ON THE TRANSMIT BEAMPATTERN DESIGN USING MIMO AND PHASED-ARRAY RADAR

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ABSTRACT

The design of the transmit beampattern of a radar system aims to focus the energy of the transmitted signals to the desired spatial section(s) in order to enhance the direction of arrival (DOA) estimation at the receiver. In this paper, we consider this problem for both multiple-input-multiple-output (MIMO) and phased-array radar from the perspective of finite impulse response (FIR) filter design. For MIMO radar, we formulate the design as a feasibility problem (FP) and show its advantages over the existing methods. For phased-array radar, we use the spectral factorization technique which has been considered to be better than conventional filter design methods. It is shown that the two systems can achieve similar transmit beampatterns for identical number of transmit antennas and waveform energy. This is contradictory to the existing argument that MIMO radar can achieve more flexible transmit beampattern design over its phased-array counterpart. Simulation results are provided and discussed.

Index Terms— Transmit beampattern design, MIMO radar, phased-array radar, FIR filter design.

1. INTRODUCTION

Researchers have intensively compared MIMO and phased-array radar in the recent decade, and demonstrated a general fact that MIMO radar transmitting orthogonal waveforms enjoys the waveform diversity to extract useful information at receiver via the use of different waveforms [1–10]. However, this comes with the loss of coherent processing gain which is preserved in phased-array radar. The comparisons have been done on the system architecture development [4–7], the signal processing techniques at the transmitter side [8, 9], and the signal processing techniques at the receiver side [10]. In this paper, we focus on the transmit beampattern design using the two systems. Via the transmit beampattern design, the energy of the transmitted waveforms are focused in the desired spatial section, as a result, the signal-to-noise ratio (SNR) at the receiver is enhanced and the performance of direction of arrival (DOA) estimation can be improved.

The transmit beampattern is characterized by the covariance matrix of the transmitted signals, which usually lies between a scaled identity matrix (orthogonal waveforms from standard MIMO radar) and a scaled all-one matrix (coherent waveforms from standard phased-array radar). Hence the objective is to optimize the covariance matrix to achieve the desired transmit beampattern. The solution to this problem for MIMO radar has been well developed in the literature [8, 9, 11–15], and it has been argued that due to the waveform diversity, MIMO radar can achieve more flexible transmit beampatterns as compared to its phased-array counterpart. In [9], the authors provided design examples using phased-array radar for comparison, and showed the inability in achieving the desired beampattern. This design is based on a minor modification to the MIMO radar problem formulation, i.e., adding a rank constraint. However, such a problem formulation may not be suitable for phased-array radar.

In this paper, we consider the transmit beampattern design for both MIMO and phased-array radar from the perspective of finite impulse response (FIR) filter design, which has not been clearly reported in the literature. We show that the MIMO radar design using transmit beamspace processing (TBP) model in [13] can be mapped to a multiple-input-single-output (MISO) FIR filter design problem, whereas the phased-array design can be mapped to a single-input-single-output (SISO) FIR filter design problem. The TBP signal model [13] has been proven for its simplicity and efficiency for MIMO radar. In this way, a feasibility problem (FP) considering all the design specifications can be formulated for MIMO radar, and the spectral factorization technique [16] for improved SISO FIR filter design can be adopted for phased-array radar. Simulation results will illustrate that via appropriate design, e.g., spectral factorization, phased-array radar can achieve similar transmit beampatterns as MIMO radar using identical number of antennas and waveform energy.

2. TRANSMIT BEAMPATTERN SIGNAL MODEL

Consider a MIMO radar system with N_{T} half wavelength uniform linear array (ULA) antennas at the transmitter, and N is the length of the waveform samples. The TBP model uses K orthogonal waveforms, $K \leq N_{\mathrm{T}}$, which can be expressed as

$$\mathbf{S} = [\mathbf{s}_0, \mathbf{s}_1, \cdots, \mathbf{s}_{K-1}]^T \in \mathbb{C}^{K \times N}, \tag{1}$$

where $\{\cdot\}^T$ is the transpose operator, and $\|\mathbf{s}_i\|^2 = 1, i \in \{0, 1, \cdots, K-1\}$ is the uniform elemental power constraint.

