COLOCATED MIMO RADAR: CRAMER-RAO BOUND AND OPTIMAL TIME DIVISION MULTIPLEXING FOR DOA ESTIMATION OF MOVING TARGETS

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ABSTRACT

Multiple-Input-Multiple-Output (MIMO) radars with colocated transmit and receive antennas offer the advantage of a larger (virtual) aperture compared to a conventional Single-Input-Multiple-Output (SIMO) radar. Hence a higher accuracy of the estimated direction of arrival (DOA) of a target can be achieved. In general, the accuracy of DOA estimators decreases in a MIMO radar if the target moves relative to the radar, because the motion causes an unknown phase change of the baseband signal due to the Doppler effect. We compute the Cramer-Rao bound (CRB) of DOA estimation of a non-stationary target for a MIMO radar with colocated antennas for a general time division multiplexing (TDM) scheme. This allows a quantitative comparison of different MIMO and SIMO radars. Moreover, we derive an optimal TDM scheme such that the CRB is as small as in the stationary case. The results are confirmed by simulations.

***Index Terms*—** MIMO Radar, Cramer-Rao Bound, Time Division Multiplexing, DOA Estimation

1. INTRODUCTION

MIMO radars have been intensively studied recently. Compared to a conventional phased array or SIMO radar, they offer several advantages, e.g. a more flexible transmit beampattern design, a higher number of detectable targets and a higher accuracy in parameter estimation, especially in DOA estimation [1, 2, 3]. High performance in DOA estimation, despite of a small number of antennas and a small geometric aperture, is important in automotive radars for example [4].

The CRB is a lower bound on the covariance matrix of all unbiased estimators, independent of the used estimation algorithm. It enables to compare the maximum accuracy of different radar systems. The CRB has already been derived for different settings. The CRB of DOA estimation for SIMO radar was derived in [5] and an extended version for non-local errors has been established in [6]. The investigation of the CRB for non-stationary targets is given in [7, 8], and for SIMO radar with only one receiving channel in [9]. The CRB of DOA estimation with a MIMO radar for general transmitted signals has been computed in [1] and [3] under the assumption of stationary targets. The authors of [10, 11] investigate target parameter estimation of moving targets and different transmission schemes in ground moving target indication, and in [12, 13] the CRB for DOA and Doppler estimation is derived for a MIMO radar with simultaneously transmitting antennas.

In general, MIMO radars require a more complex hardware than SIMO radars. They can be realized by code, frequency or time division multiplexing. All multiplexing variants have several advantages and disadvantages. We focus in this paper on TDM-MIMO radars, since they are easier to implement compared to more complicated coding schemes. Moreover, since the antennas do not transmit simultaneously, the transmitted signal can be kept as in a SIMO radar, e.g. by using a linear frequency modulated continuous wave (LFMCW). Experimental investigation of a TDM-MIMO radar was reported in [14, 15], but quantitative comparisons to SIMO radars in the non-stationary case, especially in DOA estimation, are still missing.

We fill this gap by investigating the DOA estimation of a moving target by computing the CRB of a TDM-MIMO radar. Note that since the signals are transmitted successively, the resulting baseband signal model has a different structure than that in [12, 13]. This CRB enables us to compare TDM-MIMO radars with different TDM schemes and SIMO radars for moving targets.

In a SIMO radar, the DOA can be estimated by analyzing the phase relation of the baseband signal of the receiving channels. In a MIMO system, the phase relation between the transmitting channels is used as well which makes it possible to construct a virtual array with a larger aperture than the receiving or transmitting array alone [16]. In the non-stationary case, the target is moving, resulting in another phase change of the baseband signal. This phase change has to be estimated as well, in order to exploit the phase relation between the transmitting channels. We derive the CRB for this situation.

We present the signal model in Section 2 and derive the CRB in Section 3. In Section 4, the implications of the CRB are discussed and an optimal TDM scheme is derived. The theoretical results are confirmed by simulations in Section 5.

