ON PASSIVE TDOA AND FDOA LOCALIZATION USING TWO SENSORS WITH NO TIME OR FREQUENCY SYNCHRONIZATION

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ABSTRACT

Traditional passive localization based on Time-Difference of Arrival (TDOA) or Frequency-Difference of Arrival (FDOA) usually involves several remote sensors, which require precise time-synchronization and frequency-locking among them. The need for such time or frequency alignment sometimes poses a serious operational challenge on the system. In addition, it is often desired to keep the number of sensors to a minimum. In this work we look into the operationally-simplest scenario in this context: using only two sensors, without any synchronization or locking. When at least one of the sensors, or the transmitting target, is moving at some considerable speed, it is still possible to localize the target, based on a few TDOA and / or FDOA measurements, by considering the time- and frequency-offsets as additional unknown parameters. We analyze the associated performance bound and propose a Maximum Likelihood estimation approach. The attainable accuracy and its dependence on geometry are demonstrated numerically and in simulation.

***Index Terms*—** TDOA, FDOA, passive localization, two sensors, unsynchronized, unlocked.

1. INTRODUCTION

Passive localization systems aim to estimate the location of a non-cooperating transmitter, based on reception of the transmitted signal in several, spatially diverse sensors. Some methods rely on estimating localization-related parameters locally at each sensor, and then combining these estimates into an estimate of the transmitter's location. Such methods are based, for example, on the Received Signal Strength (RSS) or Direction of Arrival (DOA) estimates, and do not require any joint processing of the raw signals intercepted at the sensors. However, other, more advanced methods, require joint processing of the signals intercepted at two or more sensors - these are methods based, e.g., on Time-Difference of Arrival (TDOA), Frequency Difference of Arrival (FDOA) or Direct Position Determination (DPD, e.g., [1], [2]).

For TDOA-based methods, common practice requires precise time-synchronization between sensors. Likewise, for FDOA-based methods precise frequency-locking is required. Obviously, for joint TDOA- and FDOA-based methods (and also for DPD-based methods) both types of synchronization are typically required. Such a requirement for precise time / frequency alignment (especially in the context of electromagnetic signals, as opposed to acoustic signals) often complicates the system design, especially when the sensors are remote and only a relatively low-bandwidth internal communication channel is available. Reliance on auxiliary systems (such as a beacon signal at a known position, or GPS) for synchronization and locking is sometimes not a desirable or feasible option, either. Nevertheless, in some scenarios precise time synchronization or frequency locking are not imperative.

For example, in the context of self-localizing wireless sensor-networks (WSN), e.g., [3], several methods have been proposed for non-synchronized TDOA-based self localization, e.g. by Rydstrom *et al.* [4] and by Fan *et al.* [5] (based on "differential TDOA" (dTDOA)), or by Xu *et al.* (based on specially transmitted signals by one or more anchor nodes). Joint synchronization and Time of Arrival (TOA) based passive localization is considered by Jean and Weiss in [6]. However, in such WSN-contexts the large number of sensors / transmitters and the cooperative nature of at least some of the transmitters play a key role which is absent in scenarios of passive localization (by very few sensors) of a single non-cooperating transmitter.

Similarly, uncertainties in the sensors' positions in the context of passive localization were addressed by Ho *et al.* in [7] and [8]. Such uncertainties are partly equivalent to synchronization uncertainties, but are of a different nature and bear different effects on the resulting estimation accuracy.

In this work we consider the minimal required number of sensors for passive TDOA/FDOA-based localization, namely two sensors only. If at least one of the sensors is moving at a sufficient (known) velocity, and the sensors' positions are known, then normally a single TDOA and FDOA measurement between these two sensors is sufficient for localizing a static transmitter, assuming precise time-synchronization and frequency-locking of the sensors. Nevertheless, if several consecutive TDOA/FDOA measurements (sufficiently separated in time) are available in this scenario, the transmitter's location can also be estimated to within reasonable accuracy even in the absence of any time or frequency alignment between the two sensors. An only requirement is that the time and frequency offsets between the sensors are constant throughout the measurements period, so that these offsets can be considered as additional fixed (nuisance) parameters. Such an assumption can be satisfied by using sufficiently stable (slowly-drifting) but independent time and frequency references at each sensor. A somewhat similar approach is also possible when the sensors are static but the target is moving at a constant (but unknown) velocity.

