# JOINT SOURCE LOCALIZATION AND SENSOR POSITION REFINEMENT FOR SENSOR NETWORKS

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**ABSTRACT**

Modern localization systems/platforms such as sensor networks often experience uncertainty in the sensor positions. Improving the sensor positions is necessary in order to achieve better localization performance. This paper proposes a joint estimator for locating multiple unknown sources and refining the sensor positions using TOA measurements. Rather than resorting to the traditional iterative non-linear least-squares approach that requires careful initializations, the proposed estimator is algebraic and computationally attractive. The small noise analysis shows that the proposed estimator is able to attain the CRLB performance for both the unknown sources and the sensor positions. Simulations support the efficiency of the proposed estimator.

***Index Terms*—** Sensor network, sensor position refinement, source localization, time of arrival

## 1. INTRODUCTION

Locating one or multiple sources via measurements from a network of sensors has been an active research for the past few years. It has wide varieties of applications in radar/sonar [1], wireless sensor networks (WSN) [2] and cellular communications [3].

Localization uses a number of spatially separated sensors at known positions to determine the location of an emitting source through measurements in the form of Time Difference of Arrival (TDOA) [4], Time of Arrival (TOA) [5], etc. Different from traditional applications such as in radar, modern localization platforms including sensor networks do not have precise knowledge of the sensor positions [6]. Accurate sensor positions are necessary for event monitoring and geographic routing applications [7]. The sensor position uncertainty could contribute to considerable amount of degradation in localization accuracy [8].

We can improve the sensor positions by using one or multiple sources, often called calibration sources or anchors, that are at known locations. Deploying a calibration source could be costly. A more practical approach is to refine the sensor positions upon the localization of an unknown emitting source [9, 10]. Based on the available, although inexact, sensor positions we can identify the location of the unknown source. The estimated unknown source location can be exploited to refine the inaccurate sensor positions. This is the technique taken in [11] which has shown that such a refinement scheme can achieve the CRLB accuracy for the sensor positions.

This paper takes a different approach than the previous estimation-refinement scheme and performs joint estimation of the unknown source locations and the inaccurate sensor positions together. The joint estimation is expected to tolerate higher noise level before the thresholding effect caused by non-linear estimation starts to occur.

Joint estimation of source and sensors for TDOA location has been examined in [12]. The present work extends the method for TOA location in sensor networks and compares the performance with the sequential estimation-refinement approach. Rather than resorting to the traditional Maximum Likelihood Estimator (MLE) which requires good initialization and high complexity [9] or sub-optimum estimator [10], we shall develop here a computationally efficient algebraic solution. Also included in the paper is the performance analysis of the proposed method in reaching the CRLB performance for Gaussian noise over the small error region.

In the following, Section 2 states the localization scenario and develops the proposed method. Section 3 analyzes the performance of the proposed solution and shows that it is able to reach the CRLB accuracy. Section 4 provides simulation results to support the theoretical developments and Section 5 concludes the paper.

## 2. PROBLEM FORMULATION AND PROPOSED SOLUTION

Let us consider the localization scenario as shown in Fig. 1, which consists of $M$ sensors to locate $K$ independent sources. The sources can be unknown emitters of interests or newly added sensor nodes. The true locations of the unknown sources to be found are denoted by $N \times 1$ vectors $\mathbf{u}_i^o$, $i = 1, 2, \cdots, K$, of Cartesian coordinates, where $N = 2$ for $2D$ localization or $N = 3$ for $3D$ localization. The precise positions of the sensors $\mathbf{s}_j^o, j = 1, 2, \cdots, M$, are not known and the available inaccurate sensor positions are $\mathbf{s}_j = \mathbf{s}_j^o + \Delta\mathbf{s}_j$, where $\Delta\mathbf{s}_j$ represents the position error of sensor $j$. They are collected to form a $NM \times 1$ sensor position vector $\mathbf{s} = [\mathbf{s}_1^T, \mathbf{s}_2^T, \cdots, \mathbf{s}_M^T]^T = \mathbf{s}^o + \Delta\mathbf{s}$, where $\mathbf{s}^o$ is the true sensor position vector and $\Delta\mathbf{s} = [\Delta\mathbf{s}_1^T, \Delta\mathbf{s}_2^T, \cdots, \Delta\mathbf{s}_M^T]^T$ is the random error vector. We shall model $\Delta\mathbf{s}$ as a zero-mean Gaussian random vector with covariance matrix $\mathbf{Q}_s$.

