SPARSITY-AWARE TDOA LOCALIZATION OF MULTIPLE SOURCES

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ABSTRACT

The problem of source localization from time-difference-of-arrival (TDOA) measurements is in general a non-convex and complex problem due to its hyperbolic nature. This problem becomes even more complicated for the case of *multi-source* localization where TDOAs should be assigned to their respective sources. We simplify this problem to an ℓ_1-norm minimization by introducing a novel TDOA fingerprinting model for a multi-source scenario. Moreover, we propose an *innovative trick* to enhance the performance of our proposed fingerprinting model in terms of the number of identifiable sources. An interesting by-product of this enhanced model is that under some conditions we can convert the given underdetermined problem to an overdetermined one and efficiently solve it using classical least squares (LS) approaches. Our simulation results illustrate a good performance for the introduced TDOA fingerprinting.

Index Terms— Multi-source localization, TDOA fingerprinting, sparse reconstruction.

1. INTRODUCTION

Precise localization of multiple sources is a fundamental problem which has received an upsurge of attention recently [1]. Many different approaches have been proposed in literature to recover the location of the sources based on time-of-flight (ToF), time-difference-of-arrival (TDOA) or received-signal-strength (RSS) measurements. A traditional wisdom in RSS-based localization tries to extract distance information from the RSS measurements. However, this approach fails to provide accurate location estimates due to the complexity and unpredictability of the wireless channel. This has motivated another category of RSS-based positioning, the so-called location fingerprinting. This technique discretizes the physical space into grid points (GPs) and creates a map representing the space by assigning to every GP a set of location-dependent RSS parameters, one for every access point (AP). The location of the source is then estimated by comparing real-time measurements with the fingerprinting map at the source or APs, for instance using K-nearest neighbors (KNN) [2] or Bayesian classification (BC) [3]. A closer look at the grid-based fingerprinting localization reveals that the source location is unique in the spatial domain, and can thus be represented by a 1-sparse vector. This motivated the use of compressive sampling (CS) [4] to recover the location of the source using only a few measurements by solving an ℓ_1-norm minimization problem [5, 6, 7, 8, 9]. In [10], we have proposed to reformulate the sparse localization problem by making use of the (not previously exploited) cross-correlations of the signal readings at different APs which leads to a considerable improvement in terms of the number of identifiable sources.

On the other hand, the problem of TDOA-based localization for a single (multiple) source(s) has been investigated from different perspectives in literature [11, 12, 13, 14, 15, 16, 17, 18]. In the speech and acoustic domain, some of these studies consider disjoint sources such as [12] and in many others linear array receivers are assumed and thus the problem basically boils down to direction of arrival (DOA) estimation [15]. In a big line of research, the conversion of phase to TDOA leads to aliasing effects at high frequencies for large receiver spacings [13, 15]. In [14], a fingerprinting-like approach is proposed and the area is discretized into a set of GPs for which an acoustic map function is defined. Through a proper processing of the acoustic map and de-emphasizing the effect of the dominant source, they illustrate a good performance in localizing two sources, but in some situations their performance drops if the number of targets is larger than three. Surprisingly, none of the aforementioned studies exploits CS or sparse reconstruction ideas. In [19], the source sparsity is exploited to simplify the hyperbolic source localization problem into an ℓ_1-norm minimization. However, the algorithm in [19] is single-source and treats different sources separately.

The contributions of this work are as follows. Firstly, we formulate the problem of sparsity-aware multi-source localization by defining a novel TDOA fingerprinting model. Secondly, we propose an *innovative trick* to enhance our proposed paradigm in terms of the number of identifiable sources which leads to a significant detection gain. In Section 2, the TDOA network model as well as our measurement model are explained. Section 3 introduces our novel sparse multi-source TDOA localization idea. Section 4 presents the trick to enhance the performance of our proposed multi-source algorithm. Simulations in Section 5 corroborate our analytical claims.