In the following section we formulate the operational model and the associated assumptions. Then, in Section 3 we derive the Cramér-Rao bound (CRB) on the localization accuracy and outline an iterative Maximum Likelihood (ML) estimation approach. The resulting performance, its relation to the CRB and its dependence on the geometric parameters are illustrated by simulation in Section 4. Conclusions are summarized in Section 5.

2. THE CONSIDERED OPERATION MODELS

We assume that two sensors are available, whose known positions at time t are denoted $\boldsymbol{p}_1(t)$ and $\boldsymbol{p}_2(t)$, moving at known velocities $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ (resp.) - which for simplicity are assumed constant for the duration of the measurements period (and one of which might be zero for a static sensor). The transmitter's unknown position is denoted $\boldsymbol{q}$. In the basic scenario we shall assume that the transmitter is static; However, the model can be extended to the case of a transmitter which is moving at a constant (unknown) speed, a scenario which enables the use of two static sensors.

The difference vectors between the positions of the sources and the transmitter at time t are given by

$$\boldsymbol{d}_i(t;\boldsymbol{q}) = \boldsymbol{q} - \boldsymbol{p}_i(t)\ , \quad i = 1, 2. \tag{1}$$

and the respective ranges are given by

$$r_i(t;\boldsymbol{q}) = |\boldsymbol{d}_i(t;\boldsymbol{q})| = \sqrt{\boldsymbol{d}_i^T(t;\boldsymbol{q})\boldsymbol{d}_i(t;\boldsymbol{q})}\ , \quad i = 1, 2. \tag{2}$$

The resulting delay differences in the arrivals to the sensors at time t is $\tau(t;\boldsymbol{q}) = (r_1(t;\boldsymbol{q}) - r_2(t;\boldsymbol{q}))/c$, where c is the propagation speed[1].

If the bandwidth of the transmitted signal is small relative to its carrier frequency (denoted f_c), the Doppler effect due to the relative motion of the sensors with respect to (w.r.t.) the transmitter reduces to a frequency-shift of the received carrier frequency (with negligible effect on the signal's waveform). The frequency-shifts observed by the sensors (at time t) are given by

$$\nu_i(t;\boldsymbol{q}) = \frac{f_c}{c} \cdot \boldsymbol{v}_i^T \cdot \frac{\boldsymbol{d}_i(t;\boldsymbol{q})}{r_i(t;\boldsymbol{q})}\ , \quad i = 1, 2, \tag{3}$$

and the difference between these shifts is given by $\nu(t;\boldsymbol{q}) = \nu_1(t;\boldsymbol{q}) - \nu_2(t;\boldsymbol{q})$.

[1] $299,792,458$ $[m/s]$ for electromagnetic waves in free space.

The TDOA and FDOA between the two sensors is estimated at N time-instances $\{t_n\}_{n=1}^N$, where it is assumed that the duration of each measurement-interval is sufficiently short, so that the TDOA and FDOA variations within each interval are small (as opposed to their possible variations between intervals). In the absence of time-synchronization and frequency-locking at the sensors, the estimated TDOA and FDOA at the n-th interval are modeled (resp.) as

$$\begin{aligned}\widehat{\tau}_n &= \tau(t_n;\boldsymbol{q}) + \delta_\tau + w_n^\tau \\ \widehat{\nu}_n &= \nu(t_n;\boldsymbol{q}) + \delta_\nu + w_n^\nu\end{aligned} \tag{4}$$

(for $n = 1, \ldots, N$), where δ_τ and δ_ν are the inherent, unknown time-offset and frequency-offset between sensors, resulting from the mis-synchronization and mis-lock, and where w_n^τ and w_n^ν are zero-mean estimation errors, which are assumed to be Normally distributed and mutually statistically independent in each measurement and between measurements. Note that the assumption of independence of w_τ and w_ν in each measurement is justified, e.g., when the transmitted signal is a Wide-Sense Stationary (WSS) process, but may be unjustified otherwise, see [9]. However, we assume WSS signals in here, hence the independence assumption.