TOA measurements are commonly used in sensor networks. Assuming each sensor can acquire the signal from each source, we have, after multiplying with the signal propagation speed, the $MK \times 1$ measurement vector $\mathbf{r} = [\mathbf{r}_1^T, \mathbf{r}_2^T, \cdots, \mathbf{r}_K^T]^T = \mathbf{r}^o + \mathbf{n}$, where $\mathbf{r}^o$ is the true range vector and $\mathbf{n}$ is the noise vector. $\mathbf{r}_i = [r_{1,i}, r_{2,i}, \cdots, r_{M,i}]^T$ is the measurement vector from source $i$ and $r_{j,i}$ is the TOA measurement of source $i$ to sensor $j$. $\mathbf{n}$ is modeled as a zero-mean Gaussian random vector with covariance matrix $\mathbf{Q}_r$. We shall assume $\Delta\mathbf{s}$ and $\mathbf{n}$ are independent of each other for ease of illustrations.

Our goal is to estimate the source locations and at the same time improve the inaccurate sensor positions as good as possible using the TOA measurements. The unknown parameter vector is $\boldsymbol{\theta} = [\mathbf{u}_1^{oT}, \mathbf{u}_2^{oT}, \cdots, \mathbf{u}_K^{oT}, \mathbf{s}^{oT}]^T$.

The proposed method makes use of a hypothetical source locations $\tilde{\mathbf{u}}_i = \mathbf{u}_i^o + \Delta\tilde{\mathbf{u}}_i$, where $\Delta\tilde{\mathbf{u}}_i$ is the difference between the hypothetical and the actual source location. The hypothetical locations are easy to obtain, please refer to [8] for details.

We begin the algorithm development from the parametric form of $r_{j,i}^o$:

$$r_{j,i}^o = ||\mathbf{u}_i^o - \mathbf{s}_j^o||. \tag{1}$$

Squaring both sides of (1), substituting $r_{j,i}^o = r_{j,i} - n_{j,i}, \mathbf{s}_j^o = \mathbf{s}_j - \Delta\mathbf{s}_j, \mathbf{u}_i^o = \tilde{\mathbf{u}}_i - \Delta\tilde{\mathbf{u}}_i$ and ignoring the second order terms of $n_{j,i}, \Delta\mathbf{s}_j$, and $\Delta\tilde{\mathbf{u}}_i$ yield

$$\begin{aligned} r_{j,i}n_{j,i} = \frac{1}{2}\big[r_{j,i}^2 - \mathbf{s}_j^T(\mathbf{s}_j - 2\tilde{\mathbf{u}}_i)\big] - \mathbf{s}_j^T\Delta\tilde{\mathbf{u}}_i \\ - \frac{1}{2}\mathbf{u}_i^{oT}\mathbf{u}_i^o - (\tilde{\mathbf{u}}_i - \mathbf{s}_j)^T\Delta\mathbf{s}_j. \end{aligned} \tag{2}$$

We shall consider $\mathbf{u}_i^{oT}\mathbf{u}_i^o$ as a new independent unknown so that (2) becomes as a pseudo linear equation.

Other than the TOA measurements, the statistical knowledge of the sensor position errors $\Delta\mathbf{s}$ can also be utilized in the estimation. Following the technique from [13] and putting (2) together for $j = 1, 2, \cdots, M$ and $i = 1, 2, \cdots, K$ yield the matrix equation

$$\boldsymbol{\epsilon}_1 = \mathbf{h}_1 - \mathbf{G}_1\boldsymbol{\varphi}_1^o, \tag{3}$$

