SINGLE TARGET DETECTION AND TRACKING USING DIRECTION-OF-ARRIVAL SPECTRUM

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ABSTRACT

This paper presents a track-before-detect algorithm based on the particle filter for simultaneously detecting and tracking a single target with nearly constant velocity. The measurement data available to the algorithm is the raw direction-of-arrival spectrum of the received wideband signals at a single observation platform. The proposed method does not require prior information on the target existence probability and it is capable of detecting a target of very low signal-to-noise ratio (-30dB). Simulation results show that the proposed algorithm demonstrates good performance in detecting and tracking a target with reasonably low errors in the estimated target states over the observation period.

Index Terms— Track-before-detect, modal analysis, minimum variance distortionless response, particle filter

1. INTRODUCTION

The track-before-detect (TBD) approach [1, chapter 11] aims to simultaneously detect and track a target using raw (unthresholded) measurement data [2, 3]. It is particularly useful for targets of low signal-to-noise ratio (SNR) for example in optical or infrared applications [4, 5] and radar systems [6, 7]. Most TBD algorithms were designed based on an optical sensor model which produces a sequence of two-dimensional grey-scale images [8–10]. However, this sensor model may not be applicable in several applications such as sonar [11, 12] and acoustic source localization systems [13].

Recently, sequential Monte Carlo methods, known as particle filters [14], have become popular in recursively tracking targets under the Bayesian framework for nonlinear/non-Gaussian systems. It has also been designed for TBD problems and has been found to be able to detect and track targets of low SNR [15–18]. However, these methods require a statistical model for each pixel of the received measurements, making the algorithms computationally complex. Besides that, they rely on the target appearance/disappearance probabilities which may not be known a priori in practice.

In this paper, we propose a TBD algorithm for a single target under the particle filtering framework with the following properties:

- The received wideband signals at the sensor array on a single observation platform undergo a modal preprocessing step following the description recently presented in [19]. It transforms the signals received at different sensor locations into signals at different modes as described by the Jacobi-Anger expansion of the wavefield [20].
- The TBD algorithm takes as measurement data the direction-of-arrival (DOA) spectrum of the modal preprocessed signals.
- The algorithm proposed redistribute the particles at each recursion so that the particles are concentrated in the area of high likelihood of target existence.
- The target existence probability is estimated directly from the measurements obtained and any prior knowledge is not required.

We use the following notations in this paper: $\mathbb{E}\{\cdot\}$ is the expectation operator, $(\cdot)^{\#}$ is the Moore-Penrose pseudo inverse of a matrix, $[\cdot]^T$ denotes vector or matrix transpose and $(\cdot)^H$ denotes complex conjugate transpose.

2. SYSTEM MODEL

We consider a two-dimensional (2D) detection and tracking system where wideband signals emitted from a single target are received by a sensor array on an observation platform. It is assumed that the target is located on the same plane as the sensor array. Besides that, it is assumed that at most one target could possibly exist at a given time instance, and if the target exists, it moves with a nearly constant velocity and it corresponds to the mainlobe of the DOA spectrum of the received signals. In this paper, we assume that the receiving sensor array is designed following the method proposed in [19] and the signals received would go through a modal preprocessing step before further processing for detecting and tracking the target is carried out. During the modal preprocessing step, the received signals at different sensor locations would be transformed into signals at different modes as described by the Jacobi-Anger expansion of the plane wave equation, so that they can be further processed using narrowband techniques. The details of the measurement model and the target dynamics model are described below.

2.1. Measurement Model

It is assumed that the signals of interest are bandlimited to a frequency band of an octave, *i.e.*, $f = \omega/(2\pi) \in [f_\ell, 2f_\ell]$Hz where f_ℓ is the lowest frequency component of interest. For good operation of the modal preprocessing step, the receiving sensor array consists of two concentric uniform circular arrays with radii respectively chosen as [19]

$$R_1 = \frac{2c}{3f_\ell} \qquad \text{and} \qquad R_2 = 0.5R_1,$$

where c is the speed of signal propagation. It is required that the concentric circular arrays have the same set of sensor angles φ_q for $q = 1, ..., Q$, where Q is the number of sensors in each circular array. The received signal at each sensor contains the signal emitted from the target of interest, $S(\omega)$ and it is corrupted by an additive noise that is assumed to be spatially and temporally white and independent of the signal of interest.