The following notations are used in this paper: $\underline{1}_K$ is a vector of length K with all elements equal 1, and $\mathbf{I}$ is the identity matrix. In addition, * stands for conjugate, ^{T} stands for transpose, ^{H} stands for conjugate transpose, $\underline{y} = \exp(\underline{x})$ is understood as an element-by-element operation $y_i = \exp(x_i)$. $\otimes$ is the Kronecker tensor product and $\odot$ the entrywise Hadamard product.

2. SIGNAL MODEL

We consider a colocated MIMO radar consisting of a linear receiver (RX) and transmitter (TX) array with isotropic antennas. The moving target is modeled as a point source and the transmitted signal is narrowband. The positions of the N_{RX} RX antennas and N_{TX} TX antennas are given in $\underline{d}^{\text{RX}} \in \mathbb{R}^{N_{\text{RX}}}$ and $\underline{d}^{\text{TX}} \in \mathbb{R}^{N_{\text{TX}}}$, respectively, in units of $\lambda/2\pi$ where λ is the carrier wavelength. The positions of the TX antennas in the sequence in which they transmit are given in $\underline{d}^{\text{Pulse}} \in \mathbb{R}^{N_{\text{Pulse}}}$, i.e. some TX antennas can occur several times in $\underline{d}^{\text{Pulse}}$, if they transmit several pulses. N_{Pulse} is the number of transmitted pulses in a cycle. The time instances at which the antennas transmit in a cycle are given in $\underline{t} \in \mathbb{R}^{N_{\text{Pulse}}}$. This means, the antenna with position d_i^{Pulse} transmits at time t_i, see Fig. 1 for an example.

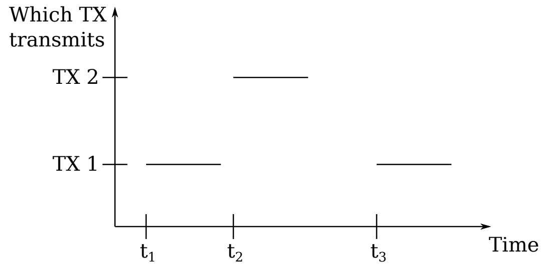

Fig. 1. Example of a TDM scheme: 2 transmitters transmitting at times $\underline{t} = [t_1, t_2, t_3]^T$. Here, $\underline{d}^{\text{Pulse}} = [d_1^{\text{TX}}, d_2^{\text{TX}}, d_1^{\text{TX}}]^T$.

The antenna positions of the virtual array of the TDM-MIMO radar is given by

$$\underline{d}^{\text{Virt}} := \underline{1}_{N_{\text{Pulse}}} \otimes \underline{d}^{\text{RX}} + \underline{d}^{\text{Pulse}} \otimes \underline{1}_{N_{\text{RX}}} \in \mathbb{R}^{N_{\text{Virt}}} \tag{1}$$

with $N_{\text{Virt}} = N_{\text{Pulse}} N_{\text{RX}}$. Note that the transmitting sequence is already incorporated in $\underline{d}^{\text{Pulse}}$. The steering vector of the virtual array, which incorporates the phase change of the baseband signal of all transmitter-receiver combinations, is given by

$$\underline{a}(u) = \exp(j \cdot \underline{d}^{\text{Virt}} u) = \exp(j \cdot \underline{d}^{\text{Pulse}} u) \otimes \exp(j \cdot \underline{d}^{\text{RX}} u). \tag{2}$$

Here $u = \sin(\Theta)$ is the electrical angle with Θ being the DOA of the target measured perpendicular to the array axis. We assume a number of L measurement cycles where one cycle consists of N_{Pulse} pulses. The complex baseband signal $\underline{X}(l) \in \mathbb{C}^{N_{\text{Virt}}}$ of cycle l is given as

$$\underline{X}(l) = \exp(j\underline{\gamma}\omega_d) \odot \underline{a}(u) \frac{1}{\sqrt{N_{\text{Pulse}}}} s(l) + \underline{N}(l), \;\; l = 1, \ldots, L \tag{3}$$

with the unknown, deterministic, complex target signal $s(l) \in \mathbb{C}$ and

$$\underline{\gamma} = \underline{t} \otimes \underline{1}_{N_{\text{RX}}}. \tag{4}$$