where

$$\begin{aligned}
&\boldsymbol{\epsilon}_1 = \big[(\mathbf{B}_1\mathbf{n})^T, -\Delta\mathbf{s}^T\big]^T, \\
&\mathbf{B}_1 = \text{diag}\big\{\mathbf{B}_{1,1}, \mathbf{B}_{1,2}, \cdots, \mathbf{B}_{1,K}\big\}, \\
&\mathbf{B}_{1,i} = \text{diag}\big\{r_{1,i}, r_{2,i}, \cdots, r_{M,i}\big\}, \\
&\mathbf{h}_1 = \big[\boldsymbol{\eta}_1^T, \boldsymbol{\eta}_2^T, \cdots, \boldsymbol{\eta}_K^T, \mathbf{0}_{NM\times 1}^T\big]^T, \\
&\boldsymbol{\eta}_i = \frac{1}{2}\big[r_{1,i}^2 - \mathbf{s}_1^T(\mathbf{s}_1 - 2\tilde{\mathbf{u}}_i), \cdots, r_{M,i}^2 - \mathbf{s}_M^T(\mathbf{s}_M - 2\tilde{\mathbf{u}}_i)\big]^T, \\
&\boldsymbol{\varphi}_1^o = \big[\Delta\tilde{\mathbf{u}}_1^T, \mathbf{u}_1^{oT}\mathbf{u}_1^o, \cdots, \Delta\tilde{\mathbf{u}}_K^T, \mathbf{u}_K^{oT}\mathbf{u}_K^o, \Delta\mathbf{s}^T\big]^T, \\
&\mathbf{G}_1 = \begin{bmatrix} \mathbf{G}_{1,1} & \cdots & \mathbf{O}_{M\times(N+1)} & \mathbf{D}_1 \\ \vdots & \ddots & \vdots & \vdots \\ \mathbf{O}_{M\times(N+1)} & \cdots & \mathbf{G}_{1,K} & \mathbf{D}_K \\ \mathbf{O}_{NM\times(N+1)} & \cdots & \mathbf{O}_{NM\times(N+1)} & \mathbf{I}_{NM\times NM} \end{bmatrix}.
\end{aligned} \tag{4}$$

$\mathbf{D}_i$ and $\mathbf{G}_{1,i}$ in $\mathbf{G}_1$ each has $M$ rows and their $j$th rows, $j = 1, 2, \cdots, M$, are equal to $\big[\mathbf{0}_{N(j-1)\times 1}^T, (\tilde{\mathbf{u}}_i - \mathbf{s}_j)^T, \mathbf{0}_{N(M-j)\times 1}^T\big]$ and $\big[\mathbf{s}_j^T, \frac{1}{2}\big]$.

The weighted least-squares (WLS) solution of $\boldsymbol{\varphi}_1^o$ from the matrix equation (3) is

$$\boldsymbol{\varphi}_1 = \big(\mathbf{G}_1^T\mathbf{W}_1\mathbf{G}_1\big)^{-1}\mathbf{G}_1^T\mathbf{W}_1\mathbf{h}_1, \tag{5}$$

where the weighting matrix $\mathbf{W}_1$ is chosen to minimize the parameter estimation mean-square error:

$$\mathbf{W}_1 = \text{diag}\big\{\mathbf{B}_1\mathbf{Q}_r\mathbf{B}_1, \mathbf{Q}_s\big\}^{-1}. \tag{6}$$

The estimation accuracy is characterized by the covariance of $\boldsymbol{\varphi}_1$, which is equal to

$$\text{cov}(\boldsymbol{\varphi}_1) \simeq \big(\mathbf{G}_1^T\mathbf{W}_1\mathbf{G}_1\big)^{-1} \tag{7}$$

when the sensor position noise is relatively small and can be neglected in $\mathbf{G}_1$.