2. TDOA NETWORK MODEL

Consider M APs distributed over an area which is discretized into N GPs. Note that the APs can be located anywhere, not necessarily on the GPs. We consider K source nodes (SNs) which are randomly located on any of these GPs. Note that extensions of this work to deal with "*off-grid*" sources using the concept of *grid mismatch* can be found in [20]. We assume that the APs are connected to each other in a wireless or wired fashion so that they can cooperate by exchanging their signal readings. Now, if the k-th source broadcasts a time domain signal $s_k(t)$, the received signal at the i-th AP can be expressed by

$$x_i(t) = \sum_{k=1}^{K} h_{i,k} s_k(t - \tau_{i,k}) + n_i(t), \tag{1}$$

where $h_{i,k}$ and $\tau_{i,k}$ respectively are the channel coefficient and time-delay from the k-th source to the i-th AP. Here, for the sake of simplicity, we have considered a single-tap flat fading channel. Further, $n_i(t)$ (with variance $\sigma^2_{n_i}$) represents the additive white Gaussian noise (AWGN). In a classical TDOA scenario, for each selection of a reference AP, we can collect $M-1$ TDOA measurements. However, the maximum number of distinct TDOA measurements, the so-called full set, is $(M-1)M/2$. Here, we choose a non-redundant

This work was supported by NWO-STW under the VICI program (10382).

set of $M-1$ TDOA measurements by always considering the first AP as the reference. Since we consider a passive source localization scenario, taking cross-correlations of the received signals is the optimal approach for extracting the TDOAs under an AWGN assumption [21]. The signals $s_k(t)$ and $n_i(t)$ are assumed to be ergodic, mutually uncorrelated white sequences, i.e., $\int_t s_k(t)s_{k'}(t-\Delta)dt = \delta(\Delta)\delta(k-k')$, $\int_t n_i(t)n_j(t-\Delta)dt = \sigma_{n_i}^2\delta(\Delta)\delta(i-j)$, and $\int_t s_k(t)n_i(t)dt = 0$. Therefore, the cross-correlation between the received signal at the i-th AP and the reference AP is given by

$$\begin{aligned} r_i(\Delta) &= \int_t \left(\sum_{k=1}^{K} h_{i,k}s_k(t-\tau_{i,k}) + n_i(t)\right) \\ &\quad \times\left(\sum_{k'=1}^{K} h_{1,k'}s_{k'}(t-\Delta-\tau_{1,k'}) + n_1(t-\Delta)\right)dt \\ &= \sum_{k=k'=1}^{K}\int_t \left(h_{i,k}s_k(t-\tau_{i,k}) + n_i(t)\right) \\ &\quad \times\left(h_{1,k}s_k(t-\Delta-\tau_{1,k}) + n_1(t-\Delta)\right)dt \\ &= \sum_{k=1}^{K} h_{i,k}h_{1,k}\delta(\Delta-\Delta_{i,k}), \end{aligned} \tag{2}$$

where $\Delta_{i,k} = \tau_{1,k} - \tau_{i,k}$ is the TDOA of the k-th source w.r.t. the AP pair $(\text{AP}_1,\ \text{AP}_i)$. As is shown by (2), for a single-tap channel as considered here, the K dominant peaks of $r_i(\Delta)$ return the TDOA values $\{\Delta_{i,k}\}_k$ related to the K sources.

Remark *(Multipath Scenarios):* In fact, TDOAs coming from reflective paths can be confused with the direct-path ones in a multipath scenario. In such scenarios, TDOA-disambiguation techniques (see e.g. [11]) could be used as an add-on module to our proposed algorithms to discriminate between direct-path and reflective-path TDOAs before performing localization. Having pointed out one solution, we will not consider the multipath issue in this paper. ■

The main problem with (2) is that even though we can estimate the set of TDOAs $\{\Delta_{i,k}\}_k$, we do not know the source indices of the TDOAs. This leaves us with solving an assignment problem to relate the TDOAs to the sources. To make it more clear, as shown in Fig. 1, we define the $\Delta_i^{(k)}$'s as the TDOAs ordered in an increasing fashion ($\Delta_i^{(1)} \le \cdots \le \Delta_i^{(K)}$). These $\Delta_i^{(k)}$'s can be measured for $i = 2, \cdots, M$ and they are stacked in the measurement vectors $\mathbf{y}^{(k)} = [\Delta_2^{(k)}, \cdots, \Delta_M^{(k)}]^T$. Note the difference with the $\Delta_{i,k}$'s, which denote the TDOA values ordered according to the source indices leading to the vectors $\mathbf{y}_k = [\Delta_{2,k}, \cdots, \Delta_{M,k}]^T$. It is worth mentioning that while the $\mathbf{y}^{(k)}$ vectors are perfectly known, the $\mathbf{y}_k$ vectors are not. Now, the problem considered herein can be stated as follows. Having computed the TDOAs for the explained multi-source scenario, locate all the sources simultaneously.

