IMPROVED MIMO RADAR CHANNEL ESTIMATION USING SPATIAL CODING

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ABSTRACT

A spatial coding of the transmitted waveforms in a distributed MIMO radar system is proposed for reducing the impact of noise and interference on the channel matrix estimate. The channel matrix is needed in target parameter estimation as well as transmitter resource allocation and target recognition, for example. It is shown that when noise or interference are correlated after filtering at the receiver, it is possible decrease the error of the channel coefficient estimates by using the proposed coding method. Numerical results shown here demonstrate the benefits of the method in practical scenarios.

***Index Terms*—** MIMO radar, spatial coding, channel estimation, interference suppression

1. INTRODUCTION

In a distributed MIMO radar system, multiple transmitters and receivers are positioned over a wide area so that targets can be seen simultaneously from several different angles[1]. Estimating the coefficients of scattering for a target is a necessary step in several key tasks of MIMO radar including target velocity estimation with pulse-Doppler radar[2], beamforming for a distributed MIMO radar[3, 4], and resource allocation for target tracking[4] as well as optimizing the use of the waveform diversity in the radar transmitter. The scattering coefficient estimation is equal to estimating the instantaneous channel matrix of the radar system after factoring in the transmit power and the propagation losses. Naturally, the estimation accuracy is limited by the signal power and interference plus noise power at the receiver. Furthermore, it also depends on how fast the RCS varies in time compared to the integration time.

Typically, noise and interference cancellation methods rely on having two correlated measurements of the noise and interferecene one of which is signal-free[5]. In a distributed MIMO radar system, obtaining a signal-free measurement requires turning the transmitter off. Unless the interference has a very long correlation time, the measurements obtained while not transmitting are no longer useful for noise and interference cancellation after the transmitter has been turned on again. On the other hand, temporal filtering methods relying on the second-order statistics of the noise plus interference also affect the received waveforms. Therefore, we propose a spatial coding method to mitigate the noise and interference based on their second-order statistics. If the second-order statistics do not change considerably after the noise and interference measurements have been obtained while not transmitting, it is shown that the proposed spatial coding approach can be used to reduce the estimation error of the channel matrix, and consequently, improve the performance in target parameter estimation.

Space-time coding for MIMO radar has been previously proposed in [6] and [7], but for the purpose of obtaining unitary waveforms in a colocated MIMO radar. Space-time coding was used in [8] to reduce the waveform cross-correlation. Altough this reduces the channel coefficient error, it does not mitigate the interference. In [9], space-time coding was used to reduce clutter in colocated MIMO radar. The space-time coding was also studied in [10–13] in the context of optimizing the number of linearly independent transmitted waveforms for detection and ranging. The interference mitigation method proposed in this paper is a novel application of spatial coding.

This paper is organized as follows: Section 2 discusses the signal model. The coding method for improved channel estimation is presented in Section 3, and numerical results are provided in Section 4. Finally, conclusions are drawn in Section 5.

2. SIGNAL MODEL

We consider a distributed MIMO radar system with M transmitters and N receivers. The signal transmitted by the m-th transmitter and received by the n-th receiver can be written in baseband as

$$\begin{aligned} r_{nm}(t) = \sqrt{P_{nm}} c_{nm} s_m(t - \tau_{t,m} - \tau_{r,n}) \\ \times e^{j2\pi f_{nm} t} e^{-j2\pi f_c(\tau_{t,m} + \tau_{r,n})} + \nu_n(t), \end{aligned} \tag{1}$$

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where P_{nm} is a power parameter, c_{nm} the scattering coefficient, s_m the signal transmitted by the m-th transmitter, $\tau_{t,m}$ the time delay from the transmitter to the target, $\tau_{r,n}$ the time delay from the the target to the receiver, f_{nm} the Doppler shift, f_c the carrier frequency, and ν_n is the noise and interference term. The power parameter P_{nm} depends on the transmit power, propagation losses, and the antenna gain at the receiver. Any unknown oscillator phase terms can be included into c_{nm}.