Let $\mathbf{W} \in \mathbb{C}^{N_\text{T} \times K}$ be a weight matrix on $\mathbf{S}$, then the covariance matrix of the waveforms, and the covariance matrix of the weighted waveforms are denoted as $\mathbf{R}_0 \approx \mathbf{SS}^H = \mathbf{I}$, and $\mathbf{R} \approx \mathbf{WSS}^H\mathbf{W}^H = \mathbf{WW}^H$, respectively, where $\{\cdot\}^H$ is the transpose conjugate operator, and $\mathbf{I}$ is an identity matrix with appropriate dimension. Let

$$\mathbf{e}(\theta) = [1, e^{-j\pi \cos\theta}, \cdots, e^{-j\pi(N_\text{T}-1)\cos\theta}]^T \tag{2}$$

be the steering vector (90° corresponds to the broadside), thus the transmit beampattern for MIMO radar can be expressed as

$$\begin{aligned} P_\text{MM}(\theta) &= \mathbf{e}^T(\theta)\mathbf{R}\mathbf{e}^*(\theta) \\ &= \mathbf{e}^T(\theta)\mathbf{WW}^H\mathbf{e}^*(\theta). \end{aligned} \tag{3}$$

This equation indicates that the design of the transmit beampattern becomes the design of the weight matrix $\mathbf{W}$, which can be easily obtained via the eigenvalue decomposition of $\mathbf{R}$.

Without loss of generality, let $\mathbf{s} = \mathbf{s}_0$ be the waveform that phased-array radar transmits, and

$$\mathbf{w} = [w(0), w(1), \cdots, w(N_\text{T}-1)]^T \tag{4}$$

be the weight vector for $\mathbf{s}$, then the transmit beampattern for phased-array radar is

$$\begin{aligned} P_\text{PH}(\theta) &= \mathbf{e}^T(\theta)\tilde{\mathbf{R}}\mathbf{e}^*(\theta) \\ &= \mathbf{e}^T(\theta)\mathbf{ws}^T\mathbf{s}^*\mathbf{w}^H\mathbf{e}^*(\theta) \\ &= \mathbf{e}^T(\theta)\mathbf{ww}^H\mathbf{e}^*(\theta), \end{aligned} \tag{5}$$

where $\tilde{\mathbf{R}}$ denotes the covariance matrix of $\mathbf{ws}^T$.

3. THE DESIGN OF THE WEIGHT MATRICES

The weight matrix for MIMO radar can be easily obtained via the eigenvalue decomposition of $\mathbf{R}$, and the design of $\mathbf{R}$ has been intensively discussed in the literature [8,9,11–15]. However, these solutions rarely consider the quantitative control of the ripples within the spatial section for energy focusing, the attenuation outside this section, and the width of the transition region. Alternatively, here we propose to consider the design as an FIR filter design problem, which quantitatively specifies all the parameters that affect the performance. Fig. 1 indicates that MIMO radar transmit beampattern in (3) can be mapped to the squared norm of a MISO FIR filter output, where the tap delay τ is mapped to $e^{j\pi\cos\theta}$.

The phased-array transmit beampattern design can be considered as the design of a single branch of the MISO filter, which is a conventional SISO FIR filter. The spectral factorization technique discussed in [16] is an efficient technique for the design of the FIR filter coefficients, i.e., the weight vector for transmitted waveform.