The term $\exp(j\underline{\gamma}\omega_d)$ incorporates the phase change of the baseband signal due to the motion of the target between successive transmission pulses with the Doppler frequency ω_d, normalized to the units of $\underline{t}$. The factor $\frac{1}{\sqrt{N_{\text{Pulse}}}}$ adjusts the signal strength of one cycle, consisting of N_{Pulse} pulses, according to a constant transmitting energy. $\underline{N}(l)$ is additive noise. With the definition of a new steering vector

$$\underline{b}(u, \omega_d) := \exp(j\underline{\gamma}\omega_d) \odot \exp(j\underline{d}^{\text{Virt}} u), \tag{5}$$

the signal model can be written as

$$\underline{X}(l) = \underline{b}(u, \omega_d) \cdot \frac{1}{\sqrt{N_{\text{Pulse}}}} s(l) + \underline{N}(l). \tag{6}$$

Note that this signal model does not satisfy the sufficient conditions of space-time separability in [17] in contrast to the SIMO radar model in [8]. This is due to the coupling of the transmitting time t_i to the position d_i^{Pulse} of the transmitter. Hence we cannot conclude that the DOA and Doppler frequency estimations are decoupled in general.

We make the following assumptions:

1. $\underline{N}(l)$ is circular complex Gaussian with zero mean, spatially and temporally uncorrelated with $\text{E}\left(\underline{N}(l)\ \underline{N}^H(m)\right) = \delta_{l,m}\ \sigma^2 \mathbf{I}$.
2. The target's distance is much larger than the geometric dimension of the radar (far-field). Hence the radar receives a plane wave and the radar cross section as well as the DOA Θ of the target is the same for all antennas.
3. The DOA Θ does not change significantly during the complete measurement, i.e. the change is much smaller than the accuracy of the radar system and can be ignored.
4. The target moves with constant relative radial velocity. Hence the Doppler frequency ω_d is constant.

As an example, consider a radar in an automobile with a carrier frequency of 77 GHz, a geometrical array size of $A = 4\,\lambda = 15.6$ mm and $L = 1$ cycle with a cycle time of 5 ms. The target is a car at a radial distance $r = 20$ m on the neighboring driving lane with a lateral distance of 4 m, which corresponds to a DOA $\Theta = 11.8°$. It is moving with a relative velocity $v = 50$ km/h. During the cycle the phase change due to the Doppler effect is $34.9 \cdot 2\pi$, which is significantly. The geometrical size of the radar is much smaller than the radial distance, $A \ll r$, hence the far-field assumption is fulfilled. The DOA changes during the measurement about $4.2 \cdot 10^{-2}\,°$, which is much smaller than the typical accuracy of such a radar system. The radial velocity changes by $1.5 \cdot 10^{-2}\%$, which is negligible.

3. CRAMER-RAO BOUND

We compute the Cramer-Rao bound $\mathbf{J}^{-1}$ for the unknown parameter vector $\underline{\Theta} = [u, \omega_d]^T$. The CRB is a lower bound for the covariance matrix of any unbiased estimator $\widehat{\underline{\Theta}}$ [18]

$$\text{E}\left[(\widehat{\underline{\Theta}} - \underline{\Theta})(\widehat{\underline{\Theta}} - \underline{\Theta})^T\right] \geq \mathbf{J}^{-1} \tag{7}$$

where $\mathbf{J} = [J_{ij}]$ is the Fisher Information Matrix (FIM). It can be computed by

$$J_{ij} = \text{E}\left[\frac{\partial \ln f(\underline{x}; \underline{\Theta})}{\partial \Theta_i} \frac{\partial \ln f(\underline{x}; \underline{\Theta})}{\partial \Theta_j}\right] \tag{8}$$

where $f(\underline{x}; \underline{\Theta})$ is the likelihood of $\underline{\Theta}$ given the observation $\underline{x}$. With the result of [19] adapted to our notations, the FIM is given by

$$\mathbf{J} = \frac{2\,L}{\sigma^2} \frac{\sigma_s^2}{N_{\text{Pulse}}} \text{Re}\,(\mathbf{C}) \tag{9}$$

with

$$\mathbf{C} = \mathbf{D}^H \mathbf{P}_{\underline{b}}^{\perp} \mathbf{D}, \tag{10}$$