After $\boldsymbol{\varphi}_1$ is obtained, the estimates of $\Delta\tilde{\mathbf{u}}_i$ and $\Delta\mathbf{s}$ can be represented as

$$\begin{aligned}
\boldsymbol{\varphi}_{1,i} &\triangleq \boldsymbol{\varphi}_1\big((N+1)(i-1)+1 : (N+1)(i-1)+N\big) \\
&= \Delta\tilde{\mathbf{u}}_i + \delta\tilde{\mathbf{u}}_i, \\
\boldsymbol{\varphi}_{1,s} &\triangleq \boldsymbol{\varphi}_1\big((N+1)K+1 : (N+1)K+NM\big) \\
&= \Delta\mathbf{s} + \delta\mathbf{s},
\end{aligned} \tag{8}$$

where $\delta\tilde{\mathbf{u}}_i$ and $\delta\mathbf{s}$ are the estimation errors of $\Delta\tilde{\mathbf{u}}_i$ and $\Delta\mathbf{s}$. Subtracting $\boldsymbol{\varphi}_{1,i}$ and $\boldsymbol{\varphi}_{1,s}$ from the hypothetical source location $\tilde{\mathbf{u}}_i$ and the sensor position vector $\mathbf{s}$ will provide the source and sensor position estimates. They are, however, not able to reach the CRLB accuracy. This is because we have introduced $K$ additional variables $\mathbf{u}_i^{oT}\mathbf{u}_i^o$, $i = 1, 2, \cdots, K$, in $\boldsymbol{\varphi}_1$. We next explore these $K$ additional variables to improve the estimation accuracy.

Though $\mathbf{u}_i^{oT}\mathbf{u}_i^o$ is not related to $\mathbf{s}^o$, the estimation errors of $\Delta\tilde{\mathbf{u}}_i$ and $\Delta\mathbf{s}$ in $\boldsymbol{\varphi}_1$ are correlated. As a result, when the accuracy of source location estimates is improved through the additional variables, the sensor position estimates can also be enhanced. In our stage-2 solution, we will estimate the error terms $\delta\tilde{\mathbf{u}}_i$ and $\delta\mathbf{s}$ in (8) in order to provide more accurate estimations of the source locations and sensor positions.

The $(N+1)i$th, $i = 1, 2, \cdots, K$, element of $\boldsymbol{\varphi}_1$ is the estimate of $\mathbf{u}_i^{oT}\mathbf{u}_i^o$

$$\boldsymbol{\varphi}_1\big[(N+1)i\big] = \mathbf{u}_i^{oT}\mathbf{u}_i^o + \Delta\boldsymbol{\varphi}_1\big[(N+1)i\big]. \tag{9}$$

Putting $\mathbf{u}_i^o = \tilde{\mathbf{u}}_i - \boldsymbol{\varphi}_{1,i} + \delta\tilde{\mathbf{u}}_i$ into (9) gives

$$\begin{aligned} \Delta\boldsymbol{\varphi}_1\big[(N+1)i\big] = \boldsymbol{\varphi}_1\big[(N+1)i\big] - (\tilde{\mathbf{u}}_i - \boldsymbol{\varphi}_{1,i})^T(\tilde{\mathbf{u}}_i - \boldsymbol{\varphi}_{1,i}) \\ - 2(\tilde{\mathbf{u}}_i - \boldsymbol{\varphi}_{1,i})^T\delta\tilde{\mathbf{u}}_i. \end{aligned} \tag{10}$$

Since $\delta\tilde{\mathbf{u}}_i = \Delta\boldsymbol{\varphi}_1\big[(N+1)(i-1)+1 : (N+1)(i-1)+N\big]$ and $\delta\mathbf{s} = \Delta\boldsymbol{\varphi}_1\big[(N+1)K+1 : (N+1)K+NM\big]$, together with (10) we have the linear matrix equation

$$\boldsymbol{\epsilon}_2 = \mathbf{h}_2 - \mathbf{G}_2\boldsymbol{\varphi}_2^o, \tag{11}$$