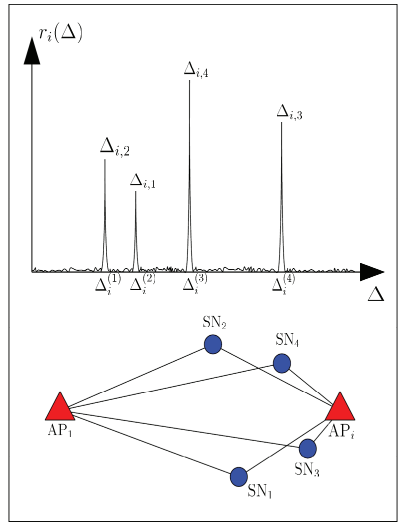

Fig. 1. Assignment problem; definition of $\Delta_i^{(k)}$ and $\Delta_{i,k}$. Note that SN_2 produces the smallest TDOA while SN_3 produces the largest.

3. SPARSITY-AWARE TDOA LOCALIZATION

The problem of TDOA localization becomes highly non-trivial for the case of multiple sources since on top of the non-linear nature of the problem, the assignment of the TDOAs to the different sources has to be resolved. Therefore, it is of special interest to be able to simultaneously localize the sources using a novel TDOA paradigm.

3.1. Single-Source Scenario

For the TDOA setup under consideration, the location-dependent parameter set used for fingerprinting will consist of the TDOA measurements from the APs. The location of the source is then (depending on the scenario) estimated by comparing the *runtime phase* measurements with the fingerprinting map recorded in the *training phase*, at the source (single-source problem) or as in our case at a central unit connected to the APs. One way to carry out this comparison is by exploiting the source sparsity and considering that the source can only be located at a single GP. This way the single-source localization problem can be cast into a sparse representation framework given by $\mathbf{y} = \boldsymbol{\Psi}\boldsymbol{\theta} + \boldsymbol{\epsilon}$ where for a single-source problem we simply have[1] $\mathbf{y} = \mathbf{y}^{(1)} = [\Delta_2^{(1)}, \cdots, \Delta_M^{(1)}]^T$ and $\boldsymbol{\Psi}$ is the $(M-1)\times N$ fingerprinting matrix of the form

$$\boldsymbol{\Psi} = \begin{bmatrix} \Delta_{2,1}^{\text{g}} & \cdots & \Delta_{2,N}^{\text{g}} \\ \vdots & \ddots & \vdots \\ \Delta_{M,1}^{\text{g}} & \cdots & \Delta_{M,N}^{\text{g}} \end{bmatrix}, \tag{3}$$

where $\Delta_{i,n}^{\text{g}}$ represents the TDOA of the received signal at the i-th AP and the reference AP from a source located at the n-th GP. Note the difference with $\Delta_{i,k}$ which is the measured TDOA from the k-th source w.r.t. the $(\text{AP}_1,\ \text{AP}_i)$ pair. Further, $\boldsymbol{\theta}$ is an $N\times 1$ vector with all elements equal to zero except for one element corresponding to the location of the source which is equal to 1. Thus, $\mathbf{y}$ will be a 1-sparse TDOA vector characterized by the sparsity basis $\boldsymbol{\Psi}$ and the ultimate goal is to recover $\boldsymbol{\theta}$ only by determining the index of its non-zero element. Solving $\mathbf{y} = \boldsymbol{\Psi}\boldsymbol{\theta} + \boldsymbol{\epsilon}$ with classical LS produces a poor estimate due to the under-determined nature of the problem ($M-1 \ll N$). Instead, sparse reconstruction techniques (or CS) aim to reconstruct $\boldsymbol{\theta}$ from $\mathbf{y}$, by taking the source sparsity concept into account. Now, as long as every 2 columns of $\boldsymbol{\Psi}$ are independent, $\boldsymbol{\theta}$ can be well-recovered by solving the following ℓ_1-norm minimization problem (similar to [19]) $\min_{\boldsymbol{\theta}} \|\mathbf{y} - \boldsymbol{\Psi}\boldsymbol{\theta}\|_2^2 + \lambda\|\boldsymbol{\theta}\|_1$ where λ is the sparsity-regularizing parameter.