$$\mathbf{D} = \left[\frac{\partial \underline{b}(u, \omega_d)}{\partial u}, \quad \frac{\partial \underline{b}(u, \omega_d)}{\partial \omega_d}\right], \tag{11}$$

$$\mathbf{P}_{\underline{b}}^{\perp} = \mathbf{I} - \underline{b}(\underline{b}^H\ \underline{b})^{-1}\underline{b}^H, \tag{12}$$

$$\sigma_s^2 = \frac{1}{L}\sum_{l=1}^{L} |s(l)|^2. \tag{13}$$

After some computations, the FIM can be expressed as

$$\mathbf{J} = 2\,L\,N_{\text{RX}} \frac{\sigma_s^2}{\sigma^2} \cdot \begin{bmatrix} \text{Var}^{\text{S}}(\underline{d}^{\text{Virt}}) & \text{Cov}^{\text{S}}(\underline{d}^{\text{Virt}}, \underline{\gamma}) \\ \text{Cov}^{\text{S}}(\underline{d}^{\text{Virt}}, \underline{\gamma}) & \text{Var}^{\text{S}}(\underline{\gamma}) \end{bmatrix} \tag{14}$$

using the following abbreviations

$$\text{sample mean} \quad \underline{\bar{x}} := \mathrm{E}^{\mathrm{S}}(\underline{x}) := \frac{1}{K}\sum_{k=1}^{K} x_k, \tag{15}$$

$$\text{sample covariance} \quad \mathrm{Cov}^{\mathrm{S}}(\underline{x},\underline{y}) := \frac{1}{K}\sum_{k=1}^{K} (x_k - \underline{\bar{x}})\left(y_k - \underline{\bar{y}}\right)^*, \tag{16}$$

$$\text{sample variance} \quad \mathrm{Var}^{\mathrm{S}}(\underline{x}) := \mathrm{Cov}^{\mathrm{S}}(\underline{x},\underline{x}) \tag{17}$$

for vectors $x, y \in \mathbb{C}^K$. One can show that

$$\mathrm{Var}^{\mathrm{S}}(\underline{d}^{\mathrm{Virt}}) = \mathrm{Var}^{\mathrm{S}}(\underline{d}^{\mathrm{RX}}) + \mathrm{Var}^{\mathrm{S}}(\underline{d}^{\mathrm{Pulse}}), \tag{18}$$

$$\mathrm{Var}^{\mathrm{S}}(\underline{\gamma}) = \mathrm{Var}^{\mathrm{S}}(\underline{t}\otimes \underline{1}_{N_{\mathrm{RX}}}) = \mathrm{Var}^{\mathrm{S}}(\underline{t}), \tag{19}$$

$$\mathrm{Cov}^{\mathrm{S}}(\underline{d}^{\mathrm{Virt}},\underline{\gamma}) = \mathrm{Cov}^{\mathrm{S}}(\underline{d}^{\mathrm{Pulse}},\underline{t}). \tag{20}$$

Using this, (14) reads

$$\mathbf{J} = 2\,L\,N_{\mathrm{RX}}\frac{\sigma_s^2}{\sigma^2}\cdot\begin{bmatrix}\mathrm{Var}^{\mathrm{S}}(\underline{d}^{\mathrm{RX}}) + \mathrm{Var}^{\mathrm{S}}(\underline{d}^{\mathrm{Pulse}}) & \mathrm{Cov}^{\mathrm{S}}(\underline{d}^{\mathrm{Pulse}},\underline{t})\\ \mathrm{Cov}^{\mathrm{S}}(\underline{d}^{\mathrm{Pulse}},\underline{t}) & \mathrm{Var}^{\mathrm{S}}(\underline{t})\end{bmatrix}. \tag{21}$$

