HIGH RESOLUTION MIMO RADAR WITH UNITARY WAVEFORM MATRIX SCHEDULING

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ABSTRACT

In this paper, we present a method of detecting the range and Doppler phase of a point target using multiple antennas. As a key illustrative example, we consider a 4×4 system employing a unitary matrix waveform set, e.g., formed from Golay complementary sequences. When a non-negligible Doppler shift is induced by the target motion, the waveform matrix formed from the complementary sequences is no longer unitary, resulting in significantly degraded target range estimates. To solve this problem, a novel Doppler estimation and compensation scheme based on a clever application of DFT is developed that provides notable improvements, both in detection performance, and processing times. Proof-of-concept simulations are presented verifying the efficacy of the proposed Doppler compensation and estimation technique for our unitary waveform matrix designs.

1. INTRODUCTION

In [1], Howard et al. proposed a new multi-channel radar scheme employing polarization diversity for obtaining multiple independent views of the target. In this scheme, Golay pairs of phase coded waveforms are used to provide synchronization while Alamouti coding is used to coordinate transmission of these waveforms on the horizontal and vertical polarizations and this enables unambiguous radar polarimetry on a pulse-by-pulse basis, thereby reducing signal processing complexity. In [2], the 2×2 case was extended to multiple antennas, and more general waveforms families were developed that allowed for perfect separation in the case of negligible Doppler. In particular, scheduling for Golay pairs was described for a 4×4 system and it was demonstrated that Golay pairs achieve both perfect separation and perfect reconstruction . However, in the presence of Doppler, Golay pairs are known to perform poorly and this is the primary reason that Golay sequences have not found widespread use in radar.

In [3], PTM sequences were used to make the Golay sequence transmissions resilient against Doppler shifts. The method achieves good results for small Doppler shifts, but the fact that this method makes the Golay sequences insensitive to Doppler means that target resolution based on differing Doppler signatures is not possible. In [4], we proposed a Doppler compensation and estimation scheme that exploits the subspace structure of the received waveform matrix to minimize the effects of Doppler. This scheme provides very good delay resolution, but as the number of targets grows, it becomes more computationally complex, and the fact that we need to find processing vectors for different target Doppler shifts from the null-space starts to affect the performance. To alleviate these effects, a new Doppler compensation and estimation scheme is proposed, that is based on a clever application of the DFT via the computationally efficient FFT algorithm. Our proposed algorithm exploits the autocorrelation property of the complementary sequences along with the fact that DFT can be used for match-filtering with the Doppler shifted copies of the received signal. The proposed algorithm shows a marked improvement in performance and complexity over the null-space algorithm developed earlier. Initial simulations show very good results with multiple targets.

2. GOLAY COMPLEMENTARY SEQUENCES AND TARGET DETECTION

A pair of sequences $s_1(n)$ and $s_2(n)$ of length N_c satisfy the Golay property if the sum of their autocorrelation functions satisfy

$$R_{s_1 s_1}(l) + R_{s_2 s_2}(l) = \begin{cases} 2N_c & if \quad l = 0 \\ 0 & if \quad l \neq 0 \end{cases} \tag{1}$$

for $l = -N_c - 1, ..., N_c - 1$. If we take the DFT of the above equation, we get

$$|S_1(k)|^2 + |S_2(k)|^2 = 2N_c \tag{2}$$

In [2], we showed that if $s_1(n)$ and $s_2(n)$ are Golay complementary, then so are $s_1^*(-n)$ and $s_2^*(-n)$. Using this fact, we can develop a 4-waveform family using Golay complementary sequences by defining

$$s_3(n) = s_1^*(-n) \tag{3}$$

and

$$s_4(n) = s_2^*(-n) \tag{4}$$

Now, in the case of negligible Doppler, the received signal over 4 PRIs is given by

$$\mathbf{R}(n) = \mathbf{H}^T\mathbf{S}(n) + \mathbf{N}(n) \tag{5}$$

where $\mathbf{S}(n)$ is the 4×4 transmitted waveform matrix given by [2]. $\mathbf{H}$ is the channel matrix which contains the various round-trip path gains from each transmit antenna to each receive antenna. and $\mathbf{N}(n)$ is the noise matrix. To detect the presence of the target in the delay resolution bin n, We process the received waveform matrix as