3.2. Multi-Source Scenario

The key question here is how we can extend this single-source localization scheme to a multi-source one. Before explaining the idea, we would like to remind the reader of a natural phenomenon in RSS fingerprinting. Different from TDOA measurements, the RSSs of the source signals will sum up at the APs [7, 10]. On the other hand,

[1]Note that only for a single-source scenario $\mathbf{y}^{(1)} = \mathbf{y}_1$, but this cannot be generalized to a multi-source scenario, i.e., in that case $\mathbf{y}^{(k)} \neq \mathbf{y}_k$.

TDOA measurements do not simply follow this pattern. Nevertheless, this motivated us to sum up the measured $\Delta_i^{(k)}$ values for different sources at the APs, i.e., $\mathbf{y} = \sum_k \mathbf{y}^{(k)}$. Note that this vector is equal to $\mathbf{y} = \sum_k \mathbf{y}_k$ and thus automatically leads to a similar formulation as for the single-source case

$$\mathbf{y} = \boldsymbol{\Psi}\boldsymbol{\theta} + \boldsymbol{\epsilon}, \tag{4}$$

where $\boldsymbol{\theta}$ is now a K-sparse vector (containing all zeros except for K ones) to accommodate the K sources. We would like to emphasize again that in practice we can only measure the $\mathbf{y}^{(k)}$ vectors because it is still unknown to which source they belong, i.e., the $\mathbf{y}_k$ vectors cannot be separately calculated. However, the *beauty* of the proposed sparsity-aware multi-source TDOA localization (SMTL) framework is that since we work with $\mathbf{y} = \sum_k \mathbf{y}^{(k)} = \sum_k \mathbf{y}_k$, it does not really require such assignment information. Therefore, similar to the single-source scenario, (4) can also be solved using an ℓ_1-norm minimization (with λ as defined earlier) as

$$\min_{\boldsymbol{\theta}} \|\mathbf{y} - \boldsymbol{\Psi}\boldsymbol{\theta}\|_2^2 + \lambda \|\boldsymbol{\theta}\|_1 . \tag{5}$$

4. ENHANCED SPARSITY-AWARE MULTI-SOURCE TDOA LOCALIZATION (ESMTL)

The proposed SMTL algorithm has a limited source detection capability which comes from the concept of sparse reconstruction. This basically limits the number of detectable sources (K) through the number of measurements (here only $M-1$). This detection capability can significantly be improved if we could somehow add extra rows to the existing $\boldsymbol{\Psi}$ defined by (3). The question is how to add additional rows to $\boldsymbol{\Psi}$ without taking additional measurements. The *innovative trick* we use here is to consider not just the sum of the TDOAs $\mathbf{y} = \sum_k \mathbf{y}^{(k)} = \sum_k \mathbf{y}_k$, but any sum of a function of the TDOAs as

$$\mathbf{y}_{f_l} = \sum_k f_l(\mathbf{y}^{(k)}) = \sum_k f_l(\mathbf{y}_k), \tag{6}$$

where $f_l(\mathbf{y}^{(k)}) = [f_{l,1}(\Delta_2^{(k)}), \cdots, f_{l,M-1}(\Delta_M^{(k)})]^T$ with $f_{l,i}(.)$ being any possible *measurement function*. If we combine a set of L such sums, i.e., $\tilde{\mathbf{y}} = [\mathbf{y}_{f_1}^T, \mathbf{y}_{f_2}^T, \cdots, \mathbf{y}_{f_L}^T]^T$, this newly defined measurement vector $\tilde{\mathbf{y}}$ calls for a new fingerprinting map $\tilde{\boldsymbol{\Psi}}$ which can accordingly be defined as