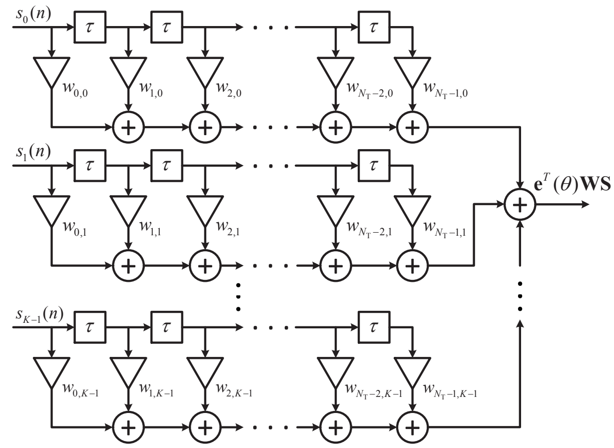

Fig. 1. MIMO radar transmit beampattern design based on TBP, expressed as MISO FIR filter design.

3.1. Feasibility Problem for MIMO radar

We formulate the design of the covariance matrix for MIMO radar as an FP. According to the properties of trace and Kronecker product, (3) can be rewritten in inner product form:

$$\begin{aligned} P(\theta) &= \mathbf{e}^T(\theta)\mathbf{R}\mathbf{e}^*(\theta) \\ &= \text{tr}\{\mathbf{e}^*(\theta)\mathbf{e}^T(\theta)\mathbf{R}\} \\ &= \left[\text{vec}\{\mathbf{e}^*(\theta)\mathbf{e}^T(\theta)\}\right]^H \text{vec}\{\mathbf{R}\} \\ &= \left[\mathbf{e}^H(\theta) \otimes \mathbf{e}^T(\theta)\right] \text{vec}\{\mathbf{R}\}. \end{aligned} \tag{6}$$

Let $\Delta\theta$ be an appropriately chosen step-size for the angle in the design, then more explicitly we have

$$\begin{bmatrix} P(0^\circ) \\ P(\Delta\theta) \\ P(2\Delta\theta) \\ \vdots \\ P(180^\circ) \end{bmatrix} = \begin{bmatrix} \mathbf{e}^T(0^\circ) \otimes \mathbf{e}^H(0^\circ) \\ \mathbf{e}^T(\Delta\theta) \otimes \mathbf{e}^H(\Delta\theta) \\ \mathbf{e}^T(2\Delta\theta) \otimes \mathbf{e}^H(2\Delta\theta) \\ \vdots \\ \mathbf{e}^T(180^\circ) \otimes \mathbf{e}^H(180^\circ) \end{bmatrix} \text{vec}\{\mathbf{R}\}, \tag{7}$$

which represents a set of linear equations. Denote (7) as

$$\mathbf{p} = \mathbf{A}\,\text{vec}\{\mathbf{R}\}, \tag{8}$$

where $\mathbf{A} \in \mathbb{C}^{[(180/\Delta\theta)+1] \times N_\text{T}^2}$, then the linear equations corresponding to the passband can be extracted to form $\mathbf{p}_\text{P} = \mathbf{A}_\text{P}\,\text{vec}\{\mathbf{R}\}$, and those corresponding to the stopband to form $\mathbf{p}_\text{S} = \mathbf{A}_\text{S}\,\text{vec}\{\mathbf{R}\}$. For simplicity, the desired beampatterns are chosen symmetrical w.r.t. 90°. The transition bandwidth is denoted as ΔB. Let the passband edges be θ_P ($90^\circ < \theta_\text{P} < 180^\circ$) and $180^\circ - \theta_\text{P}$, thus the stopband edges are $\theta_\text{S} \triangleq \theta_\text{P} + \Delta B$ and $180^\circ - \theta_\text{S}$. Let the passband ripple be δ, and the stopband attenuation be ε, then the FP is formulated as

$$\begin{aligned}
&\text{find} \;\; \mathbf{R} \\
&\;\;\text{s.t.} \;\; \mathbf{R} \succeq \mathbf{0}, \\
&\qquad \operatorname{trace}\{\mathbf{R}\} = N_{\mathrm{T}}, \\
&\qquad \max\{\mathbf{A}_{\mathrm{P}}(\theta_{\mathrm{P}}) \operatorname{vec}\{\mathbf{R}\}\} - \min\{\mathbf{A}_{\mathrm{P}}(\theta_{\mathrm{P}}) \operatorname{vec}\{\mathbf{R}\}\} \le \delta, \\
&\qquad \max\{\mathbf{A}_{\mathrm{S}}(\theta_{\mathrm{P}}, \Delta B) \operatorname{vec}\{\mathbf{R}\}\} \le \varepsilon. \qquad (9)
\end{aligned}$$

