A WAVEFORM COVARIANCE MATRIX FOR HIGH SINR AND LOW SIDE-LOBE LEVELS

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ABSTRACT

In this work to exploit the benefits of both multiple-input multiple-output (MIMO)-radar and phased-array a waveform covariance matrix is proposed. Our analytical results show that the proposed covariance matrix yields gain in signal-to-interference-plus-noise ratio (SINR) compared to MIMO-radar while the gain in SINR is close to phased-array and recently proposed phased-MIMO scheme. Transmitted waveforms with the proposed covariance matrix, at the receiver, significantly supress the side-lobe levels compared to phased-array, MIMO-radar, and phased-MIMO schemes . Moreover, in contrast to phased-MIMO our proposed scheme allows same power transmission from each antenna. Simulation results validate the analytical results.

Index Terms— MIMO-radar and Toeplitz matrix.

1. INTRODUCTION

Recently several researchers have considered the application of multiple-input multiple-output (MIMO) techniques developed for wireless communication systems to the radar systems [1–3]. Like MIMO communications, which revolutionized the design, development and deployment of wireless networks over the last decade, MIMO-radar offers a new paradigm for signal processing research. MIMO-radars have many advantages over their phased-array counterparts: improved spatial resolution, better parametric identifiability, and greater flexibility to achieve the desired transmit beampattern.

MIMO-radars can be classified into two categories: widely distributed [1] and colocated [2]. In the widely distributed case the transmitting antennas are separated so that each antenna may view a different aspect of the target. This topology can increase the spatial diversity of the system. In colocated systems the transmitting antennas are spaced so that all the transmit antennas view the same aspect of the target. The colocated antenna radar cannot provide spatial diversity but can increase the spatial resolution of the system. In contrast to phased-array, MIMO-radar allows each transmitting antenna to transmit independent waveforms, which provide extra degrees-of-freedom (DOF) that can be exploited to improve system performance [4, 5]. Therefore, in MIMO radar, waveform design is the focus of research from past few years. The waveform design methods to achieve specific goals for widely distributed radars are discussed in [6] (and the references therein) while the waveform design methods for colocated-radars to achieve a desired beampattern are discussed in [7–10].

In phased-array radars the transmitted signals are coherent between different elements of the array that yields gain in signal-to-noise ratio (SNR) but it has poor parametric identifiability problem. MIMO-radar has better parametric identifiability but compared to phased-array radar it shows loss in SNR due to non-coherent processing. To exploit the benefits of both MIMO-radar and phased-array the configuration in both [11] and [12] divides the given transmit antennas into K overlapping sub-arrays, where $1 \leq K \leq n_T$. Each sub-array transmits the waveform, which is orthogonal to the waveforms transmitted by the other sub-arrays. This configuration requires different powers to be transmitted from different antennas. Radio-frequency amplifiers (RFA)'s have non-linear relationships between their input and output and they cannot have maximum power efficiency at all power levels. If each antenna is required to transmit at a different power level then, for maximum power efficiency, multiple different RFA's with different bias voltage levels will be required. A better solution is to have identical RFA's all working at the same maximum power level.

In this work, a covariance matrix is proposed for the transmit waveforms. To generate it, the values of cosine function from 0 to π with the step size of π/n_T are used to form a positive semidefinite Toeplitz matrix. Proposed covariance matrix yields significant gain in signal-to-interference-plus-noise ratio (SINR) compared to MIMO-radar and the SINR is close to the phased-array and phased-MIMO schemes. Secondly, the proposed method has much lower side-lobe levels (SLL)'s compared to phased-array, MIMO-radar and phased-MIMO schemes. Moreover, in contrast to phased-MIMO scheme, it allows equal power transmission from all antennas.

The remainder of this paper is organised as follows. In the following section the problem formulation and some background are given. The proposed algorithm is developed in section 3. Simulation results are given in section 4, followed by our conclusions in section 5.

Notation: Bold upper case letters, $\mathbf{X}$, and lower case letters, $\mathbf{x}$, respectively denote matrices and vectors. Conjugate transposition of a matrix is denoted by $(.)^H$.