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Let $\boldsymbol{X}_1(\omega)$ and $\boldsymbol{X}_2(\omega)$ be the received signals at the first and second uniform circular array respectively. A modal preprocessing step is carried out on the sum of the received signals at the two arrays by evaluating

$$\boldsymbol{Z}(\omega) = \boldsymbol{J}^{\#}(\omega)\left[\boldsymbol{X}_1(\omega) + \boldsymbol{X}_2(\omega)\right], \quad (1)$$

where $\boldsymbol{J}(\omega)$ is a matrix of size $Q \times (2N+1)$ (N is the number of modes in the received signal[1]) written as

$$\boldsymbol{J}(\omega) = \boldsymbol{K}\left[\boldsymbol{L}_{R_1}(\omega) + \boldsymbol{L}_{R_2}(\omega)\right],$$

where

$$\boldsymbol{K} = \begin{bmatrix} e^{-jN(\varphi_1+\pi/2)} & \dots & e^{jN(\varphi_1+\pi/2)} \\ \vdots & & \vdots \\ e^{-jN(\varphi_Q+\pi/2)} & \dots & e^{jN(\varphi_Q+\pi/2)} \end{bmatrix},$$

$\boldsymbol{L}_{R_i}(\omega)$ for $i = 1, 2$ is a diagonal matrix containing the Bessel functions of different orders with arguments proportional to the radius R_i of the circular array, written as

$$\boldsymbol{L}_{R_i}(\omega) = \begin{bmatrix} J_{-N}\left(\frac{\omega}{c}R_i\right) & & \boldsymbol{0} \\ & \ddots & \\ \boldsymbol{0} & & J_N\left(\frac{\omega}{c}R_i\right) \end{bmatrix}, \, i = 1, 2$$

and $J_n(\cdot)$ are the Bessel functions of the first kind.

As it is assumed that there could be at most one target in the region of interest at any time instance, the received signal vector of size $(2N+1) \times 1$ after the modal preprocessing step (1) can be rewritten as

$$\boldsymbol{Z}(\omega) = \boldsymbol{a}(\phi_{\mathrm{T}})\gamma S(\omega) + \check{\boldsymbol{N}}(\omega), \quad (2)$$

where γ is the gain of the signal $S(\omega)$ emitted from the target located at direction ϕ_{T} from the sensor array, $\boldsymbol{a}(\phi_{\mathrm{T}})$ is the frequency-independent steering vector of the target written as

$$\boldsymbol{a}(\phi_{\mathrm{T}}) = \begin{bmatrix} e^{jN\phi_{\mathrm{T}}} \\ \vdots \\ e^{-jN\phi_{\mathrm{T}}} \end{bmatrix},$$

and $\check{\boldsymbol{N}}(\omega)$ is the noise at the different modes.

Let $\boldsymbol{Z}(\boldsymbol{\omega}_b)$ for $b = 1, ..., B$ denote the signals at the different modes available after the modal preprocessing step (2) at the b^{th} frequency subband where B is the total number of frequency subbands of interest. A beamforming method, minimum variance distortionless response (MVDR) is carried out on $\boldsymbol{Z}(\boldsymbol{\omega}_b)$ for $b = 1, ..., B$, and the power spectrum of the b^{th} frequency subband over a range of DOA values $\phi \in (-\pi, \pi]$ is obtained as

$$P^b(\phi) = \frac{1}{\boldsymbol{a}^H(\phi)\left(\boldsymbol{R}_{zz}^b\right)^{-1}\boldsymbol{a}(\phi)},$$

where $\boldsymbol{R}_{zz}^b = \mathbb{E}\left\{\boldsymbol{Z}(\boldsymbol{\omega}_b)\boldsymbol{Z}^H(\boldsymbol{\omega}_b)\right\}$. As the steering matrix is frequency-invariant, the MVDR spectrum over different frequency subbands can be averaged, *i.e.*, obtaining

$$P(\phi) = \frac{1}{B}\sum_{b=1}^{B} P^b(\phi). \quad (3)$$

The power spectrum $P(\phi)$ in (3) is then used to obtain the likelihood of the samples in the proposed particle-filter-based algorithm according to the DOA value of the samples.

