A NEW PASSIVE SOURCE LOCALIZATION METHOD USING AOA-GROA-TDOA IN WIRELESS SENSOR ARRAY NETWORKS AND ITS CRAMÉR-RAO BOUND ANALYSIS

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ABSTRACT

In this paper, a new Cramér-Rao lower bound (CRLB) is derived for passive source localization based on angles-of-arrival (AOAs), gain ratios of arrival (GROAs) and time differences of arrival (TDOAs) in a wireless sensor array network. The derived CRLB using AOA-GROA-TDOA (AGT) is reduced to the one using AOA-GROA if no coherence exists across the arrays and lower than the CRLB using AOA-only. When the coherence is considered, the CRLB using AGT measurements is consistently lower than the other known bounds using AOA-only, TDOA-only and AOA-TDOA.

***Index Terms*—** Passive source localization, Cramér-Rao lower bound (CRLB), angles-of-arrival (AOAs), gain ratios of arrival (GROAs), time differences of arrival (TDOAs).

1. INTRODUCTION

Passive source localization is one of significant applications in sensor networks that has been a research of focus for the past over ten years [1, 2, 3, 4]. When a large number of sensors are deployed in a field, two common techniques for passive source localization is to measure the time difference of arrival (TDOA) [5, 6] and the source signal energy [7].

As the arrays are regarded as a whole sensor node in the sensor networks, the additional angle of arrival (AOA) measurements derived by array processing techniques can be used in the localization tasks. Basically, the AOA-based localization methods consist of two steps [8, 9]: 1) the bearing of the source signal is estimated in each array and then transmitted to the fusion center [10, 11]; 2) those bearings are intersected to localize the target [12, 13]. Such scheme is simple to implement, has minimum communication load and requires coarse synchronization across the network [14]. However, it is suboptimal because it totally ignores the wavefront coherence between the spatial separated arrays [15]. With regard to localization accuracy, the optimal solution is joint processing all raw data sent by the sensors in the fusion center. This method has maximum communication load and requires accurate time synchronization. A compromising scheme that combines AOA and time delay estimation (TDE) is proposed in [2]. Each array transmits the bearing estimate to the fusion center. Besides, the raw data from one senor in each array is also transmitted. Thus, the coherence across the arrays is also investigated.

When the passive source signal is received by the sensors, both time delay and signal strength information can be obtained. Ho *et al.* [4] presents a hybrid source localization method that combines the gain ratios of arrival (GROAs) and TDOAs together. The energy measurements add new information to the TDOA-only source localization approaches and therefore the Cramér-Rao lower bound (CRLB) of the hybrid localization method is improved compared with that of the TDOA-only method. Recently, Gu [16] proposes a power-bearing (PB) method for target motion analysis (TMA) and shows that the CRLB of PB-TMA is lower than that of the bearing-only TMA.

In this paper, we consider the passive source localization problem using AOA-GROA-TDOA (AGT) measurements in a wireless sensor array network. We first present a joint AGT statistical signal model. Then the CRLB of a stationary source location estimate is developed using AOAs, GROAs and TDOAs. Without the coherence across the arrays, the derived CRLB using AGT is reduced to the one using AOA-GROA measurements and it is lower than the one using AOAs only. When the coherence is considered, the CRLB using AGT is consistently lower than other bounds using AOA-only, TDOA-only and AOA-TODA.

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2. STATISTICAL MODEL

The problem of passive source localization begins with a model for a single stationary narrowband source radiating

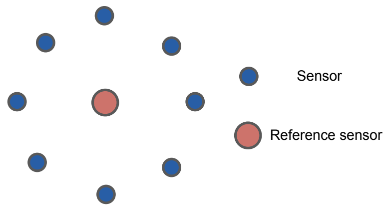

Fig. 1. A circular array consists of eight sensors on the edge and a reference sensor in the center.

source signals that is measured by a network of H arrays, each with N_h omni-directional sensors, $h = 1, \ldots, H$ and a reference sensor denoted as R_h, see Fig. 1. Assume that the N_h sensors are collaborated to estimate the source bearing and transmits the bearing estimate to the fusion center. Besides, the reference sensor in each array transmits the raw samples to the fusion center. All targets and arrays are assumed to be located in the xy-plane. Let us assume R_h is located at (r_{h1}, r_{h2}), and the location of sensor $n \in \{1, \ldots, N_h\}$ on the hth array is at coordinate $(r_{h1} + u_{hn1}, r_{h2} + u_{hn2})$. The sources are supposed to be located in the far field with respect to each array. We denote $\mathbf{p} = [p_1, p_2]^T$ as the location of the source, and $\mathbf{r}_h = [r_{h1}, r_{h2}]^T$ is the position of R_h at the hth array. The measurement of the nth sensor on array h at time t is modeled as