2. PROBLEM AND PREVIOUS WORK

Consider a uniform linear array of n_T transmit and n_R receive antennas, the inter-element-spacing between any two adjecent antennas is half of a wavelength of transmitted waveform. In the given scenario, there is a target of interest located at an angle θ_t, and L interferers located at angles θ_1 to θ_L. For the best detection, the receiver should be able to maximise the received power from the target direction and minimise it from all the other directions. In addition to this, it should be able to place deep nulls in the direction of interferers. To design such receiver, if $x_m(n)$ is the baseband signal transmitted from antenna m then by defining $\mathbf{a}_T(\theta_p) = \begin{bmatrix} 1 & e^{j\pi\sin(\theta_p)} & \cdots & e^{j(n_T-1)\pi\sin(\theta_p)} \end{bmatrix}^T$ and $\mathbf{a}_R(\theta_p) = \begin{bmatrix} 1 & e^{j\pi\sin(\theta_p)} & \cdots & e^{j(n_R-1)\pi\sin(\theta_p)} \end{bmatrix}^T$ transmit and receive steering vectors corresponding to a target at location θ_p, and $\mathbf{x}(n) = \begin{bmatrix} x_1(n) & x_2(n) & \cdots & x_{n_T}(n) \end{bmatrix}^T$ a vector of transmitted symbols at time index n, the received signals at n_R antennas in vector form can be written as

$$\mathbf{y}(n) = \beta_t\mathbf{a}_R(\theta_t)\mathbf{a}_T^T(\theta_t)\mathbf{x}(n)+\sum_{i=1}^{L}\beta_i\mathbf{a}_R(\theta_i)\mathbf{a}_T^T(\theta_i)\mathbf{x}(n)+\mathbf{v}(n), \quad (1)$$

where β_q is the reflection coefficient of the target/interferer at location θ_q and $\mathbf{v}(n) = \begin{bmatrix} v_1(n) & v_2(n) & \cdots & v_{n_R}(n) \end{bmatrix}^T$ is the vector of circularly symmetric white Gaussian noise samples each of zero mean and σ_n^2 variance. Generally, in MIMO-radar, the transmitted symbols from all antennas are fully uncorrelated i.e., $\mathrm{E}\{x_p(n)x_q^*(n)\} = 0$, for $p \neq q$. At each receive antenna, the received signal is passed through a matched-filter and the output samples are correlated with n_T transmitted waveforms. The n_T outputs after correlation are collected from each receive antenna and cascaded into a vector after which the $n_T n_R \times 1$ received signal vector can be written as

$$\mathbf{y}_m = \beta_t\mathbf{a}_T(\theta_t)\otimes\mathbf{a}_R(\theta_t)+\sum_{i=1}^{L}\beta_i\mathbf{a}_T(\theta_i)\otimes\mathbf{a}_R(\theta_i)+\mathbf{v}_m. \quad (2)$$

To maximise the SINR of the received signal a beamformer is designed at the receiver. To design a beamformer weight vector, $\mathbf{b}$, define $\mathbf{s}_m(\theta_q) = \mathbf{a}_T(\theta_q)\otimes\mathbf{a}_R(\theta_q)$ a virtual steering vector corresponding to the target/ interferer at location θ_q. By multiplying the received signal in (2) with the beamformer we can write

$$\mathbf{b}^H\mathbf{y}_m = \underbrace{\beta_t\mathbf{b}^H\mathbf{s}_m(\theta_t)}_{\text{signal term}}+\underbrace{\sum_{i=1}^{L}\beta_i\mathbf{b}^H\mathbf{s}_m(\theta_i) + \mathbf{b}^H\mathbf{v}_m}_{\text{noise+interference terms}}. \quad (3)$$

From (3), the SINR can be defined as

$$\text{SINR} = \beta_t^2\frac{|\mathbf{b}^H\mathbf{s}_m(\theta_t)|^2}{\mathbf{b}^H\mathbf{R}_{in}\mathbf{b}}, \quad (4)$$

where $\mathbf{R}_{in} \in \mathcal{C}^{n_Tn_R\times n_Tn_R}$ is the covariance matrix of interference plus noise samples and is defined as

$$\mathbf{R}_{in} = \underbrace{\sum_{i=1}^{L}|\beta_i|^2\mathbf{s}_m(\theta_i)\mathbf{s}_m^H(\theta_i)}_{\text{interferers covariance matrix}} + \underbrace{\sigma_n^2\mathbf{I}_{n_Rn_T}}_{\text{noise covariance matrix}}.$$