[1]The frequency band over an octave is separated into a number of five frequency subbands, where each subband has a different number of modes [19]

2.2. Target Model

Let $\boldsymbol{x}_k^t = [x_k^t \; y_k^t \; \dot{x}_k^t \; \dot{y}_k^t]^T$ and $\boldsymbol{x}_k^o = [x_k^o \; y_k^o \; \dot{x}_k^o \; \dot{y}_k^o]^T$ denote the target and observer states (2D positions and velocities) respectively in the Cartesian coordinates at time k. We assume a nearly constant velocity motion dynamics for the target, *i.e.*, the dynamics model can be written as

$$\boldsymbol{x}_k = \boldsymbol{F}\boldsymbol{x}_{k-1} + \boldsymbol{v}_{k-1} - \boldsymbol{U}_{k-1,k}, \quad (4)$$

where $\boldsymbol{x}_k \triangleq \boldsymbol{x}_k^t - \boldsymbol{x}_k^o = [x_k \; y_k \; \dot{x}_k \; \dot{y}_k]^T$, is the relative state vector, $\boldsymbol{F}$ is the state transition matrix given by

$$\boldsymbol{F} = \begin{bmatrix} 1 & 0 & \Delta & 0 \\ 0 & 1 & 0 & \Delta \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

Δ is the sampling interval, $\boldsymbol{v}_{k-1}$ is a 4×1 i.i.d. zero mean Gaussian process noise vector and

$$\boldsymbol{U}_{k-1,k} = \begin{bmatrix} x_k^o - x_{k-1}^o - \Delta\dot{x}_{k-1}^o \\ y_k^o - y_{k-1}^o - \Delta\dot{y}_{k-1}^o \\ \dot{x}_k^o - \dot{x}_{k-1}^o \\ \dot{y}_k^o - \dot{y}_{k-1}^o \end{bmatrix}$$

is a vector of deterministic inputs that accounts for the effects of observer accelerations.

The DOA of the target at time k, referenced clockwise positive to the y-axis, can be calculated as

$$\phi_{\mathrm{T},k} = h(\boldsymbol{x}_k) = \arctan\left(\frac{x_k}{y_k}\right).$$

3. PROPOSED PARTICLE FILTER FOR SINGLE TARGET TRACK-BEFORE-DETECT

The particle filter (PF) [14] uses a set of N_{p} weighted samples $\{\boldsymbol{x}_k^i, w_k^i\}_{i=1}^{N_{\mathrm{p}}}$ to approximate the posterior density of the target of interest at each time step k given measurements up to that time step. The target state is then estimated as the expected value of the sample set. The PF has also been designed for TBD applications, *e.g.*, in [17] where a target existence parameter ξ_k is used to represent the probability that a target exist at time instance k.

In this section, an algorithm based on the PF is proposed for simultaneously detecting and tracking a single target. Different from existing algorithms such as [17, 21], prior knowledge of the target existence probability is not required in the proposed algorithm and the measurement data used is the complete DOA spectrum of the received signals $P_k(\phi)$ at each time step k. The proposed algorithm adaptively redistributes the particles at the end of each recursion so that the particles are concentrated in the area where it is more likely for the target to exist. Besides that, resampling and regularization of the particles are carried out when required to improve the distribution of the particles. The pseudo-code of the proposed method is presented in Algorithm 1.

The proposed algorithm is initialized with random samples $\boldsymbol{x}_0^i, i = 1, ..., N_{\mathrm{p}}$ uniformly distributed over the region of interest (*e.g.*, a circular region with a minimum and maximum range of interest, $[r_{\min}, r_{\max}]$) and the particles are given weights w_0^i based on their DOA value, $h(\boldsymbol{x}_0^i)$ proportional to the initial DOA spectrum $P_0(\phi)$, *i.e.*, $w_0^i \propto P_0(h(\boldsymbol{x}_0^i))$. The initial target existence parameter ξ_0 is set to 0. Over the observation period, from time step $k = 1, ..., K$, the algorithm recursively checks for target existence and estimates the target state if it is expected to exist.