$$\tilde{\boldsymbol{\Psi}} = \left[f_1(\boldsymbol{\Psi})^T, \cdots, f_L(\boldsymbol{\Psi})^T\right]^T, \tag{7}$$

where

$$f_l(\boldsymbol{\Psi}) = \begin{bmatrix} f_{l,1}(\Delta_{2,1}^{\mathrm{g}}) & \cdots & f_{l,1}(\Delta_{2,N}^{\mathrm{g}}) \\ \vdots & \ddots & \vdots \\ f_{l,M-1}(\Delta_{M,1}^{\mathrm{g}}) & \cdots & f_{l,M-1}(\Delta_{M,N}^{\mathrm{g}}) \end{bmatrix}, \tag{8}$$

and thus the model (4) can be extended to $\tilde{\mathbf{y}} = \tilde{\boldsymbol{\Psi}}\boldsymbol{\theta} + \tilde{\boldsymbol{\epsilon}}$. Again $\boldsymbol{\theta}$ can be found by solving $\min_{\boldsymbol{\theta}} \|\tilde{\mathbf{y}} - \tilde{\boldsymbol{\Psi}}\boldsymbol{\theta}\|_2^2 + \lambda\|\boldsymbol{\theta}\|_1$. For an identifiable multi-source localization, first, there should exist $M > 3$ APs, and second, every $2K$ columns of $\tilde{\boldsymbol{\Psi}}$ should be linearly independent. More details can be found in the extended version of this work [20].

4.1. Design of the Measurement Functions

The new $\tilde{\boldsymbol{\Psi}}$ has $L(M-1)$ rows instead of only $M-1$ rows, i.e., it is capable of detecting more sources simultaneously, if the measurement functions $f_{l,i}(.)$ own certain properties. First of all, they should be nonlinear in general since linear functions generate dependent rows in $\tilde{\boldsymbol{\Psi}}$. Moreover, these functions (suppose exponentials) can generate very large or very small values compared to the elements of $\boldsymbol{\Psi}$ which impairs the incoherence property of $\tilde{\boldsymbol{\Psi}}$ and hence degrades the reconstruction quality. Having this issue in mind, forcing the resulting $\tilde{\boldsymbol{\Psi}}$ to be as close as possible to an isometry with orthonormal columns is healing and also satisfies the requirement of sparse reconstruction, i.e., every $2K$ columns of $\tilde{\boldsymbol{\Psi}}$ should be linearly independent. In principle, the measurement functions $f_{l,i}(.)$ can be any nonlinear function. However, here we restrict ourselves to a base set of L non-linear functions denoted as $\{g_l(.)\}_{l=1}^L$ (the $g_l(.)$ functions could for example be monomials, i.e, $g_l(.) = (.)^l$) and we try to find optimal scalings of these base functions to design our measurement functions $f_{l,i}(.) = c_{l,i}g_l(.)$. The corresponding measurement matrix $\tilde{\boldsymbol{\Psi}}$ can be expressed as

Algorithm 1 Optimal $f_{l,i}(.)$ design using scalings

1: Choose an appropriate L and solve (14) using $\mathbf{T}_1$ in (12)
2: Compute $\{\mathbf{c}_l\}$ using (15)
3: Calculate $\tilde{\boldsymbol{\Psi}}$ using (9)

$$\tilde{\boldsymbol{\Psi}} = \begin{bmatrix} \mathrm{diag}(\mathbf{c}_1) \cdots & \mathbf{0} \\ \vdots \quad \ddots & \vdots \\ \mathbf{0} & \cdots \mathrm{diag}(\mathbf{c}_L) \end{bmatrix} \bar{\boldsymbol{\Psi}}, \tag{9}$$

where $\mathbf{c}_l = [c_{l,1}, \cdots, c_{l,M-1}]^T$ and $\bar{\boldsymbol{\Psi}} = [g_1(\boldsymbol{\Psi})^T, \cdots, g_L(\boldsymbol{\Psi})^T]^T$. To force $\tilde{\boldsymbol{\Psi}}$ to be an isometry, we then minimize

$$\min_{\{\mathbf{c}_l\}} \left\|\tilde{\boldsymbol{\Psi}}^T(\{\mathbf{c}_l\})\tilde{\boldsymbol{\Psi}}(\{\mathbf{c}_l\}) - \mathbf{I}\right\|_F^2. \tag{10}$$