As mentioned earlier, we assume that the time- and frequency-drifts (if any) of the sensors are slow enough to justify the assumption of constant offsets δ_τ and δ_ν throughout the operation period of N consecutive measurements.

The vector $\boldsymbol{\theta}$ of unknown parameters therefore consists of the target's fixed position $\boldsymbol{q}$, as well as of the unknown offsets δ_τ and δ_ν, namely $\boldsymbol{\theta} = [\boldsymbol{q}^T\ \delta_\tau\ \delta_\nu]^T$.

Note that the sensors' positions and velocities are assumed to be precisely known, to within negligible errors. Obviously, such knowledge implies a certain degree of time-synchronization between the sensors and/or the central processing station, so as to correctly translate the known velocities and initial positions into the sensors' positions at time t_n. Nevertheless, the precision requirements for such knowledge are far less stringent than the precision requirements for synchronization for the purpose of bias-free TDOA estimation: for the former an accuracy of the order of tens of milliseconds would usually be sufficient (implying sensors' location errors of the order of centimeters), whereas for the latter accuracies of the order of nanoseconds would usually be required.

In the next section we derive the CRB on the attainable performance and outline an iterative ML estimation approach.

3. LOCATION ESTIMATION AND BOUNDS

The model (4) can be expressed in vector form as

$$\begin{bmatrix}\boldsymbol{y}^\tau \\ \boldsymbol{y}^\nu\end{bmatrix} = \begin{bmatrix}\boldsymbol{h}^\tau(\boldsymbol{\theta}) \\ \boldsymbol{h}^\nu(\boldsymbol{\theta})\end{bmatrix} + \begin{bmatrix}\boldsymbol{w}^\tau \\ \boldsymbol{w}^\nu\end{bmatrix}, \tag{5}$$

where $\boldsymbol{y}^\tau \triangleq [\widehat{\tau}_1\ \cdots \widehat{\tau}_N]^T$ and $\boldsymbol{y}^\nu \triangleq [\widehat{\nu}_1\ \cdots \widehat{\nu}_N]^T$ are the concatenated TDOA and FDOA measurements (estimates), $\boldsymbol{w}^\tau$

and $\boldsymbol{w}^{\nu}$ are the concatenated respective estimation errors, and the elements of $\boldsymbol{h}^{\tau}(\boldsymbol{\theta})$ and $\boldsymbol{h}^{\nu}(\boldsymbol{\theta})$ are (for $n=1,...,N$)

$$h_n^{\tau}(\boldsymbol{\theta}) = \frac{1}{c}\left[r_1(t_n;\boldsymbol{q}) - r_2(t_n;\boldsymbol{q})\right] + \delta_{\tau}$$
$$h_n^{\nu}(\boldsymbol{\theta}) = \frac{f_c}{c}\left[\frac{\boldsymbol{v}_1^T\boldsymbol{d}_1(t_n;\boldsymbol{q})}{r_1(t_n;\boldsymbol{q})} - \frac{\boldsymbol{v}_2^T\boldsymbol{d}_2(t_n;\boldsymbol{q})}{r_2(t_n;\boldsymbol{q})}\right] + \delta_{\nu}. \quad (6)$$