Inverting the FIM $\mathbf{J}$, assuming $\det(\mathbf{J}) \neq 0$, yields

$$\begin{aligned}\mathbf{J}^{-1} &= \frac{1}{2\,L\,N_{\mathrm{RX}}}\frac{\sigma^2}{\sigma_s^2}\\ &\cdot \frac{1}{\left(\mathrm{Var}^{\mathrm{S}}(\underline{d}^{\mathrm{RX}}) + \mathrm{Var}^{\mathrm{S}}(\underline{d}^{\mathrm{Pulse}})\right)\mathrm{Var}^{\mathrm{S}}(\underline{t}) - \left(\mathrm{Cov}^{\mathrm{S}}(\underline{d}^{\mathrm{Pulse}},\underline{t})\right)^2}\\ &\cdot \begin{bmatrix}\mathrm{Var}^{\mathrm{S}}(\underline{t}) & -\mathrm{Cov}^{\mathrm{S}}(\underline{d}^{\mathrm{Pulse}},\underline{t})\\ -\mathrm{Cov}^{\mathrm{S}}(\underline{d}^{\mathrm{Pulse}},\underline{t}) & \mathrm{Var}^{\mathrm{S}}(\underline{d}^{\mathrm{RX}}) + \mathrm{Var}^{\mathrm{S}}(\underline{d}^{\mathrm{Pulse}})\end{bmatrix}.\end{aligned} \tag{22}$$

The element CRB_u of $\mathbf{J}^{-1}$ corresponding to the electrical angle u equals

$$\mathrm{CRB}_u = \left[\mathbf{J}^{-1}\right]_{11} = \frac{1}{2\,L}\frac{1}{S}\frac{1}{U} \tag{23}$$

with

$$S := \frac{\sigma_s^2}{\sigma^2}\cdot N_{\mathrm{RX}}, \tag{24}$$

$$U := \mathrm{Var}^{\mathrm{S}}(\underline{d}^{\mathrm{RX}}) + \mathrm{Var}^{\mathrm{S}}(\underline{d}^{\mathrm{Pulse}}) - \frac{\left(\mathrm{Cov}^{\mathrm{S}}(\underline{d}^{\mathrm{Pulse}},\underline{t})\right)^2}{\mathrm{Var}^{\mathrm{S}}(\underline{t})}. \tag{25}$$

S denotes the overall SNR of the radar. The term U depends on the sample variance of the RX antenna positions in $\underline{d}^{\mathrm{RX}}$, the sample variance of the TX antenna positions in $\underline{d}^{\mathrm{Pulse}}$ as well as on the sample covariance between $\underline{d}^{\mathrm{Pulse}}$ and the transmitting time instances $\underline{t}$.

4. DISCUSSION

4.1. Comparison

We compare the CRB of the electrical angle CRB_u in (23) to the CRB using a SIMO radar and to that of a stationary target $\mathrm{CRB}_{\mathrm{stat}}$ using a MIMO radar.

First we compute the CRB of the electrical angle for the SIMO radar $\mathrm{CRB}_{\mathrm{SIMO}}$. We use the model (6) and consider only one transmitting antenna at position d^{TX} sending N_{Pulse} pulses and set $\underline{d}^{\mathrm{Pulse}} = [d^{\mathrm{TX}}, d^{\mathrm{TX}}, \ldots]^T$. Using (23) results in

$$\mathrm{CRB}_{\mathrm{SIMO}} = \frac{1}{2\,L}\frac{1}{S}\frac{1}{U_{\mathrm{SIMO}}}, \tag{26}$$

$$U_{\mathrm{SIMO}} = \mathrm{Var}^{\mathrm{S}}(\underline{d}^{\mathrm{RX}}). \tag{27}$$

This coincides with the CRB for the SIMO radar for a stationary target computed in [6]. Hence the movement of the target has no influence on the CRB in this case. Note that we have not taken the beamforming gain of a SIMO radar into account, i.e. sending the same signal phase shifted with several transmitting antennas to focus to a certain region of DOA, since it requires a priori knowledge of the target's DOA. The CRB for such a scenario is derived in [1] and [20].