When a new DOA spectrum $P_k(\phi)$ is received at time step k, the threshold level for determining target existence is obtained as

$$\gamma_k = \frac{b_k^u - b_k^l}{2\pi}, \tag{5}$$

where b_k^u and b_k^l are the upper and lower borders of the half-power beamwidth of the mainlobe of $P_k(\phi)$ respectively. The samples from the previous time step are propagated one step forward using the target dynamics model (4) as

$$\boldsymbol{x}_k^i \sim p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1}^i).$$

The likelihoods of the new samples are obtained using the new DOA spectrum of the received signals $P_k(\phi)$ as $l_k^i = P_k(h(\boldsymbol{x}_k^i))$.

If the target was not detected during the previous time step, *i.e.*, the target existence parameter $\xi_{k-1} \not> \gamma_{k-1}$, a set of N_b birth particles $\{\boldsymbol{x}_k^i, w_k^i\}_{i=N_\text{p}+1}^{N_\text{p}+N_\text{b}}$ are generated. The range and velocity of the birth particles are uniformly generated while the DOA values of the particles are drawn from the newly received DOA spectrum, $P_k(\phi)$. The birth particles are combined with the original set of particles and they are given weights proportional to their likelihoods based on the DOA spectrum, *i.e.*, $w_k^i \propto l_k^i = P_k(h(\boldsymbol{x}_k^i))$, for $i = 1, ...N_\text{p} + N_\text{b}$. On the other hand, if the target was already detected during the previous time step, the birth particles are not required and the particles are given weights $w_k^i \propto l_k^i w_{k-1}^i$, for $i = 1, ..., N_\text{p}$. The particle weights are then normalized such that $\sum_{i=1}^{N_\text{s}} w_k^i = 1$, where N_s is the total number of particles at this stage.

Let $C_k \subseteq \{1, ..., N_\text{s}\}$ denote the collection of particle indices whose particles lie within the mainlobe of the DOA spectrum $P_k(\phi)$, *i.e.*, the particles that have DOA values within the interval $[b_k^u, b_k^l]$ and range values within the interval $[r_\text{min}, r_\text{max}]$. The sum of the weights of the particles in the mainlobe of $P_k(\phi)$, written as $\tilde{w}_k = \sum_{i \in C_k} w_k^i$, is used as an indication of target existence, *i.e.*, the target existence probability $\xi_k = \tilde{w}_k$. If $\xi_k > \gamma_k$, where γ_k is the threshold level for target existence obtained in (5), the target exists and vice versa. When the target is expected to exist, its state is estimated as

$$\hat{\boldsymbol{x}}_{k|k} = \frac{1}{\tilde{w}_k} \sum_{i \in C_k} w_k^i \boldsymbol{x}_k^i.$$

Before propagating the particles to the next time step, some further processing is carried out to ensure good distribution of the particles. In all cases, the total number of particles to be propagated to the next time step remains at N_p.

- If no target has been detected, the samples are re-initialized so that the set of N_p samples are uniformly distributed over the region of interest and they are given weights proportional to $P_k(\phi)$.
- If a new target has been detected, the particles lying outside the mainlobe of $P_k(\phi)$ are discarded and the remaining samples are resampled and regularized so that the total number of samples is N_p.
- For a target that continues to exist, the samples lying outside the mainlobe of $P_k(\phi)$ are redistributed according to the mean and covariance of the particles inside the mainlobe. The new set of particles $\breve{\boldsymbol{x}}_k^i$ are given new weights $\breve{w}_k^i \propto w_k^i \breve{l}_k^i$ where $\breve{l}_k^i = P'(h(\breve{\boldsymbol{x}}_k^i)), i = 1, ..., N_\text{p}$ is the likelihood of the redistributed set of particles and $P'(\phi)$ is the normalized spectrum such that the mainlobe has values between 0 and 1. The weights are normalized and if the effective sample size, estimated as $\hat{N}_\text{eff} = 1/\sum_{i=1}^{N_\text{p}} (\breve{w}_k^i)^2$, is less than a threshold value N_th, the particles are resampled and regularized.

