\mathcal{H}_0} &= \{\overline{g}_{qu}|\overline{g}_{qu,0}([0, \Sigma_{ww}]) = [0, \mathbf{Q}\Sigma_{ww}\mathbf{Q}^H]\}, \\ \overline{G}_s|_{\mathcal{H}_1} &= \{\overline{g}_s|\overline{g}_{s,1}([\sigma^2, \Sigma_{ww}]) = [|c|^2\sigma^2, |c|^2\Sigma_{ww}]\}, \\ \overline{G}_s|_{\mathcal{H}_0} &= \{\overline{g}_s|\overline{g}_{s,0}([0, \Sigma_{ww}]) = [0, |c|^2\Sigma_{ww}]\}. \end{aligned}$$

So the hypothesis is invariant under G which is given by the combination of two groups G_{qu} and G_s. To derive a MPI bound, we must determine a maximal invariant statistic for G. Hence first we must determine a maximal invariant statistic for G_{qu}. A maximal invariant statistic for G_{qu} is given by $m_{qu}(\mathbf{r}_l) = \| \mathbf{R}_{ss}^{-1/2}\mathbf{r}_l \|^2$. Using the composition lemma [7, ch.6 th.2], the maximal invariant statistic with respect to the composition of two groups G_{qu} and G_s for the i^{th} cell, is given by $\mathbf{m}^i = \left[\frac{\|R_{ss}^{-1/2}\mathbf{r}_1^i\|^2}{\|R_{ss}^{-1/2}\mathbf{r}_L^i\|^2}, \cdots, \frac{\|R_{ss}^{-1/2}\mathbf{r}_{L-1}^i\|^2}{\|R_{ss}^{-1/2}\mathbf{r}_L^i\|^2}\right]^T$, and then the maximal invariant statistic for G is given by $\mathbf{m}^T = [\mathbf{m}^{1^T}, \mathbf{m}^{2^T}, \cdots, \mathbf{m}^{C^T}]^T$. Since $\mathbf{r}_l^i$'s are independent, the probability density function (pdf) of $\mathbf{m}$ is directly given by the pdf of $\mathbf{m}^i$'s. It can be shown that the pdf of $\mathbf{m}^i$ under each hypothesis is given by

$$f_{\mathbf{m}^i}(\mathbf{m}^i|\mathcal{H}_\zeta) = \Gamma(L) \sum_{\kappa_1=1}^{N} \cdots \sum_{\kappa_L=1}^{N} \left(\frac{B_{\kappa_1,\xi} \cdots B_{\kappa_L,\xi}}{\lambda_{\kappa_1,\xi} \cdots \lambda_{\kappa_L,\xi}}\right) \times$$
$$\times \left(\frac{m_1^i}{\lambda_{\kappa_1,i}} + \cdots + \frac{m_{L-1}^i}{\lambda_{\kappa_{L-1},i}} + \frac{1}{\lambda_{\kappa_L,i}}\right)^{-L}, \quad (2)$$

where m_l^i is the l^{th} element of $\mathbf{m}^i$, $\zeta = 0, 1$ and $\xi = 1$ if $\zeta = 1$ and $i = c$ and $\xi = 0$ for the other cases. $\Gamma(\cdot)$ is the Gamma function, $\lambda_{\kappa,\xi}$ is the κ^{th} eigenvalue of matrix $\mathbf{R}_{ss}^{-1/2}(\xi\sigma^2\mathbf{R}_{ss} + \Sigma_{ww})\mathbf{R}_{ss}^{-1/2}$ and $B_{\kappa,\xi} = \prod_{p=1,p\neq\kappa}^{N} \frac{1}{1-\frac{\lambda_{p,\xi}}{\lambda_{\kappa,i}}}$. The UMPI test statistic is obtained by constructing the likelihood ratio of $\mathbf{m}$. Constructing the likelihood ratio of maximal invariant, the UMPI test rejects $\mathcal{H}_0$ if the condition (2) is satisfied. In this equation η_{UMPI} is set to P_{fa} requirement. Note that $\lambda_{\kappa,1} = \sigma^2 + \lambda_{\kappa,0}$, $\rho_\kappa = \frac{\lambda_{\kappa,0}}{\sigma^2}$ and $D_\kappa = \prod_{j=1,j\neq\kappa}^{N} \frac{1}{\rho_\kappa - \rho_j}$. In fact, ρ_κ deals with the ratio of interference in the κ^{th} dimension of interference to scatter and signal effects. Since the UMPI test given by (2), depends on ρ_κ's, the UMPI test does not exist. Of course, by assuming ρ_κ to be known, the decision rule in (2) gives an MPI bound for evaluating the other invariant detectors.