$$\mathbf{R}(n) * \mathbf{S}^H(-n) = \mathbf{H}^T\mathbf{S}(n) * \mathbf{S}^H(-n) + \mathbf{N}'(n) \tag{6}$$

where $*$ is the pair-wise convolution of two matrices that follows the same order as matrix multiplication. It can be easily shown [2] that

$$\mathbf{S}(n) * \mathbf{S}^H(-n) = \alpha\mathbf{I}\delta(n) \tag{7}$$

From this, it follows that

$$\mathbf{R}(n) * \mathbf{S}^H(-n) = \alpha\mathbf{H}\delta(n) + \mathbf{N}'(n) \tag{8}$$

In order to detect the presence of a target in the delay resolution bin n, consider the test stastic

$$z(n) = \left\|\mathbf{R}(n) * \mathbf{S}^H(n)\right\|_F^2 \tag{9}$$

where the subscript F stands for Frobenius norm. A plot of $z(n)$ for target SNRs of 5dB and 10dB, respectively are shown in Figure 1. As we can see from the figure, the unitary waveform matrix signal design greatly facilitates high-resolution time-localization of a target when the Doppler shift is negligible.

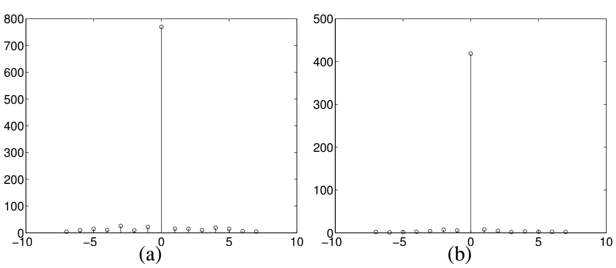

Fig. 1. Plot of $z(n)$ without Doppler for (a) SNR = 5dB (b) SNR = 10dB

3. DOPPLER COMPENSATION AND ESTIMATION

In this section, we develop a signal model that incorporates the effects of Doppler. We assume that the target is moving at a constant speed, which means that between two successive PRIs, the differential Doppler phase shift is constant.

3.1. Effects of Doppler

In the presence of Doppler, the received signal may be expressed as

$$\mathbf{R}(n) = \mathbf{H}^T\mathbf{S}(n)\mathbf{D} + \mathbf{N}(n) \tag{10}$$

The Doppler shift matrix $\mathbf{D}$ is given by

$$\mathbf{D} = diag\{1, e^{j\upsilon'}, e^{j2\upsilon'}, e^{j3\upsilon'}\} \tag{11}$$

where υ' is the Doppler-induced differential phase shift between two successive PRIs. As in the case of negligible Doppler, we process the received waveform matrix as

$$\mathbf{R}(n) * \mathbf{S}^H(-n) = \mathbf{H}^T\mathbf{S}(n)\mathbf{D} * \mathbf{S}^H(-n) + \mathbf{N}'(n) \tag{12}$$

In the presence of a non-negligible Doppler phase shift, the condition in (4) is not satisfied in general, i.e.,

$$\mathbf{S}(n)\mathbf{D} * \mathbf{S}^H(-n) \neq \alpha\mathbf{I}\delta(n) \tag{13}$$

and unambiguous range resolution becomes significantly more difficult. To illustrate this graphically, a plot of $z(n)$ for the same target SNRs of 5dB and 10dB are plotted in Figure 2 for the case of $\upsilon' = \pi/3$. For this particular set of round-trip

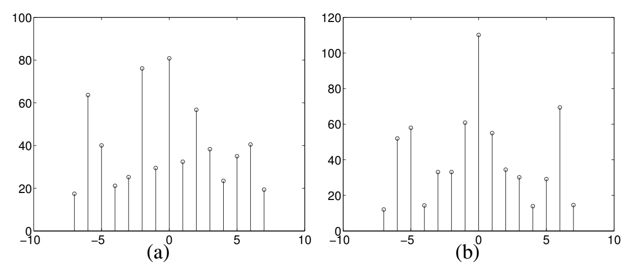

Fig. 2. Plot of $z(n)$ with Doppler ($\upsilon' = \pi/3$) for (a) SNR = 5dB (b) SNR = 10dB

channel gains, the presence of Doppler makes it impossible to detect the target. In [4], we presented a sub-space based approach for combating Doppler, which produced very good results. However, the scheme was more computationally complex, and to address that, we present a new scheme below.