$$x_{hn}(t) = s_h(t - \tau_{hn}) + w_{hn}(t), \tag{1}$$

where $x_{hn}(t)$ denotes the sensor output data, s_h is the source signal impinging on array h,

$$\begin{aligned} \tau_{hn} &= -\frac{1}{c}\left(\cos\theta_h \Delta x_{hn} + \sin\theta_h \Delta y_{hn}\right) \\ \Delta x_{hn} &= u_{hn1} - u_{h11}, \quad \Delta y_{hn} = u_{hn2} - u_{h12} \end{aligned} \tag{2}$$

is the propagation time from the first sensor to sensor n on array h, c is the signal propagation speed, θ_h is the AOA with respect to array h and $w_{hn}(t)$ represents the spatially and temporally white Gaussian noise.

The measurement data sampled at the reference sensors are represented as

$$\begin{aligned} \bar{x}_1(t) &= s_1(t) + \bar{w}_1(t) \\ \bar{x}_h(t) &= \frac{1}{\gamma_{h1}} s_1(t - \beta_{h1}) + \bar{w}_h(t) \end{aligned} \tag{3}$$

where $s_1(t)$ represents source signal observed at reference sensor 1 and β_{h1} and γ_{h1} are the time delays and attenuations of the signal received at R_h with respect to R_1.

The sensor observation data (1) contain the source location information through bearing [2] and the bearing could be estimated in each array locally. The reference sensor measurements implicate the source position by the GROA and TDOA [2, 4]. After the raw data in references sensors is transmitted, the GROA and TDOA can be estimated in the fusion center. Thus, the location of the source can be determined jointly from bearing, GROA and TDOA. In fact, the relation between source location and bearing, GROA and TDOA is given by

$$\cos(\theta_h) = \frac{p_1 - r_{h1}}{d_h}, \quad \sin(\theta_h) = \frac{p_2 - r_{h2}}{d_h} \tag{4}$$

$$\beta_{h1} = \frac{d_h - d_1}{c}, \quad \gamma_{h1} = \frac{d_h}{d_1} \tag{5}$$

where $d_h = \|\mathbf{p} - \mathbf{r}_h\|$. For convenience, we denote $\boldsymbol{\theta} = [\theta_1, \ldots, \theta_H]^T$ for AOA vector, $\boldsymbol{\gamma} = [\gamma_{21}, \ldots, \gamma_{H1}]^T$ for GROA vector and $\boldsymbol{\beta} = [\beta_{21}, \ldots, \beta_{H1}]^T$ for TDOA vector.

With regard to time delay, it is more convenient to convert the time domain measurements to the frequency domain. Let $X_{hn}(\omega)$ and $\bar{X}_n(\omega)$ be the Fourier transform of $x_{hn}(t)$ and $\bar{x}_h(t)$ respectively

$$X_{hn}(\omega) = e^{-j\omega\tau_{hn}} S_h(\omega) + W_{hn}(\omega) \tag{6}$$

$$\bar{X}_h(\omega) = \frac{1}{\gamma_{h1}} e^{-j\omega\beta_{h1}} S(\omega) + \bar{W}_h(\omega), \tag{7}$$

where $S_h(\omega)$, $S(\omega)$, $W_{hn}(\omega)$ and $\bar{W}_h(\omega)$ are the Fourier transforms of $s_h(t)$, $s(t)$, $w_{hn}(t)$ and $\bar{w}_h(t)$. We collect the measurements for bearing estimation at each array h into $N_n \times 1$ vectors

$$\mathbf{X}_h(\omega) = \left[X_{h1}(\omega), \ldots, X_{h,N_h}(\omega)\right]^T. \tag{8}$$

We then further collect all the observations from the H arrays into a vector

$$\mathbf{X}(\omega) = \left[\mathbf{X}_1^T(\omega), \ldots, \mathbf{X}_H^T(\omega), \bar{X}_1(\omega), \ldots, \bar{X}_H(\omega)\right]^T. \tag{9}$$