For a MIMO radar, the CRB of the electrical angle of a stationary target $\mathrm{CRB}_{\mathrm{stat}}$ can be derived by using the model (6), setting $\omega_d = 0$ and computing the CRB of the unknown parameter u only. This coincides with the model in [6] by replacing the RX antenna positions by the virtual array. $\mathrm{CRB}_{\mathrm{stat}}$ is given by [6]

$$\mathrm{CRB}_{\mathrm{stat}} = \frac{1}{2\,L}\frac{1}{S}\frac{1}{U_{\mathrm{stat}}}, \tag{28}$$

$$U_{\mathrm{stat}} = \mathrm{Var}^{\mathrm{S}}(\underline{d}^{\mathrm{Virt}}) = \mathrm{Var}^{\mathrm{S}}(\underline{d}^{\mathrm{RX}}) + \mathrm{Var}^{\mathrm{S}}(\underline{d}^{\mathrm{Pulse}}). \tag{29}$$

Hence $\mathrm{CRB}_{\mathrm{stat}}$ depends on the positions of the TX antennas in $\underline{d}^{\mathrm{Pulse}}$, but not on the sequence in which they transmit.

The CRBs $\mathrm{CRB}_u, \mathrm{CRB}_{\mathrm{SIMO}}$ and $\mathrm{CRB}_{\mathrm{stat}}$ differ in U, U_{SIMO} and U_{stat}. Using (25) we write

$$U = \mathrm{Var}^{\mathrm{S}}(\underline{d}^{\mathrm{RX}}) + \mathrm{Var}^{\mathrm{S}}(\underline{d}^{\mathrm{Pulse}}) - p \tag{30}$$

with the penalty term

$$p := \frac{\left(\mathrm{Cov}^{\mathrm{S}}(\underline{d}^{\mathrm{Pulse}},\underline{t})\right)^2}{\mathrm{Var}^{\mathrm{S}}(\underline{t})}. \tag{31}$$

By the Cauchy-Schwarz inequality,

$$0 \leq p \leq \mathrm{Var}^{\mathrm{S}}(\underline{d}^{\mathrm{Pulse}}) \tag{32}$$

can be shown for any $\underline{d}^{\mathrm{Pulse}}$ and $\underline{t}$. Therefore

$$\mathrm{CRB}_{\mathrm{SIMO}} \geq \mathrm{CRB}_u \geq \mathrm{CRB}_{\mathrm{stat}}. \tag{33}$$

If $p = \mathrm{Var}^{\mathrm{S}}(\underline{d}^{\mathrm{Pulse}})$, $\mathrm{CRB}_u = \mathrm{CRB}_{\mathrm{SIMO}}$. The MIMO radar has no benefit in the accuracy of DOA estimation compared to the SIMO radar. If $p = 0$, $\mathrm{CRB}_u = \mathrm{CRB}_{\mathrm{stat}}$, i.e. the movement of the target has no impact on the CRB and the whole virtual aperture can be used for the DOA estimation. Hence it is crucial for DOA estimation to choose an optimal TDM scheme to maximize U.

Note that the TDM scheme depends both on the position and sequence $\underline{d}^{\mathrm{Pulse}}$ of the selected TX antennas and the time instants $\underline{t}$ for transmission. It is characterized by the parameter vector $\underline{\vartheta}^{\mathrm{TDM}} := [(\underline{d}^{\mathrm{Pulse}})^T, \underline{t}^T]^T$. Therefore, we have to maximize U over $\underline{\vartheta}^{\mathrm{TDM}}$. Depending on the radar application, we can maximize U by varying $\underline{d}^{\mathrm{Pulse}}$ for a fixed $\underline{t}$ or by varying $\underline{t}$ for a fixed $\underline{d}^{\mathrm{Pulse}}$ or by varying both $\underline{d}^{\mathrm{Pulse}}$ and $\underline{t}$.

Below we present two TDM schemes which reach the two bounds for CRB_u in (33).

4.2. Bad Time Division Multiplexing

The worst TDM schemes are characterized by $\underline{d}^{\mathrm{Pulse}} = c_1\underline{t} + c_2$ with constants c_1, c_2. In this case, $p = \mathrm{Var}^{\mathrm{S}}(\underline{d}^{\mathrm{Pulse}})$ and $\mathrm{CRB}_u = \mathrm{CRB}_{\mathrm{SIMO}}$. This is, for example, the case if one uses a uniform linear array (ULA) of TX antennas which transmit in the order of their geometric arrangement at equally spaced time instants.