3.2. Doppler Match Filter Processing

Towards combatting this problem, consider the matrix

$$\hat{\mathbf{R}}(n) = \mathbf{R}(n) * \mathbf{S}^H(-n) \tag{14}$$

Each term of this matrix is a sum of four individual convolution sequences. Next, consider the 4×4 matrix $\mathbf{Y}_i$, given by

$$\mathbf{Y}_i(n) = \begin{bmatrix} r_{i1}(n) * s_1^*(-n) & r_{i1}(n) * s_1^*(-n) & r_{i1}(n) * s_1^*(-n) & r_{i1}(n) * s_1^*(-n) \\ r_{i2}(n) * s_2(n) & r_{i2}(n) * s_2(n) & r_{i2}(n) * s_2(n) & r_{i2}(n) * s_2(n) \\ r_{i1}(n) * s_1^*(-n) & r_{i1}(n) * s_1^*(-n) & r_{i1}(n) * s_1^*(-n) & r_{i1}(n) * s_1^*(-n) \\ r_{i2}(n) * s_2(n) & r_{i2}(n) * s_2(n) & r_{i2}(n) * s_2(n) & r_{i2}(n) * s_2(n) \end{bmatrix} \tag{15}$$

Note that column j above contains the individual convolution sequences that are summed up to yield the ij^{th} term in $\hat{\mathbf{R}}(n)$.

Consider the vector

$$\mathbf{w} = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}^T \tag{16}$$

In the case of no Doppler, and ignoring the noise, it is easy to verify that

$$\mathbf{w}^H \mathbf{Y}_i(n) = \gamma \mathbf{h}_i \delta(n) \tag{17}$$

where γ is just a scaling constant, and

$$\mathbf{h}_i = \begin{bmatrix} h_{1i} & h_{2i} & h_{3i} & h_{4i} \end{bmatrix} \tag{18}$$

The index i is associated with one of the receive antennas.

When Doppler is present, it is likewise easy to verify that

$$\mathbf{w}_D^H \mathbf{Y}_i(n) = \gamma \mathbf{h}_1 \delta(n) \tag{19}$$

where

$$\mathbf{w}_D = \begin{bmatrix} 1 & e^{j\upsilon'} & e^{2j\upsilon'} & e^{3j\upsilon'} \end{bmatrix}^T \tag{20}$$

and this holds for all i. This means that for $n \neq 0$, the matrices $\mathbf{Y}_i$ are singular, and the vector producing the desired output lies in the null-space of these matrices. Also, because of the same waveform inputs, the matrices $\mathbf{Y}_i$ share the same null-space. Thus, we can form a concatenated matrix as

$$\mathbf{Y}_C(n) = \begin{bmatrix} \mathbf{Y}_1(n) & \mathbf{Y}_2(n) & \mathbf{Y}_3(n) & \mathbf{Y}_4(n) \end{bmatrix} \tag{21}$$

It is easy to verify that

$$\mathbf{w}_D^H \mathbf{Y}_C(n) = \gamma \mathbf{h} \delta(n) \tag{22}$$

where $\mathbf{h} = [\mathbf{h}_1\ \mathbf{h}_2\ \mathbf{h}_3\ \mathbf{h}_4]$ is the 1×16 vector of path gains.