Let $\beta_t = 1$, to maximise SINR with respect to $\mathbf{b}$ using Schawarz's inequality (4) can be written as

$$\begin{aligned}\text{SINR} &= \frac{|\mathbf{b}^H\mathbf{R}_{in}^{1/2}\mathbf{R}_{in}^{-1/2}\mathbf{s}_m(\theta_t)|^2}{\mathbf{b}^H\mathbf{R}_{in}\mathbf{b}} \\ &\leq \mathbf{s}_m^H(\theta_t)\mathbf{R}_{in}^{-1}\mathbf{s}_m(\theta_t). \end{aligned} \quad (5)$$

Therefore, the optimal value of SINR is given by

$$\text{SINR}_\circ = \mathbf{s}_m^H(\theta_t)\mathbf{R}_{in}^{-1}\mathbf{s}_m(\theta_t).$$

It can be easily proved that the beamformer weight vector that brings the optimal value of SINR can be derived as [13]

$$\mathbf{b} = \frac{\mathbf{R}_{in}^{-1}\mathbf{s}_m(\theta_t)}{\mathbf{s}_m^H(\theta_t)\mathbf{R}_{in}^{-1}\mathbf{s}_m(\theta_t)}. \quad (6)$$

Finding the beamformer vector $\mathbf{b}$ requires the inversion of covariance matrix $\mathbf{R}_{in}$, which can be computed in $\mathcal{O}(n_Rn_T)^3$ computations. In the presence of only noise $\mathbf{R}_{in}$, can be replaced by a scaled identity matrix, $\sigma_n^2\mathbf{I}_{n_Rn_T}$, and the maximum value of SINR becomes

$$\text{SINR}_\circ = \frac{n_Rn_T}{\sigma_n^2}. \quad (7)$$

To improve the SINR of MIMO-radar, some other configurations are discussed in what follows.

In MIMO-radar, if the transmitted waveform $x_m(n) = x_1(n)e^{-j\pi(m-1)\sin(\theta_t)}$ then all the transmitted waveforms will be fully correlated. Such configuration of MIMO-radar is called a phased-array radar, here at each receive antenna only one matched-filter corresponding to $x_1(n)$ is required. Therefore, the received samples collected after the matched-filtering in vector form can be written as

$$\mathbf{y}_p = \beta_tn_T\mathbf{a}_R(\theta_t) + \sum_{i=1}^{L}\beta_i\mathbf{a}_T^H(\theta_t)\mathbf{a}_T(\theta_i)\mathbf{a}_R(\theta_i) + \hat{\mathbf{v}}_p. \quad (8)$$

For this configuration of MIMO-radar the optimal value of SINR, for $\beta_t = 1$, can be easily derived as

$$\text{SINR}_\circ = \mathbf{s}_p^H(\theta_t)\hat{\mathbf{R}}_{in}^{-1}\mathbf{s}_p(\theta_t),$$

where $\mathbf{s}_p(\theta_t) = n_T\mathbf{a}_R(\theta_t)$ and $\hat{\mathbf{R}}_{in} \in \mathcal{C}^{n_R\times n_R}$. The SINR for noise only case becomes

$$\text{SINR}_\circ = \frac{n_Rn_T^2}{\sigma_n^2}. \quad (9)$$

Similarly, for the noise only case the optimal SINR of the phased-MIMO radar can be derived as [14]

$$\text{SINR}_\circ = \frac{n_Rn_T(n_T-K+1)}{\sigma_n^2}. \quad (10)$$

3. PROPOSED CORRELATED MIMO-RADAR

In MIMO-radar, if the transmitted waveforms are correlated and the given waveform covariance matrix is $\mathbf{R}_x$ then by correlating the each element of $\mathbf{y}(n)$ in (1) with the n_T transmitted waveforms and cascading the outputs into a vector one can write

$$\mathbf{y}_c = \beta_t \mathbf{a}_R(\theta_t)\otimes\mathbf{R}_x\mathbf{a}_T(\theta_t)+\sum_{i=1}^{L}\beta_i\mathbf{a}_R(\theta_i)\otimes\mathbf{R}_x\mathbf{a}_T(\theta_i)+\mathbf{v}_c, \quad (11)$$