Algorithm 1 The Proposed Track-Before-Detect Algorithm

Initialization $\boldsymbol{x}_0^i, w_0^i, i = 1, ..., N_\text{p}; \xi_0$
for $k = 1$ to K **do**
  Particle propagation $\boldsymbol{x}_k^i \sim p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1}^i),\ i = 1, ..., N_\text{p}$
  if $\xi_{k-1} \not> \gamma_{k-1}$ **then**
    Draw random samples $\boldsymbol{x}_k^i,\ i = N_\text{p} + 1, ..., N_\text{p} + N_\text{b}$
    Weight $w_k^i \propto l_k^i = P_k(h(\boldsymbol{x}_k^i)),\ i = 1, ..., N_\text{p} + N_\text{b}$
  else
    Weight $w_k^i \propto l_k^i w_{k-1}^i = P_k(h(\boldsymbol{x}_k^i)) w_{k-1}^i$
  end if
  Total number of particles N_s
  Normalize weights such that $\sum_{i=1}^{N_\text{s}} w_k^i = 1$
  Obtain borders b_k^u, b_k^l of the mainlobe of $P_k(\phi)$
  Calculate target existence threshold
  $\gamma_k = (b_k^u - b_k^l)/(2\pi)$
  Find the set of indices of particles in the mainlobe C_k
  Target existence probability $\xi_k = \tilde{w}_k = \sum_{i \in C_k} w_k^i$
  if $\xi_k > \gamma_k$ **then**
    Estimated state $\hat{\boldsymbol{x}}_{k|k} = 1/(\tilde{w}_k) \sum_{i \in C_k} w_k^i \boldsymbol{x}_k^i$
    if $\xi_{k-1} \not> \gamma_{k-1}$ **then**
      Discard particles $i \notin C_k$
      Resample and regularize the particles
    else
      Redistribute the particles
      if $\hat{N}_\text{eff} < N_\text{th}$ **then**
        Resample and regularize the particles
      end if
    end if
  else
    Re-initialize the particles
  end if
end for

4. SIMULATIONS

The performance of the proposed algorithm in simultaneously detecting and tracking a single target was tested using a number of $M = 500$ Monte Carlo runs. The observer and target trajectories used in the simulations are shown in Figure 1. The sampling interval $\Delta = 1$ minute and the observation period lasts for 30 minutes. Over the observation period, the observer travels with a constant speed of 6 knots in a zigzag trajectory, alternating the direction of motion between 55° and 125° every 5 minutes, to improve the observability of the target state. The target exists from the 6th to the 25th minute and it moves with a constant velocity of 4 knots towards the direction -140°. The received signals at the observer has a very low SNR of -30dB. The target range of interest is in the interval $[0.1, 10]$ km while the minimum and maximum relative speeds of interest are 0.1 knots and 10 knots respectively. The sensor array is composed of two concentric uniform circular arrays, each with $Q = 25$ elements, and their radii are 0.5m and 0.25m respectively.

The proposed algorithm uses a number of $N_\text{p} = 2000$ particles and when no target has been detected, a number of $N_\text{b} = N_\text{p}/5$ birth particles are generated. The particles are resampled and regularized if the effective sample size $\hat{N}_\text{eff}$ is below $N_\text{th} = N_\text{p}/3$. The performance metric used to test the accuracy of the algorithm is the root mean square (RMS) position error which is defined as

$$\text{RMS}_k = \sqrt{\frac{1}{M} \sum_{i=1}^{M} [(\hat{x}_k^i - x_k^i)^2 + (\hat{y}_k^i - y_k^i)^2]},$$

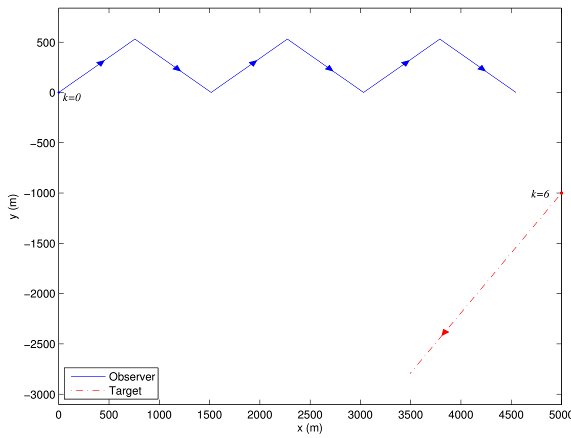

Fig. 1. The scenario used for the simulations

where M is the total number of Monte Carlo runs, $(\hat{x}_k^i, \hat{y}_k^i)$ is the estimated target position while (x_k^i, y_k^i) is the true target position at time k of the i^{th} Monte Carlo run.