Due to the Gaussianity and independence assumptions regarding $\boldsymbol{w}^{\tau}$ and $\boldsymbol{w}^{\nu}$, the log probability distribution function of the measurements $\boldsymbol{y}^{\tau}$ and $\boldsymbol{y}^{\nu}$ is given by

$$\log f(\boldsymbol{y}^{\tau},\boldsymbol{y}^{\nu};\boldsymbol{\theta}) = c - \frac{1}{2\sigma_{\tau}^2}\|\boldsymbol{y}^{\tau} - \boldsymbol{h}^{\tau}(\boldsymbol{\theta})\|^2 - \frac{1}{2\sigma_{\nu}^2}\|\boldsymbol{y}^{\nu} - \boldsymbol{h}^{\nu}(\boldsymbol{\theta})\|^2, \quad (7)$$

where c is an irrelevant constant and where σ_{τ}^2 and σ_{ν}^2 are the variances of the TDOA and FDOA estimation errors w_n^{τ} and w_n^{ν} (resp.), which are assumed constant for all n.

The CRB on unbiased estimation of $\boldsymbol{\theta}$ is given by the inverse of the Fisher Information Matrix (FIM), which in our model takes the form

$$\boldsymbol{J}_{\boldsymbol{\theta}} = \frac{1}{\sigma_{\tau}^2}\sum_{n=1}^{N}(\boldsymbol{g}_n^{\tau})^T\boldsymbol{g}_n^{\tau} + \frac{1}{\sigma_{\nu}^2}\sum_{n=1}^{N}(\boldsymbol{g}_n^{\nu})^T\boldsymbol{g}_n^{\nu}, \quad (8)$$

where $\boldsymbol{g}_n^{\tau} \triangleq \partial h_n^{\tau}(\boldsymbol{\theta})/\partial\boldsymbol{\theta}$ and $\boldsymbol{g}_n^{\nu} \triangleq \partial h_n^{\nu}(\boldsymbol{\theta})/\partial\boldsymbol{\theta}$ are gradient vectors, given as $\boldsymbol{g}_n^{\tau} = [\boldsymbol{\gamma}_n^{\tau}\ 1\ 0]$ and $\boldsymbol{g}_n^{\nu} = [\boldsymbol{\gamma}_n^{\nu}\ 0\ 1]$, with

$$\boldsymbol{\gamma}_n^{\tau} \triangleq \frac{\partial h_n^{\tau}}{\partial\boldsymbol{q}} = \frac{1}{c}\left[\frac{\boldsymbol{d}_1^T(t_n)}{r_1(t_n)} - \frac{\boldsymbol{d}_2^T(t_n)}{r_2(t_n)}\right] \quad (9)$$

$$\boldsymbol{\gamma}_n^{\nu} \triangleq \frac{\partial h_n^{\nu}}{\partial\boldsymbol{q}} = \frac{f_c}{c}\left[\left(\frac{\boldsymbol{v}_1^T}{r_1(t_n)} - \frac{\boldsymbol{v}_1^T\boldsymbol{d}_1(t_n)\boldsymbol{d}_1^T(t_n)}{r_1^3(t_n)}\right) - \left(\frac{\boldsymbol{v}_2^T}{r_2(t_n)} - \frac{\boldsymbol{v}_2^T\boldsymbol{d}_2(t_n)\boldsymbol{d}_2^T(t_n)}{r_2^3(t_n)}\right)\right] \quad (10)$$

(note that we omitted the explicit dependence of these terms on $\boldsymbol{\theta}$, just for simplifying the notations). The FIM can therefore be expressed as

$$\boldsymbol{J}_{\boldsymbol{\theta}} = N\cdot\begin{bmatrix} \frac{1}{\sigma_{\tau}^2}\boldsymbol{R}_{\tau} + \frac{1}{\sigma_{\nu}^2}\boldsymbol{R}_{\nu} & \frac{1}{\sigma_{\tau}^2}\boldsymbol{m}_{\tau} & \frac{1}{\sigma_{\nu}^2}\boldsymbol{m}_{\nu} \\ \frac{1}{\sigma_{\tau}^2}\boldsymbol{m}_{\tau}^T & \frac{1}{\sigma_{\tau}^2} & 0 \\ \frac{1}{\sigma_{\nu}^2}\boldsymbol{m}_{\nu}^T & 0 & \frac{1}{\sigma_{\tau}^2} \end{bmatrix}, \quad (11)$$