4.3. Optimal Time Division Multiplexing

A TDM scheme with the parameters $\underline{d}^{\text{Pulse}}$ and $\underline{t}$ is optimal if CRB_u in (23) is minimized or equivalently the expression U in (25) is maximized. This can be achieved by maximizing $\text{Var}^{\text{S}}(\underline{d}^{\text{Pulse}})$ and minimizing $|\text{Cov}^{\text{S}}(\underline{d}^{\text{Pulse}}, \underline{t})|$ (hopefully to zero) at the same time. Below we show one such example under the following conditions: The positions of RX antennas $\underline{d}^{\text{RX}}$ and TX antennas $\underline{d}^{\text{TX}}$ are given and fixed. Furthermore, the TX antennas transmit at uniformly spaced time instances, i.e. $\underline{t} \propto [0, 1, \ldots, N_{\text{Pulse}} - 1]^T$. Then the optimal value of $\underline{d}^{\text{Pulse}}$ is given by

$$\underline{d}^{\text{Pulse,opt}} = \arg\max_{\underline{d}^{\text{Pulse}}} \text{Var}^{\text{S}}(\underline{d}^{\text{Pulse}}) - \frac{\left(\text{Cov}^{\text{S}}(\underline{d}^{\text{Pulse}}, \underline{t})\right)^2}{\text{Var}^{\text{S}}(\underline{t})}. \quad (34)$$

$\text{Var}^{\text{S}}(\underline{d}^{\text{Pulse}})$ is maximized if only the most left and most right TX antennas with the position $d_{\text{min}}^{\text{TX}}$ and $d_{\text{max}}^{\text{TX}}$ are used and both of them occur equally often in $\underline{d}^{\text{Pulse,opt}}$. $|\text{Cov}^{\text{S}}(\underline{d}^{\text{Pulse}}, \underline{t})|$ is minimized to zero for a cycle of $N_{\text{Pulse}} = 4$ pulses if e.g.

$$\underline{d}^{\text{Pulse,opt}} = \left[d_{\text{min}}^{\text{TX}}, d_{\text{max}}^{\text{TX}}, d_{\text{max}}^{\text{TX}}, d_{\text{min}}^{\text{TX}}\right]^T. \quad (35)$$

Another possible solution is to repeat this pulse sequence several times. In this case, CRB_u achieves the same value as CRB_{stat} for a stationary target, i.e. the movement of the target has no negative effect on the DOA estimation in the sense of the CRB.

5. SIMULATIONS

In the following we present some simulations to verify the theoretical results. We determine the root mean squared error (RMSE) of the maximum likelihood (ML) estimator for the electrical angle u in different scenarios.

We consider a MIMO radar with 4 RX and 4 TX antennas, uniformly spaced with an antenna distance of $\lambda/2$, i.e. $\underline{d}^{\text{RX}} = \underline{d}^{\text{TX}} = \pi \cdot [0, 1, 2, 3]^T$. We choose 1 cycle with $N_{\text{Pulse}} = 4$ pulses and set the transmitting time instants to $\underline{t} = [0, 1, \ldots, N_{\text{Pulse}} - 1]^T$. The target's electrical angle u is set to $u = \sin(10°)$ and, for the non-stationary case, the Doppler frequency ω_d is chosen to be $\omega_d = 1.3$. The CRB CRB_{SIMO} of a SIMO radar with the same RX antennas, using only one TX antenna with $N_{\text{Pulse}} = 4$, is computed as an upper bound for CRB_u. The CRB CRB_{stat} for a stationary target with the same $\underline{d}^{\text{Pulse}}$ as the optimal TDM scheme in (35) is computed as a lower bound for CRB_u. We consider the bad TDM scheme with $\underline{d}^{\text{Pulse}} = \underline{d}^{\text{TX}}$, the optimal TDM $\underline{d}^{\text{Pulse}} = \underline{d}^{\text{Pulse,opt}}$ of (35) and the case of a stationary target using the same TX antennas, i.e. $\underline{d}^{\text{Pulse}} = \underline{d}^{\text{Pulse,opt}}$, c.f. Fig. 2. The simulations are done for overall SNR values S from 0 to 35 dB. 3000 Monte Carlo simulations are carried out for each value of S in order to determine the RMSE of the ML estimator. The ML estimates are computed by doing a 1-dimensional search in the stationary case and a 2-dimensional search in the non-stationary case on a grid, followed by a quadratic interpolation.