3. SFET BASED ON SIR

According to the results in the previous subsection, the SIR vector for this problem is given by $\text{SIR} = [1/\rho_1, \cdots, 1/\rho_N]^T = \left[\frac{\sigma^2}{\mathcal{P}_1(\mathbf{R}_s^{-1/2}\Sigma_{ww}\mathbf{R}_s^{-1/2})}, \cdots, \frac{\sigma^2}{\mathcal{P}_N(\mathbf{R}_s^{-1/2}\Sigma_{ww}\mathbf{R}_s^{-1/2})}\right]^T$, where

$$\frac{\sum_{\kappa_1=1}^{N}\cdots\sum_{\kappa_L=1}^{N}\left((\rho_{\kappa_1}+1)^{N-1}D_{\kappa_1}\cdots(\rho_{\kappa_L}+1)^{N-1}D_{\kappa_L}\right)\left(\frac{\|R_{ss}^{-1/2}\mathbf{r}_1^c\|^2}{(\rho_{\kappa_1}+1)\|R_{ss}^{-1/2}\mathbf{r}_L^c\|^2}+\cdots+\frac{\|R_{ss}^{-1/2}\mathbf{r}_{L-1}^c\|^2}{(\rho_{\kappa_{L-1}}+1)\|R_{ss}^{-1/2}\mathbf{r}_L^c\|^2}+\frac{1}{(\rho_{\kappa_L}+1)}\right)^{-L}}{\sum_{\kappa_1=1}^{N}\cdots\sum_{\kappa_L=1}^{N}\left(\rho_{\kappa_1}^{N-1}D_{\kappa_1}\cdots\rho_{\kappa_L}^{N-1}D_{\kappa_L}\right)\left(\frac{\|R_{ss}^{-1/2}\mathbf{r}_1^c\|^2}{\rho_{\kappa_1}\|R_{ss}^{-1/2}\mathbf{r}_L^c\|^2}+\cdots+\frac{\|R_{ss}^{-1/2}\mathbf{r}_1^c\|^2}{\rho_{\kappa_{L-1}}\|R_{ss}^{-1/2}\mathbf{r}_L^c\|^2}+\frac{1}{\rho_{\kappa_L}}\right)^{-L}} > \eta_{\text{UMPI}}$$

$\mathcal{P}_n(\cdot)$ is the n^{th} eigenvalue of $(\cdot)$. We define the total SIR by $\text{SIR}_{\text{tot}} \triangleq \sum_{n=1}^{N} \frac{1}{\rho_n}$. This function of unknown parameters is zero when the parameters belong to $\mathcal{H}_0$ and is positive if they belong to $\mathcal{H}_1$; hence this function is a Separating Function (SF) for this problem [13]. An SFET is given by comparing the estimate of SF by a threshold [13], hence the SFET rejects $\mathcal{H}_0$ if $\widehat{\text{SIR}_{\text{tot}}} > \eta_{\text{SFET}}$, where η_{SFET} is set to false alarm requirement. In the following, we provide an estimation of the unknown parameters to derive the SFET statistic.