where

$$\boldsymbol{R}_{\tau} \triangleq \frac{1}{N}\sum_{n=1}^{N}(\boldsymbol{\gamma}_n^{\tau})^T\boldsymbol{\gamma}_n^{\tau} \quad \boldsymbol{R}_{\nu} \triangleq \frac{1}{N}\sum_{n=1}^{N}(\boldsymbol{\gamma}_n^{\nu})^T\boldsymbol{\gamma}_n^{\nu} \quad (12)$$

$$\boldsymbol{m}_{\tau} \triangleq \frac{1}{N}\sum_{n=1}^{N}(\boldsymbol{\gamma}_n^{\tau})^T \quad \boldsymbol{m}_{\nu} \triangleq \frac{1}{N}\sum_{n=1}^{N}(\boldsymbol{\gamma}_n^{\nu})^T. \quad (13)$$

The CRB on unbiased estimation of $\boldsymbol{q}$ is given by the respective upper-left block of the inverse $\boldsymbol{J}_{\boldsymbol{\theta}}^{-1}$. Applying standard block-matrix inversion, we observe that this block can also be expressed as the inverse of Schur complement

$$\boldsymbol{J}_{\boldsymbol{q}} \triangleq N\cdot\left(\frac{1}{\sigma_{\tau}^2}\boldsymbol{R}_{\tau} + \frac{1}{\sigma_{\nu}^2}\boldsymbol{R}_{\nu} - \left[\frac{1}{\sigma_{\nu}^2}\boldsymbol{m}_{\tau} \quad \frac{1}{\sigma_{\tau}^2}\boldsymbol{m}_{\nu}\right]\begin{bmatrix}\frac{1}{\sigma_{\tau}^2} & 0 \\ 0 & \frac{1}{\sigma_{\nu}^2}\end{bmatrix}^{-1}\left[\frac{1}{\sigma_{\nu}^2}\boldsymbol{m}_{\tau} \quad \frac{1}{\sigma_{\tau}^2}\boldsymbol{m}_{\nu}\right]^T\right)$$
$$= \frac{N}{\sigma_{\tau}^2}\left(\boldsymbol{R}_{\tau} - \boldsymbol{m}_{\tau}\boldsymbol{m}_{\tau}^T\right) + \frac{N}{\sigma_{\nu}^2}\left(\boldsymbol{R}_{\nu} - \boldsymbol{m}_{\nu}\boldsymbol{m}_{\nu}^T\right). \quad (14)$$

Note that the "loss of information" due to the mis-synchronization and mis-locking is reflected in the subtraction of the respective terms containing the products of means $\boldsymbol{m}_{\tau}\boldsymbol{m}_{\tau}^T$ and $\boldsymbol{m}_{\nu}\boldsymbol{m}_{\nu}^T$, since the FIM for the case of perfect time-synchronization and frequency-locking is obviously given by the respective upper-left block of $\boldsymbol{J}_{\boldsymbol{\theta}}$, denoted

$$\boldsymbol{J}_{\boldsymbol{q}}^0 \triangleq \frac{1}{\sigma_{\tau}^2}\boldsymbol{R}_{\tau} + \frac{1}{\sigma_{\nu}^2}\boldsymbol{R}_{\nu} \succeq \boldsymbol{J}_{\boldsymbol{q}}. \quad (15)$$

This loss obviously depends on the specific geometry of the operation scenario, through the departure of the mean gradient vectors $\boldsymbol{m}_{\tau}$ and $\boldsymbol{m}_{\nu}$ from zero, which in turn depends on the path taken by the moving sensor(s) w.r.t. the target. Note further that each of the two components of $\boldsymbol{J}_{\boldsymbol{q}}$ in (14) reflects the information due to TDOA and FDOA measurements (resp.) alone, and can yield the associated TDOA-only or FDOA-only bound. However, since no hardware requirements on synchronization or locking are needed in order to exploit both modalities, it seems wasteful to exploit just one.