We assume that the transmitted waveforms s_m have sufficiently low cross-correlation and autocorrelation sidelobe values so that the delays $\tau_{t,m}$ and $\tau_{r,n}$ as well as the Doppler shifts f_{nm} can be estimated and compensated for so that the received signal can be written in a $N \times 1$ vector

$$\mathbf{r}(t) = \mathbf{H}\mathbf{s}(t) + \tilde{\boldsymbol{\nu}}(t), \tag{2}$$

where $\mathbf{H}$ is the $N \times M$ channel matrix and $\tilde{\boldsymbol{\nu}}(t)$ is the frequency-shifted and time-shifted interference plus noise vector. Performing matched filtering, the signal becomes

$$\int \mathbf{r}(t)\mathbf{s}^H(t)dt \approx \mathbf{H} + \mathbf{V}, \tag{3}$$

where $\mathbf{V} = \int \tilde{\boldsymbol{\nu}}(t)\mathbf{s}^H(t)dt$ is the filtered noise plus interference matrix. It was assumed that the waveforms are approximately orthogonal. It is shown in Section 4 that the waveforms do not have to be perfectly orthogonal in order to use the proposed coding method and consequently reduce the channel estimation error.

3. IMPROVED CHANNEL ESTIMATION

It can be seen in (3) that the matched filter output provides an estimate of the $N \times M$ channel matrix corrupted by filtered noise and interference, i.e.

$$\hat{\mathbf{H}} = \int \mathbf{r}(t)\mathbf{s}^H(t). \tag{4}$$

The mean-square error of the channel matrix estimate is

$$\mathrm{E}\big[\|\hat{\mathbf{H}} - \mathbf{H}\|_F^2\big] \approx \mathrm{E}\big[\|\mathbf{V}\|_F^2\big], \tag{5}$$

where $\|\cdot\|_F$ denotes the Frobenius norm.

If temporal filtering was used to mitigate the interference before the matched filtering, the channel matrix estimate would also be affected. Therefore, we apply instantaneus spatial precoding $\mathbf{W}$ to the transmitted waveforms so that the transmitted signal is $\mathbf{W}\mathbf{s}(t)$. The matched filter output is multiplied from the left by $\mathbf{W}^{-1}$ to decode the signal at the receiver side. Thus, the coding matrix has to be of full rank. Averaging or any other temporal filtering operations that are done after the matched filtering can be also used with this coding approach.

Assuming approximate orthogonality of the transmitted waveforms, the channel matrix estimate becomes

$$\begin{aligned}\hat{\mathbf{H}} &= \mathbf{H}\mathbf{W}\int \mathbf{s}(t)\mathbf{s}^H(t)dt\mathbf{W}^{-1} + \int \tilde{\boldsymbol{\nu}}(t)\mathbf{s}^H(t)dt\mathbf{W}^{-1} \\ &\approx \mathbf{H} + \mathbf{V}\mathbf{W}^{-1}\end{aligned} \tag{6}$$

after the decoding. Thus, we may define an error criterion

$$\mathcal{E}(\mathbf{W}) = \mathrm{E}\big[\|\mathbf{V}\mathbf{W}^{-1}\|_F^2\big] \tag{7}$$

and search for a coding matrix $\mathbf{W}$ that would minimize this error.

Using the definition of $\mathbf{V}$, one obtains

$$\begin{aligned}&\mathcal{E}(\mathbf{W}) \\ &= \mathrm{tr}\Big(\mathbf{W}^{-H}\mathrm{E}[\mathbf{V}^H\mathbf{V}]\mathbf{W}^{-1}\Big) \\ &= \mathrm{tr}\Big(\mathbf{W}^{-H}\mathrm{E}\Big[\int \mathbf{s}(t)\tilde{\boldsymbol{\nu}}^H(t)dt\int \tilde{\boldsymbol{\nu}}(t')\mathbf{s}^H(t')dt'\Big]\mathbf{W}^{-1}\Big) \\ &= \mathrm{tr}\Big(\mathbf{W}^{-H}\int\int \mathbf{s}(t)\mathrm{E}\big[\tilde{\boldsymbol{\nu}}^H(t)\tilde{\boldsymbol{\nu}}(t')\big]\mathbf{s}^H(t')dtdt'\mathbf{W}^{-1}\Big) \\ &= \mathrm{tr}\big(\mathbf{W}^{-H}\mathbf{A}\mathbf{W}^{-1}\big)\end{aligned} \tag{8}$$

where tr denotes the trace of a matrix. In practice, one would need to estimate the matrix $\mathbf{A}$ as

$$\hat{\mathbf{A}} = \frac{1}{N_s}\sum_{n=1}^{N_s}\int \mathbf{s}(t)\tilde{\boldsymbol{\nu}}_n^H(t)dt\int \tilde{\boldsymbol{\nu}}_n(t')\mathbf{s}^H(t')dt', \tag{9}$$

where N_s is the number of noise plus interference sample sequences available.