where $\mathbf{v}_c$ is no longer a white noise it is a colored. For the proposed model given in (11), similar to previous cases, the maximum SINR can be derived as

$$\text{SINR}_\circ = \mathbf{s}_c^H(\theta_t)\bar{\mathbf{R}}_{in}^{-1}\mathbf{s}_c(\theta_t), \quad (12)$$

where $\mathbf{s}_c(\theta_p) = \mathbf{a}_R(\theta_p)\otimes\mathbf{R}_x\mathbf{a}_T(\theta_p)$. From (11), the covariance matrix of interference and noise can be found as

$$\bar{\mathbf{R}}_{in} \;=\; \sum_{i=1}^{K}|\beta_i|^2\mathbf{s}_c(\theta_i)\mathbf{s}_c^H(\theta_i)+\sigma_n^2(\mathbf{I}_{n_R}\otimes\mathbf{R}_x).$$

It can be noted here that the SINR depends on $\mathbf{R}_x$. In what follows next, a waveform covariance matrix is proposed that yields lower SLL's compared to the phased-array, MIMO-radar, and phased-MIMO schemes.

To generate the waveform covariance matrix the values of cosine function from 0 to π with the step size of π/n_T are used to generate a positive-semidefinite Toeplitz matrix as given below

$$\mathbf{R}_x = \begin{bmatrix} 1 & \cos(\frac{\pi}{n_T}) & \cdots & \cos\left(\frac{(n_T-1)\pi}{n_T}\right) \\ \cos(\frac{\pi}{n_T}) & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \cos(\frac{\pi}{n_T}) \\ \cos\left(\frac{(n_T-1)\pi}{n_T}\right) & \cdots & \cos(\frac{\pi}{n_T}) & 1 \end{bmatrix}.$$

It can be easily proved that $\mathbf{R}_x$ is a positive-semidefinite matrix and has only two eigenvalues $n_T/2$ and $n_T/2$. Since the proposed waveform covariance matrix, $\mathbf{R}_x$, is real, its transmit beampattern will be symmetric about $\theta = 0$. In Fig. 1, the normalised transmit beampattern using $\mathbf{R}_x$ is compared with the normalised transmit beampatterns of phase-array, MIMO-radar and phased-MIMO schemes for $\theta_t = 0$.

Using the proposed method, a covariance matrix to illuminate a target at location $\theta_t \neq 0$ can be easily obtained as [9]

$$\tilde{\mathbf{R}}_x = \mathbf{R}_x \odot \mathbf{a}_T(\theta_t)\mathbf{a}_T^H(\theta_t), \quad (13)$$

where $\odot$ represnts Hadamard product. Now, using $\mathbf{R}_x$ in (12) and matrix algebra identities $(\mathbf{A}\otimes\mathbf{B})^{-1} = \mathbf{A}^{-1}\otimes\mathbf{B}^{-1}$ and $(\mathbf{A}\otimes\mathbf{B})(\mathbf{C}\otimes\mathbf{D}) = (\mathbf{AC}\otimes\mathbf{BD})$ (here $\mathbf{A},\mathbf{B},\mathbf{C}$ and $\mathbf{D}$ are

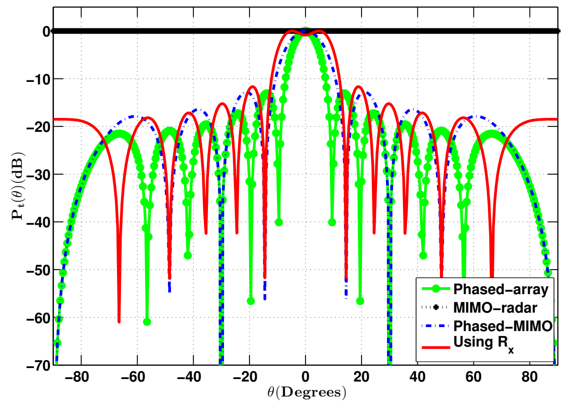

Fig. 1. Transmit beampattern of phased-array, MIMO-radar, phased-MIMO and proposed covariance matrices.