The CRB on the mean square (matrix) error in unbiased estimation of the transmitter's location is given by the inverse of $\boldsymbol{J}_{\boldsymbol{q}}$ in the unsynchronized, unlocked case, and by the inverse of $\boldsymbol{J}_{\boldsymbol{q}}^0$ in the synchronized and locked case. In the case of ML estimation, the asymptotic Normality and efficiency of ML allows to translate the CRB matrix into a "confidence ellipse" about the true location (containing the estimate with some prescribed probability p).

In order to obtain the ML estimate of $\boldsymbol{\theta}$ we need to maximize the log-likelihood in (7) w.r.t. $\boldsymbol{\theta}$. Obviously, this is a Least-Squares problem which can be minimized iteratively, e.g., using Gauss-Newton method (see, e.g., [10]). Starting with some initial guess of $\boldsymbol{\theta}$, denoted $\widehat{\boldsymbol{\theta}}^0$, the iterations proceed for $k=0,1,2,...$ according to

$$\widehat{\boldsymbol{\theta}}^{k+1} = \widehat{\boldsymbol{\theta}}^k + \boldsymbol{J}_{\boldsymbol{\theta}}^{-1}(\widehat{\boldsymbol{\theta}}^k)\cdot \left[\frac{1}{\sigma_{\tau}^2}\sum_{n=1}^{N}(\boldsymbol{g}_n^{\tau})^T(\widehat{\tau}_n - h_n^{\tau}(\widehat{\boldsymbol{\theta}}^k)) + \frac{1}{\sigma_{\nu}^2}\sum_{n=1}^{N}(\boldsymbol{g}_n^{\nu})^T(\widehat{\nu}_n - h_n^{\nu}(\widehat{\boldsymbol{\theta}}^k))\right], \quad (16)$$

such that $\boldsymbol{g}_n^{\tau}$ and $\boldsymbol{g}_n^{\nu}$ are obtained using the expressions in (9), (10), calculated at $\widehat{\boldsymbol{\theta}}^k$, and are also used for calculating the local FIM $\boldsymbol{J}_{\boldsymbol{\theta}}^{-1}(\widehat{\boldsymbol{\theta}}^k)$ using (8) or (11).

An apparently more appealing approach for obtaining the ML estimate is to exploit the linear dependence of $\boldsymbol{h}^{\tau}(\boldsymbol{\theta})$ and

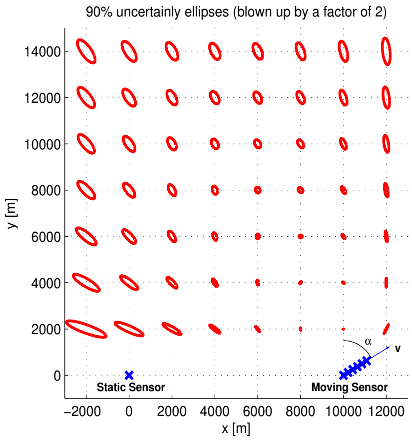

Fig. 1. A possible operation scenario: One static sensor and one moving sensor, and the resulting CRB-based confidence ellipses at various locations.

$\boldsymbol{h}^{\nu}(\boldsymbol{\theta})$ on the nuisance parameters δ_{τ} and δ_{ν} (resp.), substituting the closed-form minimizers w.r.t. these parameters into the LS criterion. Consequently, the criterion would be expressed in terms of the parameters of interest $\boldsymbol{q}$ alone (such an approach is taken, e.g., in [6]). However, it can be shown that given the same initial guess for $\boldsymbol{q}$, both iterative algorithms would yield the same sequence of estimates of $\boldsymbol{q}$. Therefore, although the latter approach may sometimes offer numerical or computational advantages (due to the reduced dimensionality), we shall not pursue that approach in more detail in here.