Assume that $\mathbf{X}(\omega)$ is a zero mean complex Gaussian random vector. The measurement noise $W_{hn}(\omega)$ and $\bar{W}_h(\omega)$, $n = 1, \ldots, N_h$, $h = 1, \ldots, H$ are modeled as zero mean complex Gaussian random variable. The source signal and measurement noise in (6) and (7) are uncorrelated with each other. Then, the covariance matrix is given by

$$\begin{aligned} \mathbf{R}(\omega) &= E[\mathbf{X}(\omega)\mathbf{X}^H(\omega)] \\ &= \begin{bmatrix} \mathbf{R}_A(\omega) & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_{GT}(\omega) \end{bmatrix}, \end{aligned} \tag{10}$$

where $\mathbf{R}_A(\omega)$ is the covariance matrix for bearing estimation, $\mathbf{R}_{GT}$ is correlation matrix for the reference sensor measurements. Assume that the signals are spatially incoherent among arrays for AOA estimation. Then

$$\begin{aligned} &\mathbf{R}_A(\omega) \\ &= \begin{bmatrix} P_1(\omega)\mathbf{a}_1(\omega)\mathbf{a}_1^H(\omega) & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & P_H(\omega)\mathbf{a}_H(\omega)\mathbf{a}_H^H(\omega) \end{bmatrix} \\ &\quad + \boldsymbol{\Sigma}_A(\omega) \end{aligned} \tag{11}$$

where

$$\mathbf{a}_h(\omega) = [e^{-j\omega\tau_{h1}}, \ldots, e^{-j\omega\tau_{h,N_h}}]^T \quad (12)$$

$$\mathbf{\Sigma}_A(\omega) = \text{blkdiag}[\sigma_1(\omega)\mathbf{I}_{N_1}, \ldots, \sigma_H(\omega)\mathbf{I}_{N_H}] \quad (13)$$

$\sigma_h(\omega)$ is the variance of the noise spectrum in the hth array, $\mathbf{I}_{N_h}$ denotes the $N_h \times N_h$ identity matrix and blkdiag denotes block diagonal operator. $\mathbf{R}_{GT}(\omega)$ utilizes the spatial signal coherence to allow GTOA and TDOA estimates,

$$\mathbf{R}_{GT}(\omega) = P_1(\omega)\boldsymbol{\alpha}(\omega)\boldsymbol{\alpha}^H(\omega) \odot \mathbf{\Upsilon}(\omega) + \mathbf{\Sigma}_{GT}(\omega) \quad (14)$$

where $\mathbf{\Upsilon}(\omega)$ is the correlation coefficient matrix for the source signals among the arrays

$$\mathbf{\Upsilon}(\omega) = \begin{bmatrix} 1 & \upsilon_{21}(\omega) & \cdots & \upsilon_{H1}(\omega) \\ \upsilon_{21}(\omega) & 1 & \cdots & \upsilon_{H-1,1}(\omega) \\ \vdots & \vdots & \ddots & \vdots \\ \upsilon_{H1}(\omega) & \upsilon_{H-1,1}(\omega) & \cdots & 1 \end{bmatrix} \quad (15)$$

$$\boldsymbol{\alpha}(\omega) = \left[1, \frac{1}{\gamma_{21}}e^{-j\omega\beta_{21}}, \ldots, \frac{1}{\gamma_{H1}}e^{-j\omega\beta_{H1}}\right]^T \quad (16)$$

$$\mathbf{\Sigma}_{GT}(\omega) = \text{diag}[\sigma_1(\omega), \ldots, \sigma_H(\omega)] \quad (17)$$

$\odot$ denotes the Hadamard product and diag denotes diagonal operator.

3. THE CRAMER-RAO LOWER BOUND ANALYSIS

In this section, we derive the CRLB for the source localization problem based on AOA, GROA and TDOA measurements. The parameter of interest is the source position $\mathbf{p}$. To develop the CRLB, we start with the probability density function (pdf) of each measurement. The log of the probability density function of $\mathbf{X}(\omega)$ is

$$\ln p(\mathbf{X}(\omega)) = C - \ln\det(\mathbf{R}(\omega)) - \mathbf{X}^H(\omega)\mathbf{R}^{-1}(\omega)\mathbf{X}(\omega) \quad (18)$$

where C represents the constant term. The Fisher information matrix (FIM) is defined as

$$\mathbf{J} = -E\left[\frac{\partial^2 \ln p(\mathbf{X}(\omega))}{\partial\mathbf{p}\partial\mathbf{p}^T}\right]. \quad (19)$$

Let us consider the narrowband source signal, where the band of frequencies ranges $\omega_0 - (\Delta\omega/2) \le \omega \le \omega_0 + (\Delta\omega/2)$. If $\Delta\omega$ is small enough, then the ω dependent terms in (19) can be approximated by their values at ω_0. For large observation period $\mathcal{T}$, the FIM is approximated by

$$\mathbf{J} \approx -\frac{\mathcal{T}\Delta\omega}{2\pi}E\left[\frac{\partial^2 \ln p(\mathbf{X}(\omega_0))}{\partial\mathbf{p}\partial\mathbf{p}^T}\right]. \quad (20)$$