where

$$\begin{aligned}
&\boldsymbol{\epsilon}_2 = \mathbf{B}_2\Delta\boldsymbol{\varphi}_1, \\
&\mathbf{B}_2 = \text{diag}\big\{\mathbf{B}_{2,1}, \mathbf{B}_{2,2}, \cdots, \mathbf{B}_{2,K}, -\mathbf{I}_{NM\times NM}\big\}, \\
&\mathbf{B}_{2,i} = \mathbf{I}_{(N+1)\times(N+1)},\ \mathbf{h}_2 = \big[\boldsymbol{\xi}_1^T, \boldsymbol{\xi}_2^T, \cdots, \boldsymbol{\xi}_K^T, \mathbf{0}_{NM\times 1}^T\big]^T, \\
&\boldsymbol{\xi}_i = \big[\mathbf{0}_{N\times 1}^T, \boldsymbol{\varphi}_1\big((N+1)i\big) - (\tilde{\mathbf{u}}_i - \boldsymbol{\varphi}_{1,i})^T(\tilde{\mathbf{u}}_i - \boldsymbol{\varphi}_{1,i})\big]^T, \\
&\mathbf{G}_2 = \text{diag}\big\{\mathbf{G}_{2,1}, \mathbf{G}_{2,2}, \cdots, \mathbf{G}_{2,K}, \mathbf{I}_{NM\times NM}\big\}, \\
&\mathbf{G}_{2,i} = \big[-\mathbf{I}_{N\times N}, 2(\tilde{\mathbf{u}}_i - \boldsymbol{\varphi}_{1,i})\big]^T, \\
&\boldsymbol{\varphi}_2^o = \big[\delta\tilde{\mathbf{u}}_1^T, \delta\tilde{\mathbf{u}}_2^T, \cdots, \delta\tilde{\mathbf{u}}_K^T, \delta\mathbf{s}^T\big]^T.
\end{aligned} \tag{12}$$

The WLS solution of $\boldsymbol{\varphi}_2^o$ is then

$$\boldsymbol{\varphi}_2 = \big(\mathbf{G}_2^T\mathbf{W}_2\mathbf{G}_2\big)^{-1}\mathbf{G}_2^T\mathbf{W}_2\mathbf{h}_2, \tag{13}$$

where the weighting matrix $\mathbf{W}_2$ is

$$\mathbf{W}_2 = \mathbf{B}_2^{-1}\text{cov}(\boldsymbol{\varphi}_1)^{-1}\mathbf{B}_2^{-1}. \tag{14}$$

Let $\boldsymbol{\varphi}_{2,i}$ be $\boldsymbol{\varphi}_2\big[N(i-1)+1 : Ni\big]$ and $\boldsymbol{\varphi}_{2,s}$ be $\boldsymbol{\varphi}_2\big[NK+1 : NK+NM\big]$. According to (8) the final source and sensor position estimates are

$$\hat{\mathbf{u}}_i = \tilde{\mathbf{u}}_i - \big(\boldsymbol{\varphi}_{1,i} - \boldsymbol{\varphi}_{2,i}\big), \qquad \hat{\mathbf{s}} = \mathbf{s} - \big(\boldsymbol{\varphi}_{1,s} - \boldsymbol{\varphi}_{2,s}\big). \tag{15}$$

## 3. PERFORMANCE ANALYSIS

In this section we shall show analytically that the proposed solution can reach the CRLB accuracy. By using $\Delta\tilde{\mathbf{u}}_i = \tilde{\mathbf{u}}_i - \mathbf{u}_i^o$ and the definitions of $\boldsymbol{\varphi}_{1,i}$, $\boldsymbol{\varphi}_{1,s}$ in (8), (15) can be expressed as

$$\begin{aligned}\hat{\mathbf{u}}_i &= \mathbf{u}_i^o - \left(\delta\tilde{\mathbf{u}}_i - \boldsymbol{\varphi}_{2,i}\right) = \mathbf{u}_i^o + \Delta\boldsymbol{\varphi}_{2,i},\\ \hat{\mathbf{s}} &= \mathbf{s}^o - \left(\delta\mathbf{s} - \boldsymbol{\varphi}_{2,s}\right) = \mathbf{s}^o + \Delta\boldsymbol{\varphi}_{2,s}.\end{aligned} \tag{16}$$