Let us index each 4-PRI set with k, and define a new matrix as

$$\mathbf{F}(n) = \begin{bmatrix} \mathbf{Y}_C^{(1)} \\ \vdots \\ \mathbf{Y}_C^{(K)} \end{bmatrix} \tag{23}$$

which is a matrix of dimension $4K \times 16$ for every n. To aid in the development of our proposed algorithm, it is instructive to write the matrix $\mathbf{F}(n)$ as

$$\mathbf{F}(n) = \begin{bmatrix} \mathbf{f}_1(n) & \cdots & \mathbf{f}_{16}(n) \end{bmatrix} \tag{24}$$

Extending (19) to $\mathbf{F}(\mathbf{n})$, we have that for the case of a single scatterer imparting a Doppler phase shift of υ

$$\tilde{\mathbf{w}}_D(\upsilon)\mathbf{f}_i(n) = 0 \tag{25}$$

for $i = 1, 2, ..., 16$ and for all $n \neq 0$, and $\tilde{\mathbf{w}}_{\mathbf{D}}$ is defined as

$$\tilde{\mathbf{w}}_D(\upsilon) = \begin{bmatrix} 1 & e^{j\upsilon} & \ldots & e^{j(4K-1)\upsilon} \end{bmatrix} \tag{26}$$

This forms the basis of our proposed algorithm; for every delay bin n, (21) will have a non-zero value only for the values of υ that correspond to the true target Doppler phase shift, and that is true for all targets. As seen from the structure of (21), we can use DFT of length P, with $P \geq 4K - 1$ to accomplish this match filtering along the Doppler axis, i.e.

$$\mathbf{f}_i^D(n) = DFT\left\{\mathbf{f}_1(n)\right\} \tag{27}$$

And we have the $P \times 16$ matrix

$$\mathbf{F}^D(n) = \begin{bmatrix} \mathbf{f}_1^D(n) & \cdots & \mathbf{f}_{16}^D(n) \end{bmatrix} \tag{28}$$

Each entry of the vector $\mathbf{f}_i^D(n)$ is the output of match filtering with a Doppler shift corresponding to that entry. Since in radar, we are only interested in measuring the energy returned in every delay-Doppler bin, we take the Hadamard product of the matrix with the conjugate of itself to form

$$\mathbf{F}_S^D(n) = \mathbf{F}^D(n) \odot \mathbf{F}^{*D}(n) \tag{29}$$

where

$$\mathbf{F}_S^D(n) = \begin{bmatrix} \mathbf{f}_{S1}^D(n) & \cdots & \mathbf{f}_{S16}^D(n) \end{bmatrix} \tag{30}$$

Now, recall our initial assumption that the target location and speed do not change appreciably over our $4K$ PRI observation interval, so given this, all vectors $\mathbf{f}_{Si}^D(n)$ provide the same information about the Doppler phase shift for every n, so we form

$$\mathbf{f}_S^D(n) = \sum_{i=1}^{16} \mathbf{f}_{Si}^D(n) \tag{31}$$

Our detection statistic for every delay-Doppler bin is now given as

$$z(l, n) = f_S^D(l, n) \tag{32}$$

where $f_S^D(l, n)$ is the l^{th} component of the vector $\mathbf{f}_S^D(n)$ for every delay bin n.

3.3. Further Sidelobe Reduction

To further reduce the background noise, we modify our transmitted waveforms by multiplying each waveform with a complex exponential of known frequency. Note that this operation is not meant to impart a frequency shift, but is used instead to modify the original waveform such that it remains unimodular, has the same autocorrelation properties, but it provides a better sidelobe and background noise reduction. To see this, consider a sequence $x(n)$ with autocorrelation function $R_x x(m)$. If we define a new sequence as

$$v(n) = e^{j\omega_0 n} x(n) \tag{33}$$

Then, we have that

$$R_{vv}(m) = e^{j\omega_0 m} R_{xx}(m) \tag{34}$$

As we will see in the simulation results, this has the effect of reducing the background noise, which is vital for target detection at low SNRs.