As all the constraints are convex, the FP problem can be efficiently solved via public domain optimization tools [17]. Let N_{bot} and N_{top} denote the minimum and maximum number of antennas that are used by a MIMO radar system respectively, then the algorithm to minimize the number of transmit antennas is summarized as follows:

Step 0: Set designing parameters: θ_{P}, ΔB, δ, and ε.

Step 1: Start at initial value $N_{\mathrm{T}} \leftarrow N_{\text{bot}}$.

Step 2: Run iterative solution to (9).

Step 3: If the problem is not feasible, then $N_{\mathrm{T}} \leftarrow N_{\mathrm{T}}+1$, and go to **Step 4**. Else if the problem is feasible, then the problem is solved, and the $\mathbf{R}$ obtained is the optimal solution.

Step 4: If $N_{\mathrm{T}} \le N_{\text{top}}$, go to **Step 2**. Else if If $N_{\mathrm{T}} > N_{\text{top}}$, then the problem cannot be solved, N_{top} should be increased.

The existing methods for MIMO radar transmit beampattern design [8, 9, 11–15] usually minimize the least squared error between the desired beampattern and the designed one, or maximize the ratio between the energy within and outside of the focusing section. These methods do not take control of the passband ripples, transition bandwidth, and stopband attenuation. Instead, our formulation is superior to the existing ones, because it specifies all the design parameters a prior, and yields the minimum number of transmit antennas.

3.2. Spectral Factorization for Phased-Array Radar

The authors in [9] convert their MIMO radar formulation to a phased-array radar formulation by adding the constraint $\operatorname{rank}\{\mathbf{R}\} = 1$. This constraint is nonconvex and complicates the solution. Moreover, the iterative solution fails to achieve the desired beampattern. In fact, such a formulation may not be suitable for phased-array radar. In this paper, we consider the transmit beampattern design problem for phased-array radar using filter design techniques.

It can be seen from Fig. 1 that each branch of the MISO filter is corresponding to a SISO filter, and can be considered as the beampattern design problem for phased-array radar. Let the autocorrelation coefficients associated with $\mathbf{w}$ be

$$r(k) = \sum_{i=-N_{\mathrm{T}}+1}^{N_{\mathrm{T}}-1} w(i)w(i+k), \qquad (10)$$

where $r(k) = r(-k)$, and the vector form of $r(k)$ be

$$\mathbf{r} = [r(0), r(1), \cdots, r(N_{\mathrm{T}}-1)]^T. \qquad (11)$$

Instead of directly solving for $\mathbf{w}$, [16] uses $\mathbf{r}$ as the optimization variable. The squared frequency response of the filter can be represented by the Fourier transform of $\mathbf{r}$, i.e.,

$$\begin{aligned}
R(f) &= \sum_{k=-N_{\mathrm{T}}+1}^{N_{\mathrm{T}}-1} r(k) e^{-j2\pi f k} \qquad (12) \\
&= \left(\sum_{k=0}^{N_{\mathrm{T}}-1} w(k) e^{-j2\pi f k}\right)\left(\sum_{k=0}^{N_{\mathrm{T}}-1} w(k) e^{-j2\pi f k}\right)^* \\
&= |W(f)|^2 \quad \text{(squared frequency response)} \qquad (13) \\
&= 1 + 2\begin{bmatrix} \cos(2\pi f) \\ \cos(2\pi 2 f) \\ \vdots \\ \cos(2\pi(N_{\mathrm{T}}-1)f) \end{bmatrix}^T \mathbf{r}, \qquad (14)
\end{aligned}$$

where $f \in [-0.5, 0.5]$ is the normalized frequency. Replacing $-2\pi f$ by $\cos\theta$, the FP for phased-array radar transmit beampattern design is formulated as:

$$\begin{aligned}
&\text{find} \;\; \mathbf{r} \\
&\;\;\text{s.t.} \;\; [1, 0, 0, \cdots, 0]\mathbf{r} = N_{\mathrm{T}}, \\
&\qquad \max\{\mathbf{B}_{\mathrm{P}}(\theta_{\mathrm{P}})\mathbf{r}\} - \min\{\mathbf{B}_{\mathrm{P}}(\theta_{\mathrm{P}})\mathbf{r}\} \le \delta, \\
&\qquad \max\{\mathbf{B}_{\mathrm{S}}(\theta_{\mathrm{P}}, \Delta B)\mathbf{r}\} \le \varepsilon. \qquad (15)
\end{aligned}$$

According to (14), a matrix $\mathbf{B}$ is formed as

$$\begin{bmatrix}
1 & 2\cos(\cos 0^\circ) & \cdots & 2\cos((N_{\mathrm{T}}-1)\cos 0^\circ) \\
1 & 2\cos(\cos \Delta\theta) & \cdots & 2\cos((N_{\mathrm{T}}-1)\cos \Delta\theta) \\
1 & 2\cos(\cos 2\Delta\theta) & \cdots & 2\cos((N_{\mathrm{T}}-1)\cos 2\Delta\theta) \\
\vdots & \vdots & \ddots & \vdots \\
1 & 2\cos(\cos 180^\circ) & \cdots & 2\cos((N_{\mathrm{T}}-1)\cos 180^\circ)
\end{bmatrix},$$

and $\mathbf{B}_{\mathrm{P}}$ and $\mathbf{B}_{\mathrm{S}}$ are extracted from $\mathbf{B}$ in the same way as $\mathbf{A}_{\mathrm{P}}$ and $\mathbf{A}_{\mathrm{S}}$ are extracted from $\mathbf{A}$. Once $\mathbf{r}$ is optimized, the spectral factorization techniques can be used to obtain $\mathbf{w}$, which are summarized in the appendix of [16].

4. SIMULATIONS

In this section, we show via simulations that i) for MIMO radar, the proposed FP formulation has better performance as compared to several commonly used methods, such as the $l1$-norm beampattern matching design [9], the discrete prolate spheroidal sequence (DPSS) based method [13], and the least squares (LS) method with eigenvalue modification [14]; ii) via an appropriate design, e.g., the spectral factorization method, phased-array radar is able to achieve similar transmit beampatterns as compared to MIMO radar, using identical number of transmit antennas and waveform energy. This is in contrary to the arguments in [9].

The simulation results are shown in Fig. 2, where the passband (spatial section for energy focusing) is $[65^\circ, 115^\circ]$,