For notation simplicity, we will drop the ω_0 index in the symbols in the following. Since we assume the complex Gaussian random vector $\mathbf{X}(\omega)$ is zero mean, the i,jth element of $\mathbf{J}$ can be written by

$$J_{i,j} = \frac{\mathcal{T}\Delta\omega}{2\pi}\text{tr}\left[\frac{\partial\mathbf{R}}{\partial p_i}\mathbf{R}^{-1}\frac{\partial\mathbf{R}}{\partial p_j}\mathbf{R}^{-1}\right]. \quad (21)$$

After all the elements of $\mathbf{J}$ are calculated, the CRLB is the inverse of the FIM

$$\text{CRLB}(\mathbf{p})_{\boldsymbol{\theta},\boldsymbol{\beta},\boldsymbol{\gamma}} = \mathbf{J}^{-1} \quad (22)$$

It can be observed from (10) and (21) that the dependency of the CRLB on $\boldsymbol{\theta}$, $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$, and hence $\mathbf{p}$, is through the correlation matrix $\mathbf{R}$. For $\mathbf{R}_A$, it does not consider any coherence information among the arrays. Yet $\mathbf{R}_{GT}$ explores the coherence information among the arrays. If $\upsilon_{21} = \cdots = \upsilon_{H1} = 0$, $\mathbf{R}_{GT}$ becomes

$$\mathbf{R}_{GT} = \text{diag}\left[\begin{array}{cccc} P_1 + \sigma_1 & \frac{P_1}{\gamma_{21}^2} + \sigma_2 & \cdots & \frac{P_1}{\gamma_{H1}^2} + \sigma_H \end{array}\right]. \quad (23)$$

We say that the CRLB using AGT is reduced to the CRLB using AOA-GROA.

4. SIMULATION RESULTS

In this section, numerical examples are presented to evaluate the CRLB on the localization accuracy using AOA, GROA and TDOA in a wireless sensor array network. In the following simulations, we consider the same array network geometry as the examples given in [2, 17]. There are $H = 3$ identical arrays are located at coordinates $(x_1, y_1) = (0, 0)$, $(x_2, y_2) = (400, 400)$, and $(x_3, y_3) = (100, 0)$. The arrays are circular with a 4-ft radius and each array contains eight omni-directional sensors, i.e., $N_1 = \cdots = N_H = 8$ and a reference sensor. Eight sensors are equally spaced around the perimeter of the array and the reference sensor is located at the center of the circle, see Fig. 1. Let the source locate at $(200, 300)$. The source is narrowband with a bandwidth of 1 Hz centered at 50 Hz. For each example, $T = 4000$ samples are measured with sampling rate $f_s = 2000$ samples per second. The observation time is $\mathcal{T} = T/f_s = 2$ second. The CRLB is calculated using the FIM [2]

$$\text{CRLB}(\mathbf{p}) = \sqrt{J_{11}^{-1} + J_{22}^{-1}} \quad (24)$$

We first consider zero coherence case. Fig. 2 plots the CRLB ellipses using AOA-TDOA and AGT. We observe that the CRLB using AGT is lower than the CRLB using AOA-TDOA. Fig. 3 plots the CRLB of the source localization using AOA-only, TDOA-only, AOA-TDOA and AGT with respect to various values of coherence υ_{21} and υ_{31}, where $\upsilon_{21} = \upsilon_{31}$. From Fig. 3, we observe that the CRLB using AGT is consistent lower than the other bounds using AOA-only, TDOA-only and AOA-TDOA. The CRLB using TDOA-only is lower

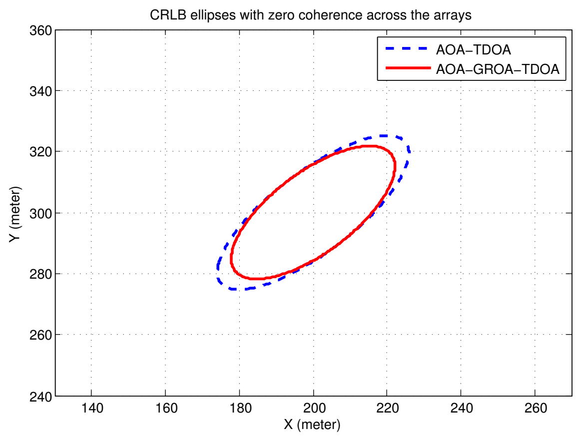

Fig. 2. The CRLB ellipses derived using AOA-TDOA and AOA-GROA-TDOA with zero coherence across the arrays.