matrices) [15], for noise only case, the optimum SINR can be easily derived as

$$\text{SINR}_\circ \;=\; \frac{n_R\otimes\mathbf{a}_T^H(\theta_t)\mathbf{R}_x^H\mathbf{a}_T(\theta_t)}{\sigma_n^2}. \quad (14)$$

Since the main lobe is symmetric about $\theta_t = 0$, the transmit steering vector correspond to $\theta_t = 0$ will become

$$\mathbf{a}_T(\theta_t) = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}^T \in \mathcal{C}^{n_T\times 1},$$

which help us to write (14) as

$$\text{SINR}_\circ = \frac{n_R}{\sigma_n^2}\sum_{p=0}^{n_T-1}\left(\sum_{n=0}^{n_T-1}\cos\left(\frac{(n-(p-1))\pi}{n_T}\right)\right). \quad (15)$$

Using the result $\sum_{n=0}^{n_T-1}\cos\left(\frac{(n-p)\pi}{n_T}\right) = \frac{2\sin\left(\frac{p\pi}{n_T}+\frac{\pi}{2n_T}\right)\sin\left(\frac{\pi}{2n_T}\right)}{1-\cos\left(\frac{\pi}{n_T}\right)}$ given by [16] in (15) allow us to write

$$\text{SINR}_\circ \;=\; \frac{2n_R}{\sigma_n^2}\left(\frac{\cos(\pi/n_T)}{1-\cos(\pi/n_T)}\right). \quad (16)$$

It can be noted in (16) that in contrast to MIMO-radar with the increase in the transmit antennas the gain in SINR increases exponetially.

For the proposed covariance matrix the conventional beamformer can be found as

$$\mathbf{b}_c = \mathbf{a}_R(\theta_t = 0)\otimes\mathbf{R}_x\mathbf{a}_T(\theta_t = 0), \quad (17)$$

and the receive power from the direction of θ at the output of a conventional receiver can be found as

$$P_r(\theta) = \left|\mathbf{b}_c^H\cdot\mathbf{a}_R(\theta)\otimes\mathbf{R}_x\mathbf{a}_T(\theta)\right|^2. \quad (18)$$

Therefore, again using trignometric and summation identities the received power in (18) can be derived as

$$P_r(\theta) = \left(\frac{n_T\sin(\frac{n_R\pi\sin(\theta)}{2})\cos\left(\frac{n_T\pi\sin(\theta)}{2}\right)\cos\left(\frac{\pi\sin(\theta)}{2}\right)}{\sin(\frac{\pi\sin(\theta)}{2})\left(\cos(\pi\sin(\theta))-\cos(\frac{\pi}{n_T})\right)}\right)^2.$$

In the above equation, it can be noted that as the value of θ increases the amplitude of the numerator term $\cos\left(\frac{n_T \pi \sin(\theta)}{2}\right) \cos\left(\frac{\pi \sin(\theta)}{2}\right)$ decreases and the amplitude of denominator term $\left(\cos(\pi \sin(\theta)) - \cos(\frac{\pi}{nT})\right)$ increases, after each null point. As a consequence the value of $P_r(\theta)$ can decrease rapidly with the increase in the value of θ in the SLL region. Here, the maximum power using the conventional receiver at $\theta = 0$ is $\frac{(n_T n_R)^2}{\left(1-\cos\left(\frac{\pi}{n_T}\right)\right)^2}$, which in the case of MIMO-radar is $(n_T n_R)^2$. Therefore, the gain using the proposed covariance matrix compared to MIMO-radar is $\left(1 - \cos\left(\frac{\pi}{n_T}\right)\right)^{-2}$.

It should be noted here that to make the above derivations simple the target is located at $\theta_t = 0$. However, the same results for SINR and received power can be obtained to detect a target at $\theta_t \neq 0$ by modifying the covariance matrix using (13). The simulation results for $\theta_t = 10$ degrees are shown in the simulation section.