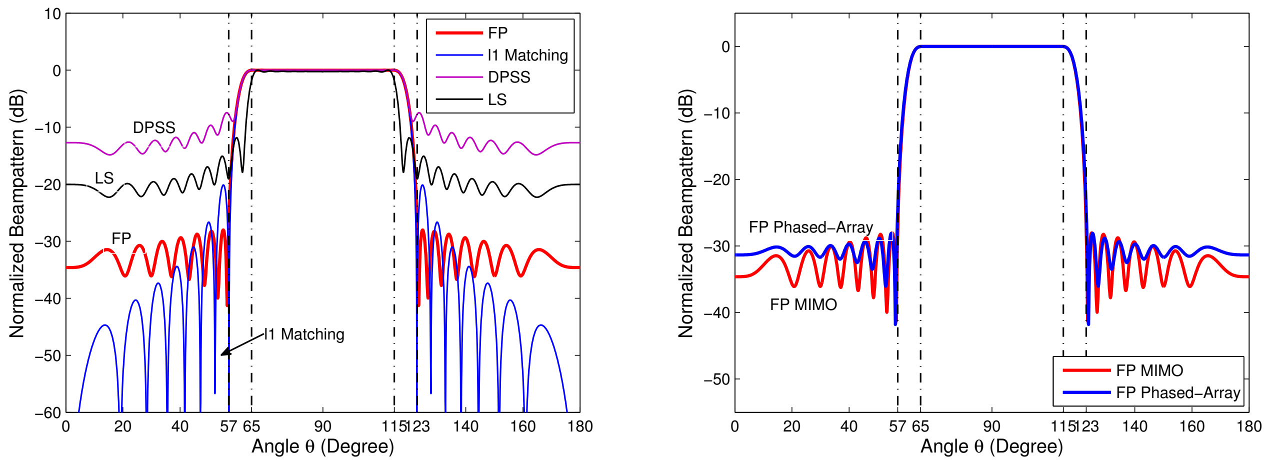

Fig. 2. Simulation results of the transmit beampatterns, where $N_{\mathrm{T}} = 29$, $\theta_{\mathrm{P}} = 115°$, $\Delta B = 8°$, $\delta = \varepsilon = 0.1$. (a) For MIMO radar. The proposed FP formulation compared with existing methods. (b) FP formulation for MIMO radar compared with FP formulation and spectral factorization for phased-array radar.

$\Delta B = 8°$, and $\delta = \varepsilon = 0.1$. The total energy of the transmitted waveforms, i.e., $\mathrm{tr}\{\mathbf{R}\} = N_{\mathrm{T}}$, and $\|\mathbf{w}\|^2 = 1$. We first use the proposed MIMO FP algorithm to obtain the minimum number of transmit antennas required to meet the specifications, which turns out to be $N_{\mathrm{T}} = 29$. For $N_{\mathrm{T}} = 29$, we have also done the design using existing methods as well as the design using phased-array radar.

The designed transmit beampatterns using the proposed FP formulation and existing methods are shown in Fig. 2 (a). It can be seen that the proposed method yields the lowest sidelobe levels. It also has the lowest ripple levels in the passband. This is because the FP formulation directly constrains the ripples while the existing methods do not. The LS solution has very sharp transition bandwidth, but the sidelobe peak is relatively high. In general, the existing methods have no control of the stopband levels, which result in sidelobe levels higher than the proposed method.

Fig. 2 (b) provides a comparison between the transmit beampatterns designed for MIMO and phased-array radar. The two solutions have nearly identical mainlobes and sidelobe peaks. This indicates that with appropriate designs, both MIMO and phased-array radar result in transmit beampatterns having similar passband, stopband, transition band, and passband/stopband ripples. The designs use same number of transmit antennas and total waveform energy.

5. CONCLUSION

This paper considers the transmit beampattern design for MIMO and phased-array radar, from the perspective of FIR filter design. For MIMO radar, we propose the FP formulation and an iterative algorithm, which takes control of all the design parameters and yields the minimum number of transmit antennas required. The proposed method is shown to be better than the existing methods as it results in lowest sidelobe levels and smallest passband/stopband ripples. It has been argued in the literature that phased-array radar is not able to have flexibility in design as MIMO radar because of the lack of waveform diversity. However we illustrate that via appropriate design, i.e., the FP formulation and the spectral factorization technique, phased-array radar can achieve a very similar transmit beampattern as the one obtained using MIMO radar.

6. RELATION TO PRIOR WORK

The methods proposed in [8, 9, 11–15] for the design of MIMO radar transmit beampattern consider various design criteria and formulations with iterative and closed-form solutions. However, they ignore several important parameters such as the passband/stopband ripples and the transition bandwidth. This problem is revealed when we consider the design as a MISO FIR filter design problem. Hence we propose an FP MISO filter design formulation with constraints on those parameters. The proposed method provides a general design which allows full control of the parameters. The work presented here furthers and completes the prior work. In addition, we reconsider the transmit beampattern design using phased-array radar, which has been argued in [9] and other works as unable to achieve attractive transmit beampatterns as compared with MIMO radar. We use the spectral factorization technique discussed in [16] for efficient SISO FIR filter design for phased-array radar. The simulation results using this method indicate that phased-array radar is able to achieve very similar transmit beampatterns as compared to MIMO radar.

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