By applying $\mathrm{vec}(\mathbf{ABC}) = (\mathbf{C}^T \otimes \mathbf{A})\mathrm{vec}(\mathbf{B})$, we can write

$$\mathrm{vec}\left(\tilde{\boldsymbol{\Psi}}^T\tilde{\boldsymbol{\Psi}}\right) = (\bar{\boldsymbol{\Psi}}^T \otimes \bar{\boldsymbol{\Psi}}^T)\,\mathrm{vec}\Bigg(\overbrace{\begin{bmatrix} \mathrm{diag}(\mathbf{c}_1 \odot \mathbf{c}_1) \cdots & \mathbf{0} \\ \vdots \quad \ddots & \vdots \\ \mathbf{0} & \cdots \mathrm{diag}(\mathbf{c}_L \odot \mathbf{c}_L) \end{bmatrix}}^{\boldsymbol{\Gamma}}\Bigg),$$

where $\otimes$ and $\odot$ respectively denote the Kronecker and Hadamard products. Let us define $\boldsymbol{\gamma} = \mathrm{vec}(\boldsymbol{\Gamma})$ and $\bar{\boldsymbol{\gamma}} = \mathrm{diag}(\boldsymbol{\Gamma})$. As a result, we can replace (10) by the following LS problem

$$\min_{\{\mathbf{c}_l\}} \left\|(\bar{\boldsymbol{\Psi}}^T \otimes \bar{\boldsymbol{\Psi}}^T)\boldsymbol{\gamma} - \mathrm{vec}(\mathbf{I})\right\|_2^2. \tag{11}$$

Now, because of the structure in $\boldsymbol{\gamma}$, we have $\boldsymbol{\gamma} = \mathbf{T}_1\bar{\boldsymbol{\gamma}}$, where $\mathbf{T}_1$ is a matrix of size $L^2(M-1)^2 \times L(M-1)$ which ensures that $\mathbf{T}_1\bar{\boldsymbol{\gamma}}$ has the same structure as $\boldsymbol{\gamma}$. Not that $\mathbf{T}_1$ can be given by

$$[\mathbf{T}_1]_{i,j} = \begin{cases} 1, & \text{if } i = (j-1)L(M-1) + j, \\ 0, & \text{otherwise}. \end{cases} \tag{12}$$

Now, since for every $\bar{\boldsymbol{\gamma}}$ we can find at least one set $\{\mathbf{c}_l\}$, the LS problem (11) is equivalent to

$$\min_{\bar{\boldsymbol{\gamma}}} \left\|(\bar{\boldsymbol{\Psi}}^T \otimes \bar{\boldsymbol{\Psi}}^T)\mathbf{T}_1\bar{\boldsymbol{\gamma}} - \mathrm{vec}(\mathbf{I})\right\|_2^2, \tag{13}$$

which is a linear LS problem with the following solution

$$\widehat{\bar{\boldsymbol{\gamma}}} = \left[(\bar{\boldsymbol{\Psi}}^T \otimes \bar{\boldsymbol{\Psi}}^T)\mathbf{T}_1\right]^\dagger \mathrm{vec}(\mathbf{I}), \tag{14}$$

Finally, we can calculate

$$\left[\hat{\mathbf{c}}_1^T, \cdots, \hat{\mathbf{c}}_L^T\right]^T = \mathrm{diag}\left(\left[\mathrm{ivec}(\mathbf{T}_1\widehat{\bar{\boldsymbol{\gamma}}})\right]^{\frac{1}{2}}\right), \tag{15}$$

where $\mathrm{diag}(\mathbf{X})$ takes the diagonal elements of the diagonal matrix $\mathbf{X}$. The overall procedure is summarized in Algorithm 1. A more elaborate way of designing the measurement functions using optimal linear combinations of $\{g_l(.)\}_{l=1}^L$ can be found in [20].