4. SIMULATION RESULTS

In our simulations, the number of transmit and receive antennas are kept equal to 12 and for the phased-MIMO scheme $K = 6$. In Fig. 2, the SINR of the proposed scheme is compared with the SINR of phased-array, MIMO-radar, and phased-MIMO schemes. It can be seen in the figure that the SINR with the proposed waveform covariance matrix is much higher than the MIMO-radar and close to the phased-array and phased-MIMO schemes. Similarly, Fig. 3 shows the normalised received power using the conventional receiver, designed to detect a target at location $\theta_t = 10$ degrees for all the schemes. It can be seen in the figure that our proposed scheme outperforms all the other schemes significantly in terms of SLL's supression. In the last simulation, the target of interest is located at $\theta_t = 10$ degrees and there are two interferers located at -20 and 30 degrees, the interference-to-noise ratio is 30-dB while SNR=10-dB. In this scenario, to detect a target a minimum-variance distortionless response (MVDR) beamformer [13] is designed for all the schemes, Fig. 4 shows the corresponding simulation results using all the schemes. Here, it can be noted that the proposed scheme has similar interferer supression capabilities to that of other schemes, however, compared to all the other schemes it has much lower SLL's. An other advantage of the proposed scheme is that it has better resolution in the SLL's compared to other schemes.

5. CONCLUSION

The proposed waveform covariance matrix yielded good SINR and significant lower SLL's compared to phased-array, MIMO-radar, and phased-MIMO schemes. Due to the limitation of space direct results are provided and the details and discussions are left for the journal version of this paper [17].

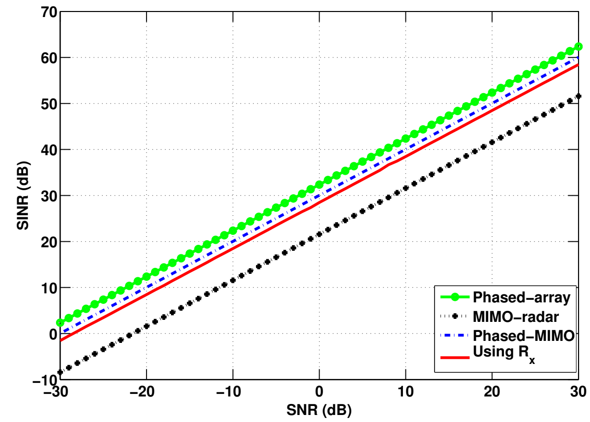

Fig. 2. Comparison of the SINR using the proposed schemes with the MIMO-radar, phased-array and phased-MIMO schemes.

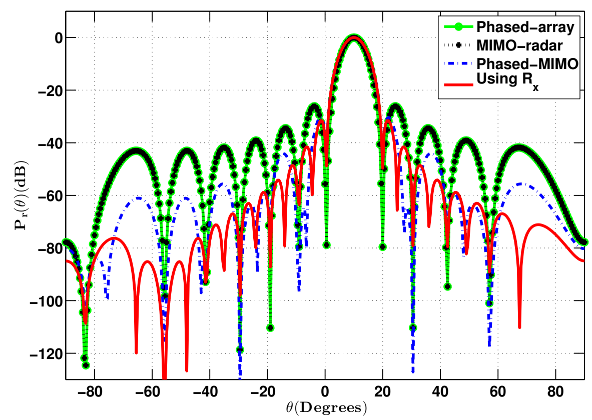

Fig. 3. Receive beampatterns using conventional beamformers of phased-array, MIMO-radar, phased-MIMO, and proposed schemes.

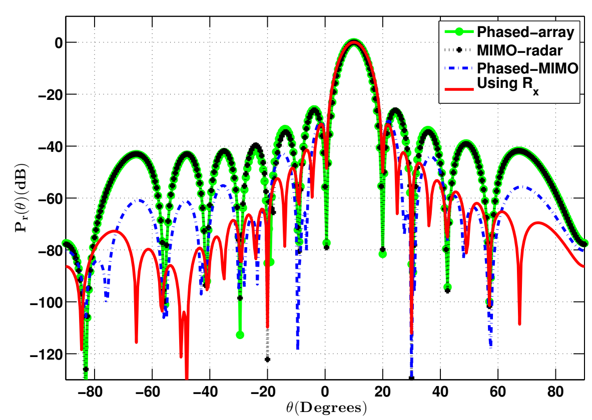

Fig. 4. Receive beampatterns using MVDR beamformers of phased-array, MIMO-radar, phased-MIMO, and proposed schemes.

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