4. ANALYSIS AND SIMULATION RESULTS

We consider an operation scenario depicted in Figure 1 (in a two-dimensional space), in which one static sensor is located at $(0, 0)$, and a second, moving sensor, is located at $(10\text{Km}, 0)$ at $t = 0$ and travels at a constant speed of 25[m/s] (90[Km/h]) at an angle $\alpha = 75^{o}$ w.r.t. the y-axis. TDOA and FDOA measurements (unsynchronized, unlocked) are taken every 10 seconds during a period of 50 seconds, amounting to $N = 6$ measurements. We assume that the estimation variances of the TDOA and FDOA measurements are $\sigma_{\tau}^2 = (10[\text{ns}])^2$ and $\sigma_{\nu}^2 = (10[\text{mHz}])^2$ (resp.). The resulting CRB-based 90% confidence-ellipses are depicted (blown-up by a factor of two on the same scale, for visibility), for a grid of possible target-locations, in Figure 1.

To demonstrate the dependence of the performance on the geometry, we present in Figure 2 the dependence of the long axis and short axis of the CRB-based 90% confidence ellipse at location $(8[\text{Km}], 8[\text{Km}])$ on the direction of motion α of the moving sensor. For comparison, we also present the same for a fully time-synchronized and frequency-locked system, observing roughly a ten-fold performance-loss (in terms of the long axis) due to lack of synchronization and locking. Although such a loss can be considered quite significant, the simplicity gain in implementing an unsynchronized system can often justify the compromised precision, as long as the attainable accuracy is acceptable. Note that, as expected, in

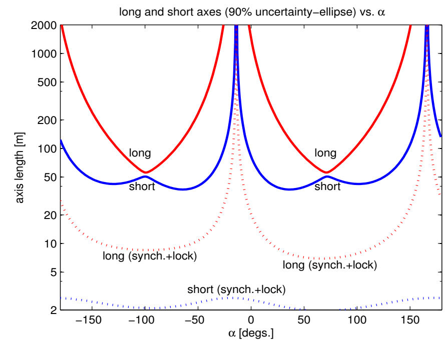

Fig. 2. Long and short axes of the confidence ellipse at (8[Km],8[Km]) vs. the angle α - unsynchronized, unlocked case (solid), synchronized and locked case (dashed).

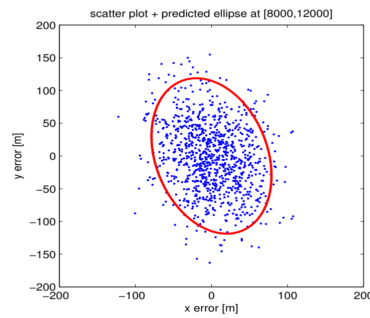

Fig. 3. A CRB-based 90% confidence-ellipse superimposed on the errors in 1000 independent localization trials.

both cases the performance deteriorates when the moving sensor moves directly towards or away from the target, since both the Doppler sensitivity and the relative geographical diversity of the sensor's track are at their worst in this case.

In the last figure we show some simulation results demonstrating the validity of the 90% confidence-ellipse. The ellipse is superimposed on the results of 1000 independent trials, in which the ML estimate of transmitter's location was obtained as outlined in Section 3. The true location was at $(8[\text{Km}], 12[\text{Km}])$, and the TDOA and FDOA measurements were generated by applying random time and frequency biases in addition to the zero-mean, independent Gaussian errors. The iterative estimation algorithm was initialized at $(5[\text{Km}], 5[\text{Km}])$ in each trial.

5. CONCLUSION

We have demonstrated the ability to operate a passive TDOA and FDOA based localization system with just two sensors, with no requirement for precise time-synchronization or frequency-locking between these sensors, as long as at least one of the two (or the target) is moving. We provided explicit expressions for the performance bound, and proposed an iterative approach for ML estimation, which was demonstrated to attain the bound in our simulation. The bound expression enables to evaluate the expected performance in any considered scenario, so as to decide whether the simplified configuration is affordable in terms of the attainable accuracy.

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