In order not to change the total transmit power, we need to place a constraint on $\|\mathbf{W}\|_F^2 = \mathrm{tr}\big(\mathbf{W}^H\mathbf{W}\big)$. Without loss of generality, we assume the total power to be equal to the number of transmitters M. Thus, we need to solve a constrained optimization problem

$$\min_{\mathbf{W}} \mathrm{tr}\big(\mathbf{W}^{-H}\mathbf{A}\mathbf{W}^{-1}\big) \text{ s.t. } \mathrm{tr}\big(\mathbf{W}^H\mathbf{W}\big) = M. \tag{10}$$

This problem can be solved using the method of Lagrange multipliers. The Lagrangian of the problem can be written as

$$\begin{aligned}&L(\mathbf{W}, \lambda) \\ &= \mathrm{tr}\big(\mathbf{W}^{-H}\mathbf{A}\mathbf{W}^{-1}\big) - \lambda\Big[\mathrm{tr}\big(\mathbf{W}^{-H}\mathbf{W}^{-1}\big) - M\Big].\end{aligned} \tag{11}$$

In order to obtain a critical point, it can be shown that this function can be differentiated with respect to $\mathbf{W}^*$ while treating $\mathbf{W}^*$ and $\mathbf{W}$ as independent variables to get the gradient[14]. The result is

$$\frac{\partial L}{\partial \mathbf{W}^*} = -\mathbf{W}^{-H}(\mathbf{A}\mathbf{W}^{-1})\mathbf{W}^{-H} + \lambda\mathbf{W}. \tag{12}$$

Setting this equal to zero and multiplying from the left by $\mathbf{W}^{-1}$ results in

$$\mathbf{W}^{-1}\mathbf{W}^{-H}\mathbf{A}\mathbf{W}^{-1}\mathbf{W}^{-H} = \lambda\mathbf{I}, \tag{13}$$

which is a form of a bi-quadratic matrix equation. Assuming that λ is nonzero, we can substitute $\mathbf{Q} = \lambda^{-1/2}\mathbf{W}^{-1}\mathbf{W}^{-H}$ to obtain

$$\mathbf{Q}\mathbf{A}\mathbf{Q} = \mathbf{I}. \tag{14}$$

Since $\mathbf{A}$ is Hermitian by definition, $\mathbf{Q}$ can be found easily using eigenvalue decomposition. Letting the eigenvalue decomposition be $\mathbf{A} = \mathbf{U}\mathbf{D}\mathbf{U}^H$, we get

$$\mathbf{Q} = \mathbf{U}\mathbf{D}^{-1/2}\mathbf{U}^H \tag{15}$$

and then the optimal precoding matrix is given by

$$\mathbf{W}_o = (\lambda^{1/2}\mathbf{Q})^{-1/2} = \lambda^{-1/4}\mathbf{U}\mathbf{D}^{1/4}\mathbf{U}^H \tag{16}$$

Substituting the result into the constraint equation yields

$$\lambda = \left[\frac{1}{M}\mathrm{tr}\big(\mathbf{D}^{1/2}\big)\right]^2. \tag{17}$$

The solution is thus valid when $\mathbf{A}$ is invertible.

We need to still show that the critical point of the Lagrangian actually reduces the error, i.e. $\mathrm{tr}\big(\mathbf{W}_o^{-H}\mathbf{A}\mathbf{W}_o^{-1}\big) \leq \mathrm{tr}\big(\mathbf{A}\big)$, which can be simplified to

$$\frac{1}{M}\left[\mathrm{tr}\big(\mathbf{D}^{1/2}\big)\right]^2 \leq \mathrm{tr}\big(\mathbf{D}\big). \tag{18}$$

Using Cauchy-Schwarz inequality, one obtains

$$\Big(\sum_{m=1}^{M} 1\, d_{mm}^{1/2}\Big)^2 \leq \Big(\sum_{m=1}^{M} 1\Big)\Big(\sum_{m=1}^{M} d_{mm}\Big), \tag{19}$$

where d_{mm} is the mth element on the diagonal of $\mathbf{D}$. This proves (18).