Using the proposed algorithm, the RMS position errors of the estimated target states, when it is expected to exist (from the 6^{th} to the 25^{th} minute) is shown in Figure 2. It can be seen that during

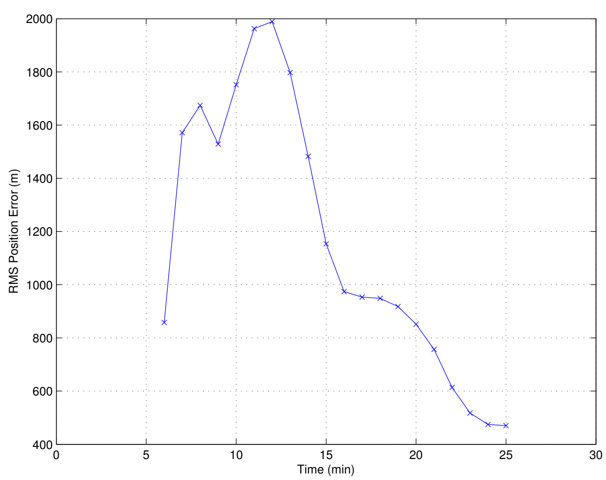

Fig. 2. The RMS position error of the proposed algorithm

the first few time steps of target detection (from the 6^{th} to the 12^{th} minute), the errors are relatively high due to the limited observability of the target range. As more measurements are obtained over the observation period, the errors drop steadily to reasonably low values (from the 12^{th} to the 25^{th} minute), showing good accuracy of the proposed algorithm in tracking the target.

The number of Monte Carlo runs (out of the 500 runs in the simulations) for the different initial target detection time step are shown in Table 1. It can be seen that in more than 97% of the runs, the target is detected within 3 time steps of existence. The disappearance of the target is successfully detected at time step $k = 26$ in 85% of the runs.

Initial Detection Time Step	Number of Runs
6	268
7	187
8	32
9	10
10	2
11	1

Table 1. The number of runs with different initial target detection time step

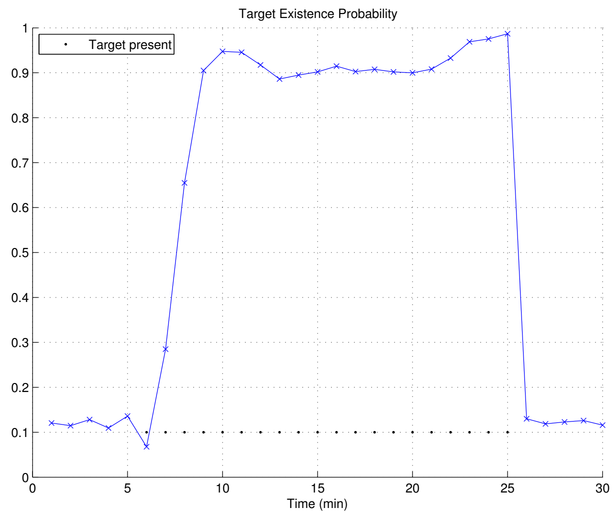

Fig. 3. The average target existence parameter

The average target existence probability ξ_k over the 500 runs is shown in Figure 3. It can be seen that the target existence probability increases rapidly from the 6^{th} minute to the 9^{th} minute where it stays most of the time at a value above 0.9 over the duration of target presence. The target existence probability drops sharply back to about 0.1 at the 26^{th} minute when the target stops to exist.

5. CONCLUSION

We have proposed an algorithm for jointly detecting and tracking a single target of low SNR traveling with a nearly constant velocity. The proposed algorithm takes as measurement data the complete DOA spectrum of the received wideband signals emitted from the target. It could reliably detect the appearance and disappearance of the target without any prior information on the target existence probability and it demonstrates good accuracy in tracking the target when it exists. Future work will include the investigation of a target with non-constant velocity and also extending the algorithm for simultaneously detecting and tracking multiple targets.

6. ACKNOWLEDGMENT

The authors would like to thank Dr Tharaka Lamahewa (DSTO, Australia) for his useful comments and discussions.

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