Consider $\Sigma'_{ww} \triangleq \frac{\Sigma_{ww}}{\sigma^2}$, then the pdf of observation under the union of $\mathcal{H}_0$ and $\mathcal{H}_1$ is given by

$$f(\mathbf{r}^c;\sigma^2,\Sigma'_{ww}) = \frac{\exp\left(\frac{-1}{\sigma^2}\sum_{l=1}^{L}\mathbf{r}_l^{cH}(\mathbf{R}_{ss}+\Sigma'_{ww})^{-1}\mathbf{r}_l^c\right)}{\pi^{NL}\sigma^{2NL}\det(\mathbf{R}_{ss}+\Sigma'_{ww})}. \quad (3)$$

Solving $\frac{\partial f(\mathbf{r}^c;\sigma^2,\Sigma'_{ww})}{\partial\sigma^2} = 0$ and $\frac{\partial f(\mathbf{r}^c;\sigma^2,\Sigma'_{ww})}{\partial\Sigma'_{ww}} = \mathbf{0}$, we have

$$\widehat{\Sigma'_{ww}} = \frac{N}{L}\sum_{l=1}^{L}\frac{\mathbf{r}_l^c\mathbf{r}_l^{cH}}{\mathbf{r}_l^{cH}(\mathbf{R}_{ss}+\widehat{\Sigma'_{ww}})^{-1}\mathbf{r}_l^c} - \mathbf{R}_{ss}, \quad (4)$$

$$\widehat{\sigma^2} = \frac{1}{NL}\sum_{l=1}^{L}\frac{\mathbf{r}_l^c\mathbf{r}_l^{cH}}{\mathbf{r}_l^{cH}(\mathbf{R}_{ss}+\Sigma'_{ww})^{-1}\mathbf{r}_l^c}, \quad (5)$$

We cannot find $\widehat{\Sigma'_{ww}}$ and $\widehat{\sigma^2}$ directly, so we propose a recursive method to find a proper solving for (4). Consider the following recursive method for Σ'_{ww} and σ^2 by

$$\widehat{\Sigma'_{ww}}^{(k+1)} = \frac{N}{L}\sum_{l=1}^{L}\frac{\mathbf{r}_l^c\mathbf{r}_l^{cH}}{\mathbf{r}_l^{cH}(\mathbf{R}_{ss}+\widehat{\Sigma'_{ww}}^{(k)})^{-1}\mathbf{r}_l^c} - \mathbf{R}_{ss}, \quad (6)$$

$$\widehat{\sigma^2}^{(k)} = \frac{1}{NL}\sum_{l=1}^{L}\frac{\mathbf{r}_l^c\mathbf{r}_l^{cH}}{\mathbf{r}_l^{cH}(\mathbf{R}_{ss}+\Sigma'^{(k)}_{ww})^{-1}\mathbf{r}_l^c} \quad (7)$$

where, $\widehat{\Sigma'_{ww}}^{(0)} = \mathbf{I}$. Hence, the statistic of SFET is given by replacing the estimations of $\Sigma_{ww} = \sigma^2\Sigma'_{ww}$ and σ^2 into the SF.

4. OPTIMAL SIGNAL DESIGN

We define $A \triangleq R_{ss}^{-1/2}\Sigma_{ww}R_{ss}^{-1/2}/\sigma^2$, the eigenvalues of A provide the SIR vector. So the total SIR is given by

$$\text{SIR}_{\text{tot}} = \frac{1}{\sigma^2}\text{trace}\{\mathbf{R}_{ss}^{1/2}\Sigma_{ww}^{-1}\mathbf{R}_{ss}^{1/2}\} = \frac{1}{\sigma^2}\text{trace}\{\mathbf{R}_{ss}\Sigma_{ww}^{-1}\}$$

We assume that the transmitting energy of each transmitter is limited to a constant E. Hence, we can describe an optimization problem for signal design as bellow:

$$\max_{\mathbf{s}_1,\cdots,\mathbf{s}_K} \text{SIR}_{\text{tot}} \ \text{,s.t.,}\ \mathbf{s}_{k'}^H\mathbf{s}_k = \begin{cases} 0 \ , & k' \neq k, \\ E \ , & k' = k. \end{cases} \quad (8)$$