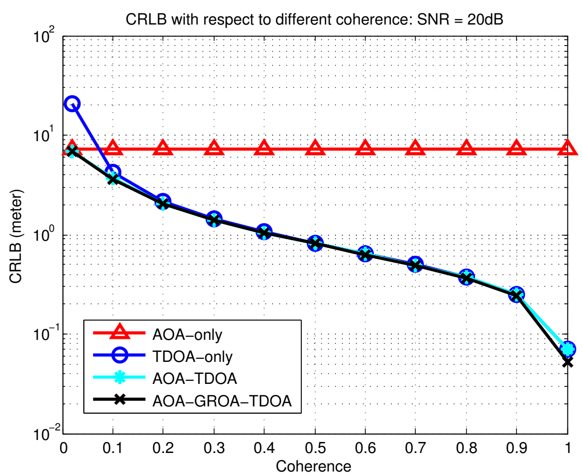

Fig. 3. The CRLBs using AOA-only, TDOA-only, AOA-TDOA and AOA-GROA-TDOA with respect to various coherence, SNR = 20 dB.

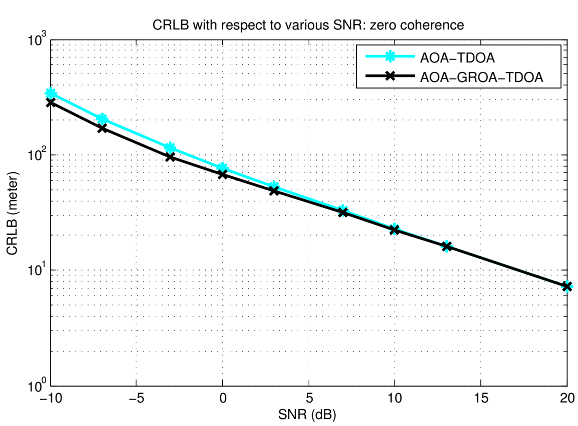

Fig. 4. The CRLBs using AOA-TDOA and AOA-GROA-TDOA with respect to different SNR, zero coherence case.

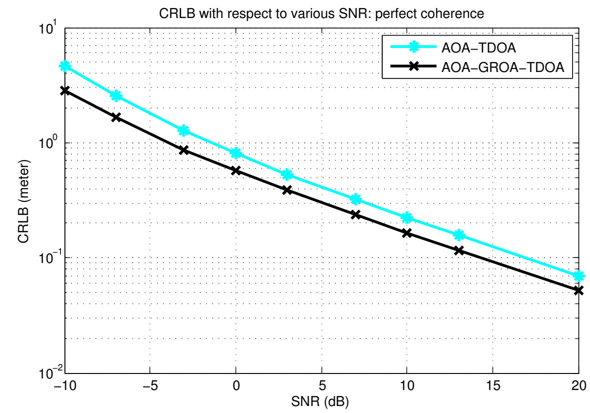

Fig. 5. The CRLBs using AOA-TDOA and AOA-GROA-TDOA with respect to different SNR, perfect coherence case.

than the one using AOA-only except when the coherence is small.

We then investigate the CRLB with respect to various signal-to-noise ratios (SNR). The SNR is controlled by varying the observation noise variance in each array:

$$\mathrm{SNR}_h = 10 \log_{10}(P_h/\sigma_h). \tag{25}$$

The comparisons between the two CRLBs using AOA-TDOA and AGT with respect to various SNR are given in Fig. 4 and Fig. 5. Fig. 4 shows the CRLBs using AOA-TDOA and AGT when the coherence is zero and Fig. 4 plots the two CRLBs for perfect coherence. Both Fig. 4 and Fig. 5 show that the GROA adds new information for source location estimate and therefore improves the CRLB greatly.

5. CONCLUSION

In this paper, we present a new passive source localization method using the AOAs, GROAs and TDOAs measurements jointly in an energy-constrained wireless sensor array network. We develop its Cramér-Rao lower bound (CRLB) of source localization estimate. To develop the CRLB, we use the scheme which transmits the bearing estimate in each array to the fusion center and communicates the raw data in the reference sensor at the same time. The raw data can be used to estimate the TDOAs and GROAs.

If there is no coherence across the arrays, only GROAs can be estimated from the raw data in fusion center. The CRLB using AOA-GROA-TDOA (AGT) is reduced to the one using AOA-GROA. The GROA measurements explicitly improve the performance of localization and the CRLB using AOA-GROA is lower than the CRLB using AOA-only. When the coherence is explored, the TDOA can be estimated and the CRLB using AGT is consistently lower than the other bounds using AOA-only, TDOA-only and AOA-TDOA.

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