Fig. 3 shows that the ML estimator with the bad TDM scheme achieves, for a moving target, only the upper bound CRB_{SIMO} and the ML estimator with the optimal TDM scheme reaches indeed the lower bound CRB_{stat}, as does the ML estimator in the stationary case. The threshold value of S at which the ML estimator reaches the CRB is approximately the same for the bad and optimal TDM scheme. Hence, with the same hardware platform, we can achieve a gain of 4.47 dB for the same DOA accuracy by just changing the transmission sequence $\underline{d}^{\text{Pulse}}$ of the TX antennas.

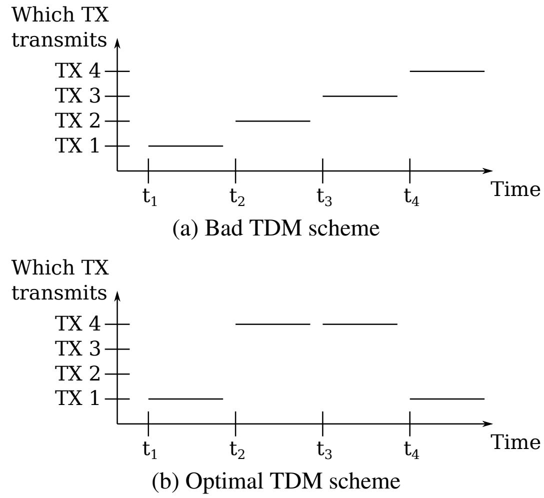

Fig. 2. Considered TDM schemes: (a) bad TDM scheme with $\underline{d}^{\text{Pulse}} = \underline{d}^{\text{TX}}$ and (b) optimal TDM scheme with $\underline{d}^{\text{Pulse}} = [d_1^{\text{TX}}, d_4^{\text{TX}}, d_4^{\text{TX}}, d_1^{\text{TX}}]^T$

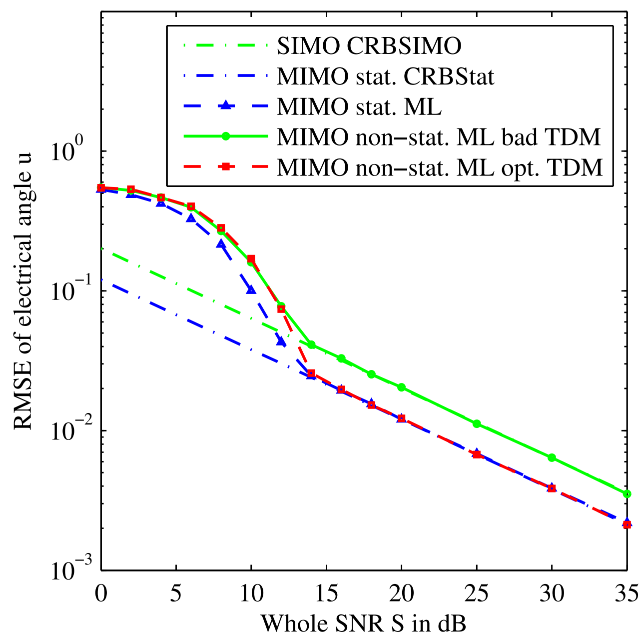

Fig. 3. Comparison of MIMO-radar DOA estimation with stationary target and non-stationary target with bad and optimal TDM scheme

6. CONCLUSIONS

We have investigated a time division multiplex MIMO radar and analyzed the DOA estimation of a moving target. We derived the CRB of the DOA and compared the CRB of MIMO radars using different TDM schemes and the CRB of a SIMO radar. Using this result, we deduced optimal TDM schemes such that the CRB is as small as for a stationary target. Simulations confirmed the theoretical results and showed that the RMSE of the maximum likelihood estimator is indeed as small as for a stationary target, if an optimal TDM scheme is used.

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