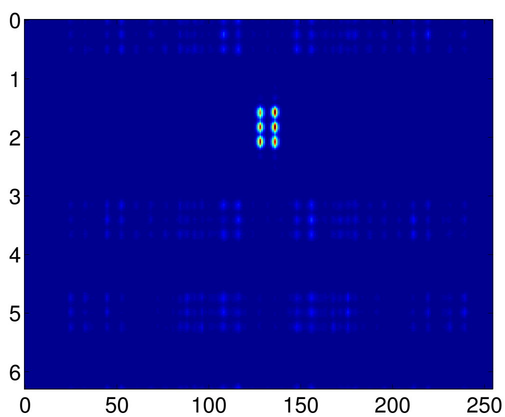

Fig. 3. Delay-Doppler plot for a target SNR of 3db

4. SIMULATION RESULTS

We present some simulation results that show how powerful this technique is for delay-Doppler resolution. We plot results for a six target scenario where three targets are at the same delay each, and 2 targets are at the same Doppler shift. In Figure 3, we plot the results for the six target case where the delay separation of the targets is 2 chips, and the Doppler separation is $1/4\pi$. In Figure 4, we see that the sidelobes levels are lower as compared to Figure 3 when we use the sidelobe reduction. In both these figures, the targets are clearly identifiable. In Figure 5, we show a magnified view of the case where the Doppler separation is the same, but the delay separation is now only 1 chip. The magnification helps to demonstrate the efficacy of our scheme in resolving closely spaced targets. As

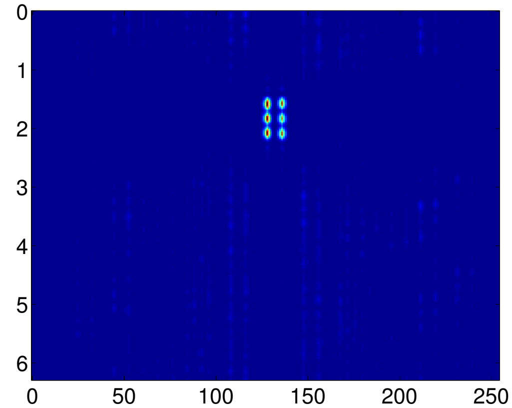

Fig. 4. Delay-Doppler plot for a target SNR of 3db with sidelobe reduction

we can see from these figures, the closely spaced targets can be uniquely identified even for a nominal target SNR of 3dB.

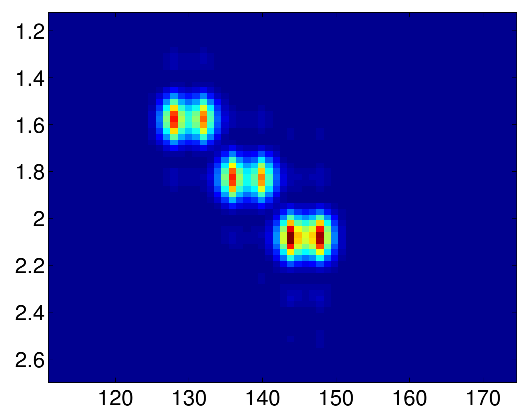

Fig. 5. Magnified Delay-Doppler plot for target SNR of 3db

5. CONCLUSIONS

We have developed a technique for accurate target ranging and Doppler phase estimation in MIMO radar using Golay complementary sequences when multiple targets are present. Complementary sequences based transmit waveforms and a clever manipulation of the data allows for the use of FFT to estimate the Doppler, resulting in a very high range resolution while also providing very good Doppler resolution. Simulation results show that our proposed technique works well even for low target SNRs.

6. REFERENCES

[1] Howard, S. D., Calderbank, A. R., Moran, W., "A Simple Polarization Dversity Scheme for Radar Detection", in *Proc. of Second Intl. Conf. on Waveform Diversity and Design*, 22-27, (2006).

[2] M. Zoltowski, R. Calderbank, T. Qureshi and W. Moran, "Unitary Design of Radar Waveform Diversity Sets", *in Conf. Rec. Forty Second Asilomar Conf. Signals, Syst., Comput.* , Pacific Grove, CA, 2008.

[3] Pezeshki, A.; Calderbank, A.R.; Moran, W.; Howard, S.D., "Doppler Resilient Golay Complementary Waveforms", IEEE Transactions on Information Theory, Volume 54, Issue 9, Sept. 2008 Page(s):4254 - 4266

[4] Qureshi, Tariq R.; Zoltowski, Michael D.; Calderbank, Robert; "Target detection in MIMO radar in the presence of Doppler using complementary sequences", *in IEEE Conf. Acoustics, Speech, Sig. Proc.*, 2010.