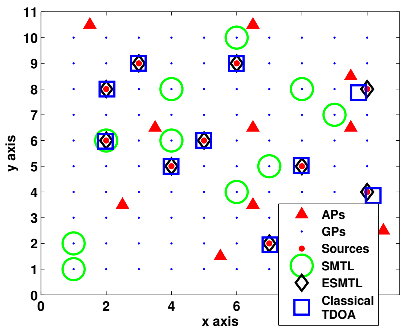

Fig. 2. Multi-source localization with 10 APs

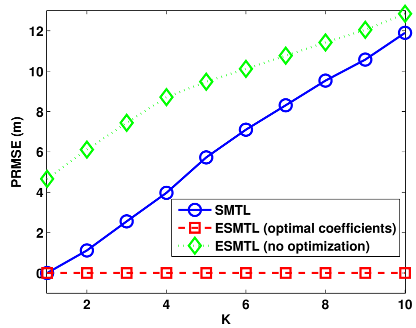

Fig. 3. PRMSE vs. K for SNR $= 20$dB

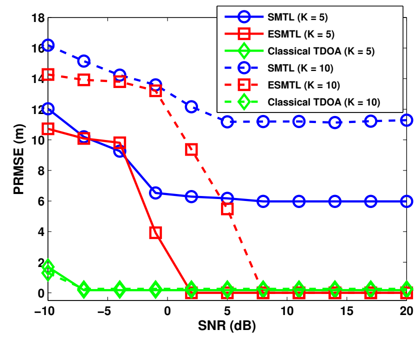

Fig. 4. PRMSE vs. SNR for $K = 5$ and 10

4.2. Advantages of ESMTL

Besides the enhanced source detection capability, there are a number of other advantages in using the ESMTL approach. First of all, an important advantage of this idea is that the recently added rows of $\tilde{\mathbf{\Psi}}$ (compared to $\mathbf{\Psi}$) are simply generated based on the existing elements of $\mathbf{\Psi}$ and *no extra measurements* are required in the training phase. The same holds for the runtime phase where the new elements of $\tilde{\mathbf{y}}$ are simply calculated based on the already measured values of $\mathbf{y}$. This important characteristic of the proposed TDOA fingerprinting avoids imposing extra *cost-prohibitive* measurements on the central unit. In some situations, Δ peaks might coincide in the output of the cross-correlations contained in $\tilde{\mathbf{y}}^{(k)}$ and this problem cannot be resolved. Another important corollary of the new $\tilde{\mathbf{\Psi}}$ is healing this issue (more details and solutions in [20]). Now that we can have several extra equations, a simple solution to heal this issue is that when computing cross-correlations, say for the $(\text{AP}_1,\ \text{AP}_i)$ pair, if we notice that some peaks are overlapping (number of dominant peaks is less than K), we can ignore the corresponding elements in $\tilde{\mathbf{y}}$ and correspondingly the rows in $\tilde{\mathbf{\Psi}}$. This means that we solve $\min_{\boldsymbol{\theta}} \|\tilde{\mathbf{y}}' - \tilde{\mathbf{\Psi}}'\boldsymbol{\theta}\|_2^2 + \lambda\|\boldsymbol{\theta}\|_1$ with $\tilde{\mathbf{y}}' = \mathbf{T}_2\tilde{\mathbf{y}}$ and $\tilde{\mathbf{\Psi}}' = \mathbf{T}_2\tilde{\mathbf{\Psi}}$ where $\mathbf{T}_2$ is a selection matrix which removes the elements and rows corresponding to the measurements with coincident peaks from $\tilde{\mathbf{y}}$ and $\tilde{\mathbf{\Psi}}$, respectively. Moreover, if there are $M > 3$ APs as is required for identifiable multi-source TDOA localization [20], by finding appropriate measurement functions we can keep on increasing the number of rows so that we can attain a full column rank $\tilde{\mathbf{\Psi}}$ matrix. In such a case, no matter what the structure of $\boldsymbol{\theta}$ might be (even not sparse), it can be efficiently recovered using classical LS as $\hat{\boldsymbol{\theta}}_{\text{LS}} = \tilde{\mathbf{\Psi}}^{\dagger}\tilde{\mathbf{y}}$.