As a result, the covariance matrix of $\hat{\boldsymbol{\theta}} = \left[\hat{\mathbf{u}}_1^T, \cdots, \hat{\mathbf{u}}_K^T, \hat{\mathbf{s}}^T\right]^T$ is the same as that of $\boldsymbol{\varphi}_2$. When the error component in $\mathbf{G}_2$ is small enough to be neglected ($\tilde{\mathbf{u}}_i$ sufficiently accurate), based on the WLS theory we have

$$\mathrm{cov}(\hat{\boldsymbol{\theta}}) \simeq (\mathbf{G}_2^T\mathbf{W}_2\mathbf{G}_2)^{-1} = \begin{bmatrix}\tilde{\mathbf{X}} & \tilde{\mathbf{Y}}\\ \tilde{\mathbf{Y}}^T & \tilde{\mathbf{Z}}\end{bmatrix}^{-1}, \tag{17}$$

where

$$\begin{aligned}&\tilde{\mathbf{X}} = \mathbf{G}_3^T\mathbf{Q}_r^{-1}\mathbf{G}_3, \tilde{\mathbf{Y}} = \mathbf{G}_3^T\mathbf{Q}_r^{-1}\mathbf{G}_4, \tilde{\mathbf{Z}} = \mathbf{G}_4^T\mathbf{Q}_r^{-1}\mathbf{G}_4 + \mathbf{Q}_s^{-1},\\ &\mathbf{G}_3 = \mathrm{diag}\left\{\mathbf{G}_{3,1}, \mathbf{G}_{3,2}, \cdots, \mathbf{G}_{3,K}\right\},\\ &\mathbf{G}_{3,i} = \mathbf{B}_{1,i}^{-1}\mathbf{G}_{1,i}\mathbf{B}_{2,i}^{-1}\mathbf{G}_{2,i},\\ &\mathbf{G}_4 = \left[\mathbf{G}_{4,1}^T, \mathbf{G}_{4,2}^T, \cdots, \mathbf{G}_{4,K}^T\right]^T, \mathbf{G}_{4,i} = -\mathbf{B}_{1,i}^{-1}\mathbf{D}_i.\end{aligned} \tag{18}$$

Following a procedure similar to that in Appendix V of [8], we can prove that when the noise is small compared to target range,

$$\frac{||\Delta\mathbf{s}_j||}{r_{j,i}^o} \simeq 0, \frac{|n_{j,i}|}{r_{j,i}^o} \simeq 0, i = 1, 2, \cdots, K,\ j = 1, 2, \cdots, M \tag{19}$$

we have

$$\mathbf{G}_3 \simeq \frac{\partial\mathbf{r}^o}{\partial\mathbf{u}^o}, \qquad \mathbf{G}_4 \simeq \frac{\partial\mathbf{r}^o}{\partial\mathbf{s}^o}. \tag{20}$$

Putting (20) into (18) and comparing (17) with the CRLB given in Appendix A of [11] yield

$$\mathrm{cov}(\hat{\boldsymbol{\theta}}) \simeq \mathrm{CRLB}(\boldsymbol{\theta}^o). \tag{21}$$

From the small noise analysis above, the proposed solution is able to attain the CRLB accuracy for both the source location and sensor position estimates.

## 4. SIMULATION

A total of 100 random localization geometries are used in the simulations. Each geometry has $K = 2$ sources and $M = 5$ sensors, where the sources and sensors are placed randomly with uniform distribution over a square area of $100 \times 100$ and $60 \times 60$ respectively. Fig. 2 shows the overlay of the 100 geometries.

The performance indices are the mean squared error (MSE) of the estimates computed by $\mathrm{mse}(\mathbf{u}) = \frac{1}{K}\sum_{i=1}^K \left(\sum_{l=1}^L \|\hat{\mathbf{u}}_i^{(l)} - \mathbf{u}^o\|^2/L\right)$ and $\mathrm{mse}(\mathbf{s}) = \sum_{l=1}^L \|\hat{\mathbf{s}}^{(l)} - \mathbf{s}^o\|^2/L$, where $L$ is the number of ensemble runs, $\hat{\mathbf{u}}_i^{(l)}$ and $\hat{\mathbf{s}}^{(l)}$ are the $i$th source location estimate and the sensor position estimates at ensemble $l$. Besides the proposed method, the sequential method [11] (estimation-refinement scheme) and the iterative MLE [14] are implemented for comparison. The sensor position estimates from [8] are also included. The approach in [8] is applied to obtain the hypothetical source locations for the proposed estimator. The same hypothetical source locations are used as the initial guesses for the MLE .