The channel estimation method then proceeds as follows:

Algorithm	
1.	Obtain noise and interference only measurements $\boldsymbol{\nu}$ while transmitters are off
2.	Detect a target and estimate time delays $\tau_{t,m}$ $\tau_{r,n}$ and the Doppler shifts f_{nm}
3.	Use the previously obtained measurements to form $\tilde{\boldsymbol{\nu}}$, then $\mathbf{V}$ and finally an estimate of $\mathbf{A}$ using (9)
4.	Transmit with the optimal precoding $\mathbf{W}_o$ in (16) and then estimate the channel matrix using (6)

It should be noted that the error criterion can be easily generalized for multiple targets as

$$\begin{aligned}\mathcal{E}(\mathbf{W}) &= \mathrm{E}\Big[\|\sum_k \mathbf{V}_k\mathbf{W}^{-1}\|_F^2\Big] \\ &= \mathrm{tr}\Big(\mathbf{W}^{-H}\sum_k \mathrm{E}[\mathbf{V}_k^H\mathbf{V}_k]\mathbf{W}^{-1}\Big).\end{aligned} \tag{20}$$

Table 1. Difference to the baseline SNR in dB for each Tx–Rx branch in the example.

	Tx 1	Tx 2	Tx 3
Rx 1	0	2.5458	-0.0819
Rx 2	-6.3071	-0.3703	6.4488
Rx 3	-4.0808	-5.6728	-0.2927
Rx 4	-0.7801	8.1298	6.9045
Rx 5	1.2571	8.1779	5.9750

The filtered noise and interference $\mathbf{V}_k$ depends on the target as the shifts needed to get the signal in the form of (2) can be different depending on the target.

4. NUMERICAL EXAMPLES

A numerical example demonstrating the use of the proposed spatial coding scheme to reduce the channel matrix estimation error is shown in this section. This example uses the signal model for the matched filter output developed in Section 2 the spatial coding matrix derived in Section 3 in estimating the channel matrix of a single target.

In this example, a widely distributed MIMO radar systems consists of three transmitters located at $(0, 0)$, $(500, 500)$, and $(700, 2000)$, and five receivers at $(-500, 159)$, $(0, 0)$, $(620, 0)$, $(700, 2998)$, and $(707, 1450)$. The target is at $(-2000, 3000)$ and moving at a velocity $(20, -20)$.

All the transmitters use 0.5 GHz carrier frequency. The polyphase sequences proposed in [15] were used as the waveforms. The first three sequences of the set with parameters $m = 1$, $n = 3$, and $p = 1$ as defined in [15] and lenght of 61 were used. These sequences have a normalized peak cross-correlation of 0.1765 and autocorrelation peak sidelobe of 0.1721. The sequences were transmitted at 10^6 chips per second.

Path losses for the transmitted waveforms were equal to the free-space propagation loss. The scattering coefficients were assumed to be circular complex Gaussians held constant throughout the estimation process. Receiver noise was assumed to be spatially and temporally white i.i.d. complex Gaussian, but since the waveforms are not perfectly orthogonal, the noise is correlated after the matched filtering. The Tx 1 – Rx 1 pair was chosen as the baseline SNR level. Differences to this baseline level for each received signal are shown in Table 1.

First, 100 samples of interference plus noise were observed. These were then used to estimate the matrix $\mathbf{A}$ for the target whose position and Doppler shifts were assumed to have been correctly estimated. The channel matrix $\mathbf{H}$ was then estimated and the normalized MSE defined as

$$\mathrm{NMSE} = \frac{1}{K}\sum_{k=1}^{K}\frac{\|\hat{\mathbf{H}}_k - \mathbf{H}\|_F^2}{\|\mathbf{H}\|_F^2} \tag{21}$$

Table 2. The normalized MSE of the channel matrix estimation for different baseline SNR values with noise only. The proposed coding method decreases the estimation error in each case.

Method \ SNR	0 dB	5 dB	15 dB
Normal	0.0236	0.0190	0.0171
Coded	0.0178	0.0132	0.0112

was averaged over 1000 independent runs. The reason for using the normalized MSE is that it depends on SNR but not the actual power levels. For example, if both the signal power plus noise and iterference powers are doubled, the MSE quadruples while the NMSE remains unchanged. It is also easily seen that the coding matrix $\mathbf{W}_o$ that is optimal with respect to the MSE channel estimation error results also in the lowest NMSE.

The results for different SNR values are shown in Table 2. It can be seen that the proposed spatial coding decreases the normalized MSE channel matrix estimate at all the tested SNR values.