Based on this maximization problem, the desired signals maximize the total SIR and then maximize the MPI bound. A typical strategy for solving this problem is based on the Lagrange coefficients. Thus, we must maximize

$$\begin{aligned} L(\mathrm{s}_k) &= \text{SIR}_{\text{tot}} + \sum_{k=1}^{K}\mu_k(\mathbf{s}_k^H\mathbf{s}_k - E) \\ &= \frac{1}{\sigma^2}\text{trace}\{\mathbf{R}_{ss}\Sigma_{ww}^{-1}\} + \sum_{k=1}^{K}\mu_k(\mathbf{s}_k^H\mathbf{s}_k - E) \end{aligned}$$

where μ_k is the k^{th} coefficient of Lagrange method. So the optimal signals will be given by solving $\frac{\partial L(\mathbf{s}_k)}{\partial\mathbf{s}_k} = 0$, i.e.,

$$\begin{aligned} &\frac{\partial L(\mathbf{s}_k)}{\partial\mathbf{s}_k} = \frac{1}{\sigma^2}\frac{\partial\text{trace}\{\mathbf{R}_{ss}\Sigma_{ww}^{-1}\}}{\partial\mathbf{s}_k} + 2\mu_k\mathbf{s}_k = \mathbf{0} \\ &\Rightarrow \frac{1}{\sigma^2}2\Sigma_{ww}^{-1}\mathbf{s}_k + 2\mu_k\mathbf{s}_k = \mathbf{0}. \end{aligned} \quad (9)$$

According to the last equation, $\Sigma_{ww}^{-1}\mathbf{s}_k = -\sigma^2\mu_k\mathbf{s}_k$, that means $\mathbf{s}_k$ must be a eigenvector of Σ_{ww}^{-1} i.e., $\mathbf{s}_k = \sqrt{E}\mathbf{u}_k$, where $\mathbf{u}_k$ is the k^{th} eigenvector of Σ_{ww}^{-1}. Note that this set of signals is also orthogonal. Now consider the total SIR by

$$\begin{aligned} \text{SIR}_{\text{tot}} &= \frac{1}{\sigma^2}\text{trace}\{\mathbf{R}_{ss}\Sigma_{ww}^{-1}\} = \frac{E}{\sigma^2}\sum_{k=1}^{K}\mathbf{u}_k^H\Sigma_{ww}^{-1}\mathbf{u}_k \\ &= \frac{E}{\sigma^2}\sum_{k=1}^{K}\delta_k, \end{aligned} \quad (10)$$

where δ_k's are the eigenvalues of Σ_{ww}^{-1}. Therefore by choosing the first K δ_k (from maximum to minimum of eigenvalues of Σ_{ww}^{-1}) and their corresponding $\mathbf{u}_k$'s, the optimal set of signals for the MIMO radar is derived such that it maximizes the invariant detection bound. Note that the covariance matrix is an unknown matrix and so it must be estimated by reference cells ($i \neq c$), $\widehat{\Sigma}_{ww} = \frac{1}{C-1}\sum_{i=1,r\neq c}^{C}\mathbf{r}_r^i\mathbf{r}_r^{i\,H}$.

In the following we simulate the performance of the tests when the transmitted signals are complex exponential and

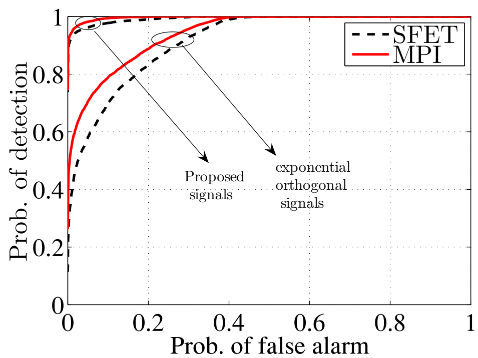

Fig. 1. P_d v.s. P_fa curves of SFET in comparison with MPI bound for orthogonal narrow bound signals and proposed set of signals.

compare the results when the proposed set of signals is used. In the first simulation, we consider $s_k(n) = e^{j\omega_k n}$ for $n = 0, \cdots, N-1$ and $k = 1, \cdots, K$, where $\omega_k = (k-1)\Delta\omega$ and $\Delta\omega$ is the two by two increment between the transmitter carriers, which assumed $\Delta\omega = 2\pi/K$ and in another experience consider the same MIMO system with the proposed set of signals. In this simulation we assume that Σ_{ww} is unknown, therefore we have calculated $\widehat{\Sigma}_{ww}$ using 120 reference cells, in derivation of the proposed set of signal. In all simulations, we consider $K = N = 10$ and $L = 15$. Fig.1 depicts P_d versus P_fa curves for the MPI test and the SFET. It is seen that the set of proposed signals has a superior performance in comparison with the orthogonal narrow band MIMO radar.