The covariance matrix of the TOA measurements (after multiplied with signal propagation speed square) is $\mathbf{Q}_r = \sigma_r^2\mathbf{I}$, where $\mathbf{I}$ is an identity matrix of size $MK$, $\sigma_r^2$ is the noise power which is fixed to $10^{-3}$ in the simulations. The covariance matrix of the sensor positions is $\mathbf{Q}_s = \boldsymbol{\sigma}_s^2\mathbf{J}$, where $\mathbf{J}$ is a $NM \times NM$ diagonal matrix whose diagonal elements are uniformly distributed between 1 and 10. We generate a different $\mathbf{J}$ for each localization geometry. $\sigma_s^2$ is a scaling proportion of the sensor position covariance matrix whose value varies between $10^{-2.25}$ and $10^{0.25}$. The number of ensemble runs $L$ is 1000 in each geometry and the results given are the average over the 100 geometries.

Fig. 3 gives the performance for source and sensor position estimates as the sensor position noise power increases. The sensor position noise power ($\boldsymbol{\sigma}_{\text{avg}}^2$) in the x-axis is $\mathrm{trace}(\mathbf{Q}_s)/(NM)$ averaged over the 100 geometries. From the source location estimate results in Fig. 3(a), when $\boldsymbol{\sigma}_{\text{avg}}^2$ is not larger than $10^{-0.25}$, the proposed method, the sequential method and the MLE give similar results and attain the CRLB accuracy. For the sensor estimates in Fig. 3(b), the proposed method is always better than [8] by more than 1 dB when $\boldsymbol{\sigma}_{\text{avg}}^2$ is not larger than 1. When $\boldsymbol{\sigma}_{\text{avg}}^2$ exceeds $10^{0.5}$, the proposed method is worse than [8] because of the joint estimation rather than the sensor position estimation only as in [8]. The proposed method also outperforms the sequential method when $\boldsymbol{\sigma}_{\text{avg}}^2$ is larger than 0.1. The MLE deviates from the CRLB slightly later than the proposed method but it requires iterations and higher computational cost.

The improvement of computation speed of the proposed method over the MLE (with an average of 3 iterations) is about a factor of two, measured using computation time in matlab for the simulations provided. The actual speed improvement could vary depending on implementations.

One purpose of refining the sensor positions is for better locating a newly appeared source. To demonstrate, we continue the simulation study as follows: for each of the 100 random geometries of two sources and five sensors, we add one new emitting source. After the positions of the two sources and five sensors are estimated, the refined sensor positions are used to locate the new source and the results are shown in Fig. 4. We observe that the CRLB of the new source location estimate is about 3 dB lower when using the refined sensor positions. The proposed method performs better than the sequential method in estimating the new source position. Interestingly enough, it yields comparable results with the MLE.

## 5. CONCLUSION

In this paper, we have developed an algebraic solution that jointly estimates the positions of multiple sources and sensors. The proposed method is able to achieve the CRLB performance for both the source and the sensor locations. The refined sensor positions can improve the localization of newly appeared sources subsequently. The good performance of the proposed estimator is shown analytically and supported by simulations. Compared to the sequential estimation-refinement technique, the proposed estimator provides better performance in sensor position estimates at higher noise level.

The robustness of the proposed estimator under large or non-Gaussian type errors are a subject for further investigation.

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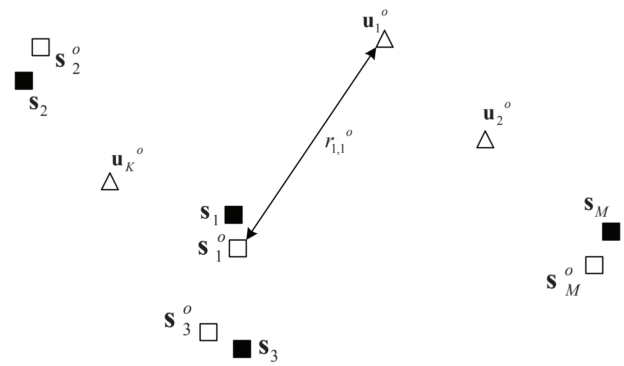


**Fig. 1**. Localization of the sources at $\mathbf{u}_i^o$ using sensors at $\mathbf{s}_j$. Open squares denote the true sensor positions that are not known and closed squares are the available sensor positions that are erroneous.