5. SIMULATION RESULTS

We consider a wireless network of size 10×10 m^2 divided into $N = 100$ GPs, $M = 10$ APs covering the whole area, and up to $K = 10$ sources. Instead of taking infinite integrals (as in (2)), we will work with discrete-time signals of limited length and hence the noise terms $n_i(t)$ will not be completely eliminated and will affect our performance through $\boldsymbol{\epsilon}$. We assume that none of the received signals is so weak that it will be considered as noise in $r_i(\Delta)$ and cannot be detected. Here, we consider a baseband ultra-wideband (UWB) BPSK signal with bandwidth $B = 1$GHz and compute the autocorrelations and cross-correlations during a time-slot of length $T = 5\mu$s. This is equal to recording $T \times B = 5 \times 10^{-6} \times 10^{9} = 5000$ BPSK symbols for our computations. We define the signal to noise ratio (SNR) at the i-th AP as the ratio of the received signal power to the noise power. For a quantitative comparison, we consider the positioning root mean squared error (PRMSE) defined by PRMSE $= \sqrt{\sum_{p=1}^{P}\sum_{k=1}^{K} e_{k,p}^2/P}$, where $e_{k,p}$ represents the distance between the real location of the k-th source and its estimated location at the p-th Monte Carlo (MC) trial. All simulations are averaged over $P = 20$ MC runs where in each run the sources are deployed on different random locations. For the ESMTL, we consider $L = 5$ monomial base functions, i.e., $g_l(.) = (.)^l,\ l = 1, \cdots, 5$ to enhance the proposed SMTL by introducing new rows. Next, we use the proposed approach in Subsection 4.1 to find proper scalings of the base functions and compute $\tilde{\mathbf{\Psi}}$. This means $\tilde{\mathbf{\Psi}}$ will be a $5(M-1) \times N = 45 \times 100$ matrix, while $\mathbf{\Psi}$ is a $(M-1) \times N = 9 \times 100$ matrix. As a benchmark, we also simulate the hyperbolic positioning method proposed in [16] (we call it classical TDOA) where we localize each source disjointly and then compute the overall PRMSE for the K sources.

In Fig. 2, we consider $K = 10$ sources randomly deployed over the covered area. The SNR is assumed to be 20dB for all the APs. As is clear from the figure, the SMTL can only localize a single source with minimum error. However, by using the ESMTL algorithm we can locate all the $K = 10$ sources and this clearly illustrates the enhanced performance of ESMTL compared to SMTL. As can be seen, ESMTL performs even (a little bit) better than the classical disjoint hyperbolic TDOA method because the classical disjoint approach is built on triangulation and thus the TDOA measurement errors are directly mapped to location errors.

Fig. 3 depicts the PRMSE of localization vs. the number of sources increasing up to $K = 10$. As can be seen, by increasing K, the PRMSE of localization for SMTL increases sharply while ESMTL can handle all the sources simultaneously with minimum error. Fig. 3 also emphasizes the effect of the proposed optimal choice of the coefficients for the measurement functions (Subsection 4.1). As is clear from the figure, when we use $\bar{\mathbf{\Psi}}$ (the dotted line marked with $\Diamond$) instead of $\tilde{\mathbf{\Psi}}$, the ESMTL is drastically degraded. This is because g_4 and g_5 generate much larger or smaller values than the values in $\mathbf{\Psi}$ which impairs the mutual incoherence in $\bar{\mathbf{\Psi}}$. Note that we do not plot the results for $K > 10$ sources since for those cases $\boldsymbol{\theta}$ is not really sparse, i.e., we do not have $K \ll N$.

Finally, we also plot the PRMSE vs. SNR for the number of sources $K = 5$ and $K = 10$ in Fig. 4. As can be seen, increasing the number of sources will degrade the performance of SMTL. However, ESMTL can attain the minimum PRMSE even with $K = 10$ sources for SNR values larger than 8dB. Notably, classical hyperbolic TDOA performs better than ESMTL for very low SNRs while for high SNRs ESMTL performs a little bit better. We would like to highlight that we do not compare our results with the KNN, the BC, or even semi-definite relaxation (SDR)-based algorithms because the superiority of ℓ_1-norm minimization approaches compared to them in similar contexts is illustrated in [7] and [19].

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