5. CONCLUSION

In this paper, we derived an MPI bound for the WSA MIMO radar target detection. Based on the eigenvalues expansion and MPI derivation, we extended the Signal and Scatter to Interference Ratio for a WSA MIMO radar problem and gave a set of signals such that it maximizes the upper performance bound for invariant tests. Furthermore, we derived an invariant SFET detector based on a recursive estimation of SIR. The simulation results showed that the performance of SFET is close to the MPI bound and it significantly improves when we use the proposed transmitted signals.

6. REFERENCES

[1] G. Scutari, D. P. Palomar, and S. Barbarossa,"Cognitive MIMO Radio", *IEEE Signal Processing Magazine*, Nov., pp. 46-59 2008.

[2] A. M. Haimovich, R. S. Blum and L.J. Cimin, "MIMO radar with widely separated antennas", *IEEE Signal Processing magazine*, vol. 25, pp. 116-129, Jan. 2008.

[3] J. Li and P. Stoica, "MIMO radar with colocated antennas", *IEEE Signal Processing magazine*, vol. 24, no. 5, pp. 106-114, Sep. 2007.

[4] E. Fishler, A. M. Haimovich, R. S. Blum, L. J. Cimini, D. Chizhik and R.A. Valenzuela, "Spatial diversity in radars-models and detection performance", *Signal Processing, IEEE Transactions on,* vol. 54, no. 3, pp. 823-838, Mar. 2006.

[5] S. M. Kay and J.R. Gabriel, "Optimal Invariant Detection of a Sinusoid With Unknown Parameters", *Signal Processing, IEEE Transactions on*, vol. 50, no. 1, pp. 27-40, Jan. 2002.

[6] A. A. Tadaion, M. Derakhtian, S. Gazor, M. M. Nayebi, and M. R. Aref, "Signal Activity Detection of Phase-Shift Keying Signals," *Communications, IEEE Transactions on* , vol.54, no.6, pp.1143, June 2006.

[7] E. L. Lehmann, *Testing Statistical Hypothesis*, Wiley, New York, 1986. S. M. Kay, *Fundamentals of Statistical Signal Processing: vol. I, Estimation Theory*, Upper Saddle River, NJ: Prentice-Hall PTR, 1998.

[8] W. Pu, L. Hongbin, and B. Himed, "Moving Target Detection Using Distributed MIMO Radar in Clutter With Nonhomogeneous Power," *Signal Processing, IEEE Transactions on* , vol.59, no.10, pp.4809-4820, Oct. 2011.

[9] M. Akcakaya, and A. Nehorai, "MIMO Radar Sensitivity Analysis for Target Detection," *Signal Processing, IEEE Transactions on* , vol.59, no.7, pp.3241-3250, Jul. 2011.

[10] N. Li, G. Cui, L. Kong, and X. Yang, "Rao and Wald tests design of multiple-input multiple-output radar in compound-Gaussian clutter," *Radar, Sonar and Navigation, IET* , vol.6, no.8. 2012

[11] D. Chaoran, J. S. Thompson, and Y. R. Petillot, "Detector and Waveform Design for MIMO Radar System with Noisy Channel Estimation," *Aerospace and Electronic Systems, IEEE Transactions on* , vol.48, no.3, pp.2332-2348, Jul. 2012.

[12] A. Ghobadzadeh, A. A. Tadaion, and M. R. Taban, "GLR approach for MIMO radar signal sampling in unknown clutter parameter," *Telecommunications, 2008. IST 2008.* International Symposium on , pp.624-628, 27-28 Aug. 2008.

[13] A. Ghobadzadeh, S. Gazor, M. R. Taban, A. A. Tadaion, and M. Gazor, "Separating Function Estimation Tests: A New Perspective on Binary Composite Hypothesis Testing," *Signal Processing, IEEE Transactions on* , vol.60, no.11, pp. 5626-5639, Nov. 2012.