The performance of the coding scheme was then tested with a jammer present at $(-922, 2216.6)$. The jamming signal was assumed to be colored Gaussian noise with a autocorrelation function $a(k) = 8 - |k|$, $k = -8, -7, \ldots, 7, 8$. Taking the path loss into account, the power of the jamming signal compared to the jammer to noise ratio (JNR) baseline at each receiver was 0, -1.1606, -2.1818, 1.3387, and 1.3389 dB. The JNR was then varied form -10 dB to 15 dB to test the performance of the proposed method under different levels of jamming. Moreover, the normal transmission mode and the proposed spatial coding scheme were compared to a theoretical case where the covariance of the noise plus interference remains unchanged but the used waveforms are orthogonal so the MSE is given by $\mathrm{tr}(\mathbf{W}^{-H}\mathbf{A}\mathbf{W}^{-1})$. The coding matrix $\mathbf{W}$ was formed using the estimate $\hat{\mathbf{A}}$ also in this case.

The NMSE of the channel coefficient estimates in the presence of jamming are shown in Fig.1. It can be seen that the spatial coding method reduces the estimation error even when there is a strong interfering signal present. However, the estimation error is not as low as in the theoretical case of orthogonal waveforms. The difference in performance is especially significant for high SNR regime. This shows that the cross-correlation of the used waveforms deteriorates the channel matrix estimation substantially.

The decrease in the normalized MSE as the JNR increases towards zero is likely caused by estimation error in $\mathbf{A}$. When the jamming signal is slightly weaker than noise, it is difficult to estimate the eigenvalues and eigenvectors of $\hat{\mathbf{A}}$ correctly, but the jamming is still strong enough to cause error in the estimation. When the JNR is increased, the NMSE initially decreases as the eigenvalues and eigenvectors are less affected by noise.

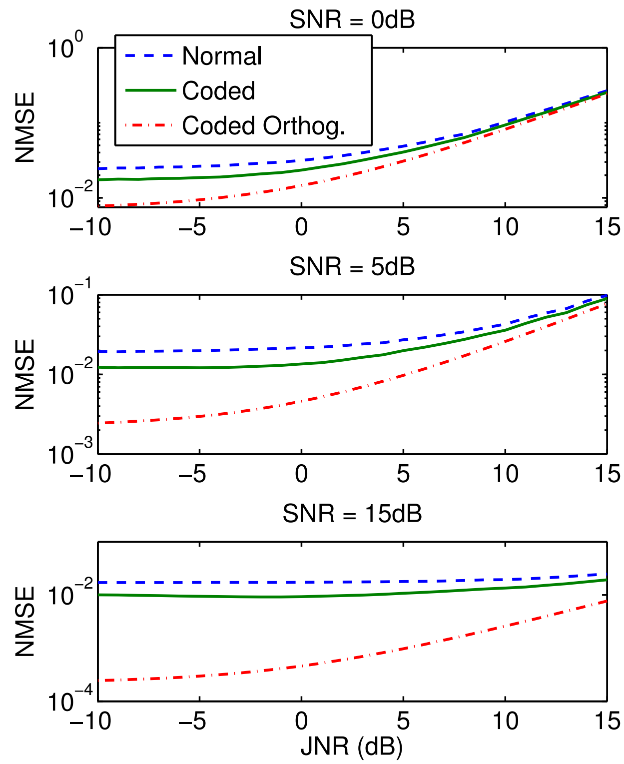

Fig. 1. Normalized mean square error of the channel matrix estimate at different JNR levels. Normal stands for direct transmission of the waveforms and coded means the proposed spatial coding method, whereas coded orthogonal corresponds to the theoretical case of orthogonal waveforms. The proposed method can improve the estimate also in the presence of jamming. The estimation error would be much lower with orthogonal waveforms, so a large part of the estimation error is caused by the waveform cross-correlation.

5. CONCLUSIONS

Estimation of the channel matrix is a necessary task in many applications of ditributed MIMO radar, including target parameter estimation, transmitter resource allocation and target recognition. We have proposed a spatial precoding scheme that reduces the channel matrix estimation error when noise plus interference is correlated after matched filtering. The correlation may be the property of the interference itself or result of using matched filtering of waveforms not exactly orthogonal. The optimal precoding matrix can be obtained using second order statistics of the interference plus noise and eigenvalue decomposition making the proposed approach computationally efficient. The proposed method was seen in the examples to reduce the channel estimation error for a wide range of interference power. However, the cross-correlation of the transmitted waveforms limits the performance significantly. Channel matrix estimation methods for correlated waveforms in should be developed in future work.

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