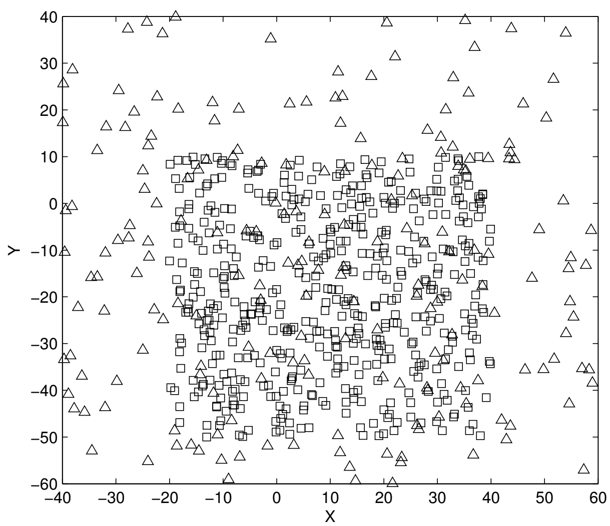


**Fig. 2**. The 100 random localization geometries, upper triangles represent the sources and open squares denote the sensors.

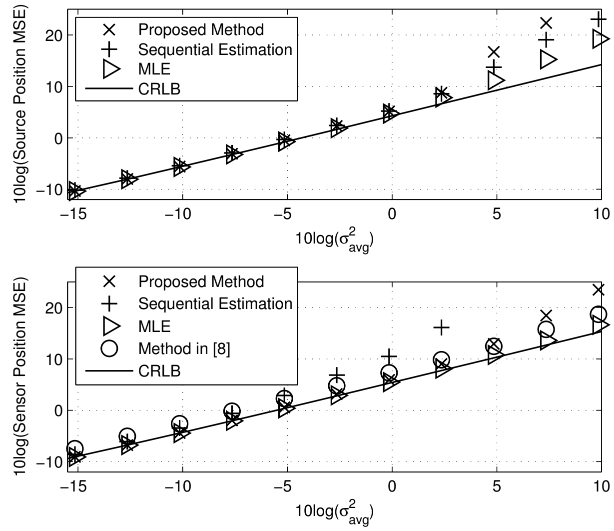


**Fig. 3**. Comparison of the estimation accuracy when $\sigma_r^2$ is fixed to $10^{-3}$ and the sensor position noise power varies. (a) Results on source location estimates. (b) Results on sensor position estimates.

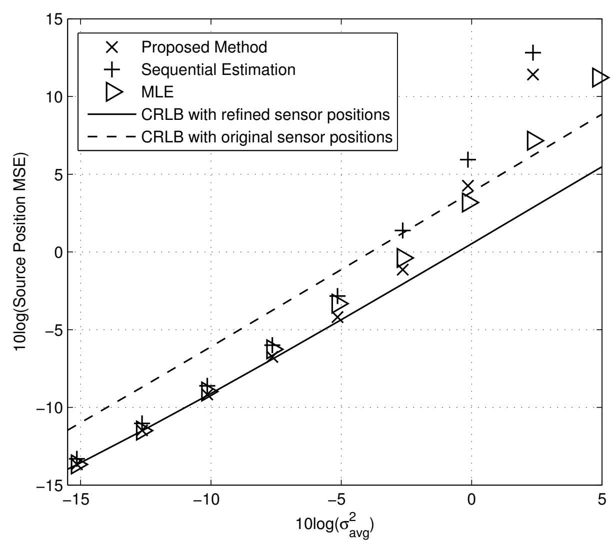


**Fig. 4**. Estimation results of the new source location when $\sigma_r^2$ is fixed to $10^{-3}$ and the sensor position noise power varies.


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